Dichroic N-subjettiness ratio

Computing and constraining jet substructure from first principles

Grégory Soyez

IPhT, CEA Saclay, CNRS

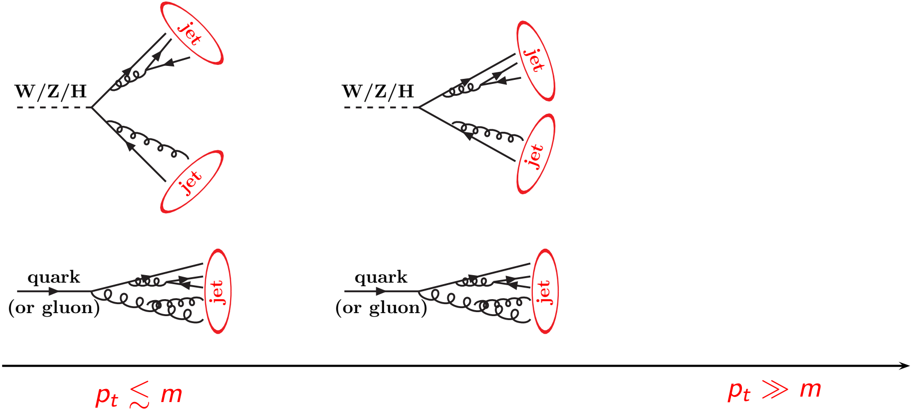
(in collaboration with Gavin Salam and Lais Schunk)
(+ earlier work with Mrinal Dasgupta and Lais Schunk)

PSR 2017 - March 27-29 2017

Tagging boosted objects

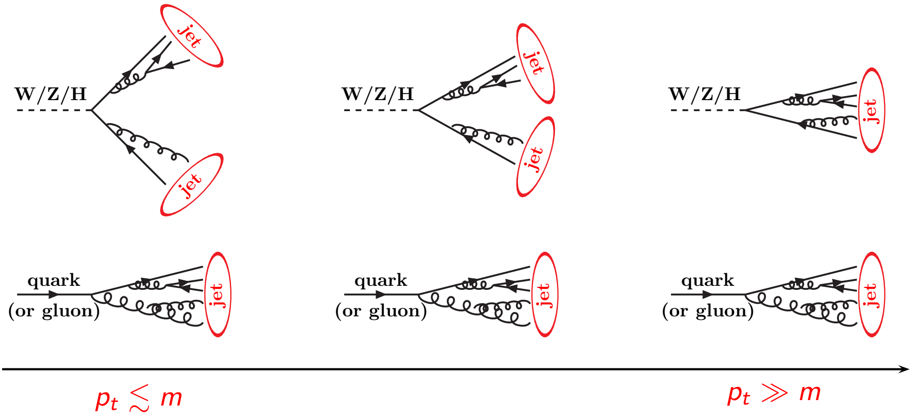
Standard lore: $W/Z/H$
reconstructed as 2 jets

Tagging boosted objects



Standard lore: $W/Z/H$
reconstructed as 2 jets

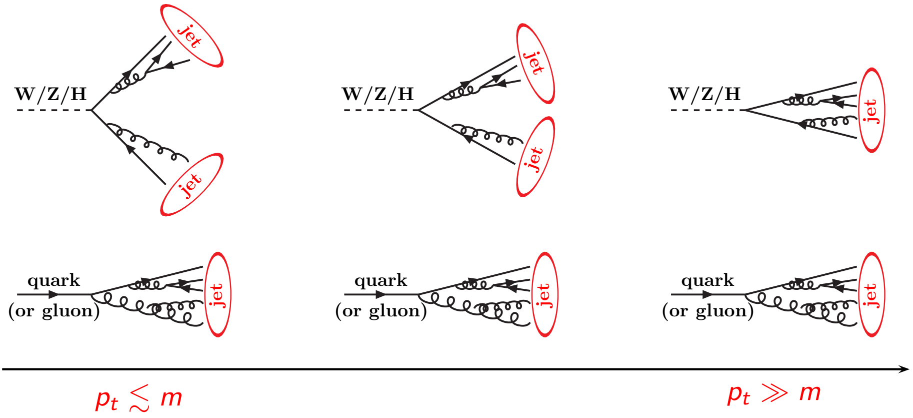
Tagging boosted objects



Standard lore: $W/Z/H$ reconstructed as 2 jets

Boosted case: $W/Z/H$ seen as 1 jet (as q/g)

Tagging boosted objects



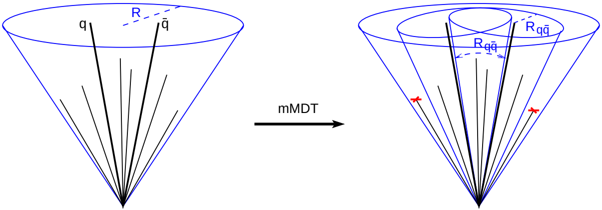
Standard lore: $W/Z/H$ reconstructed as 2 jets

Boosted case: $W/Z/H$ seen as 1 jet (as q/g)

Use jet substructure to separate $W/H/Z$ from q/g

Tag 2 hard prongs: modified Mass-Drop Tagger (mMDT)

Idea #1: find 2 hard cores/prongs in the jet



(modified) Mass-Drop tagger (mMDT)

- Cluster the jet with Cambridge/Aachen (from small to large angles)
- Iteratively undo the last recombination $j \to j_1 + j_2$
  - if $z = \min(p_{t,1}, p_{t,2})/p_t \geq z_{\text{cut}}$, we have found 2 prongs (stop)
  - otherwise, continue to iterate with the hardest of j_1 and j_2

[J.Butterworth,A.Davison,M.Rubin,G.Salam; 08]
[M.Dasgupta,A.Fregoso,S.Marzani,G.Salam; 13]

Idea #2: $W/Z/H$ and q/g have different radiation patterns

- typically: smaller radiation in $X \to q\bar{q}$ than q/g
- several measures of radiation in a jet (usually jet shapes)

N-subjettiness

$$\tau_{21} = \frac{\tau_2^{(\beta)}(\text{jet; axes})}{\tau_1^{(\beta)}(\text{jet; axes})} = \frac{\sum_{i\in\text{constits}} z_i \min(\theta^\beta_{i,a_{2,1}}, \theta^\beta_{i,a_{2,2}})}{\sum_{i\in\text{constits}} z_i\, \theta^\beta_{i,a_{1,1}}}$$

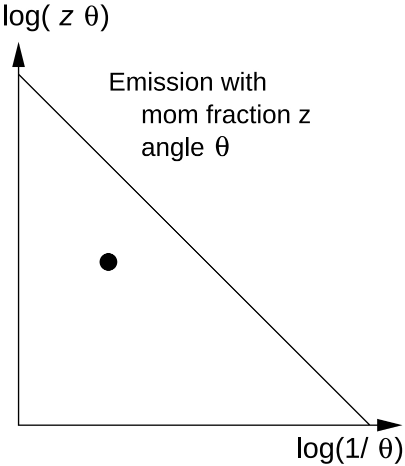
[J.Thaler,K.Van Tilburg; 11]

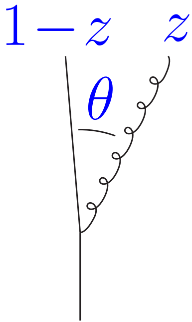
Note: Here we focus on

- $\beta = 2$: simpler because "mass-like" (brief comparison to $\beta = 1$ later)
- generalised-$k_t(p = 1/2 = 1/\beta)$ axes (close to optimal)

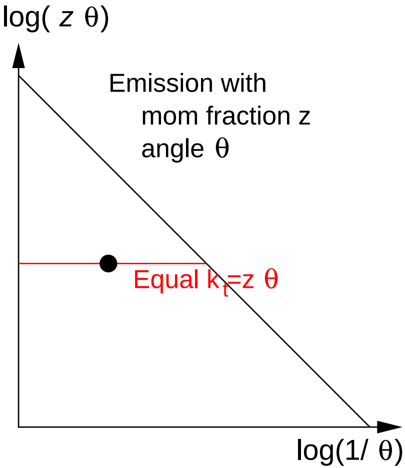
Analytic structure of taggers in cartons

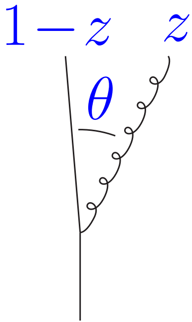
Anatomy of the phase-space (at Leading Log)



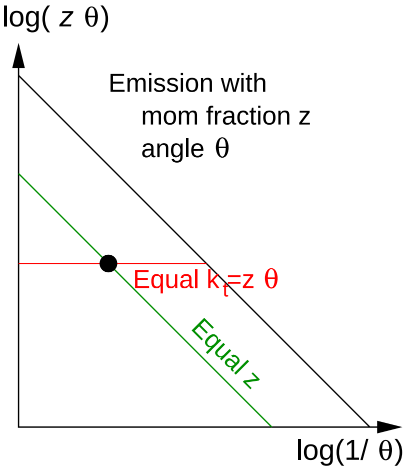


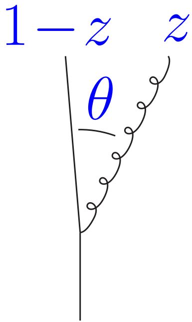
Anatomy of the phase-space (at Leading Log)



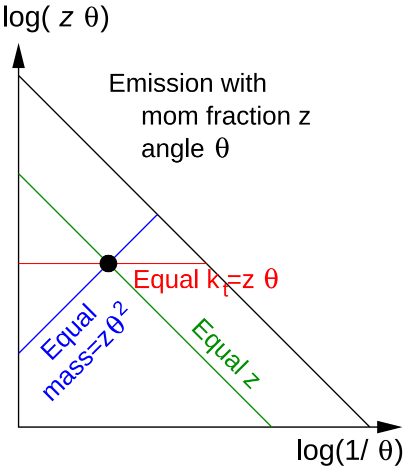


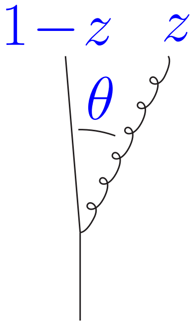
Anatomy of the phase-space (at Leading Log)

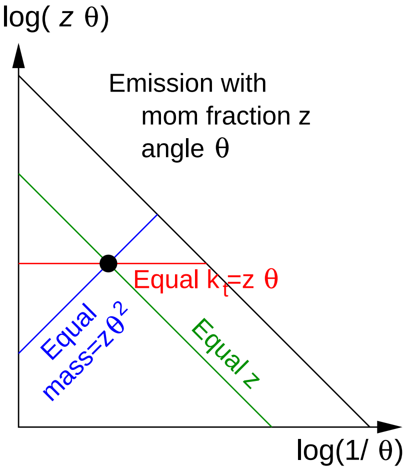




Anatomy of the phase-space (at Leading Log)







Jet "mass": $(z_1\theta_1 \gg z_2\theta_2 \gg \ldots)$

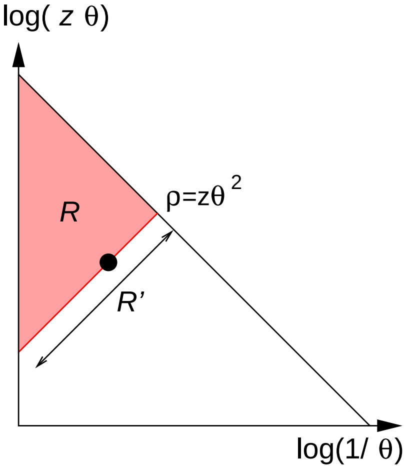
$$\rho \equiv \frac{m^2}{p_t^2 R^2} = \sum_{i \in \text{jet}} z_i \theta_i^2 \approx z_1 \theta_1^2$$

N-subjettiness:

$$\tau_1 = \rho$$

$$\tau_2 = \sum_{i=2}^{n} z_i \theta_i^2 \approx z_2 \theta_2^2$$

Anatomy of the phase-space (at Leading Log)



(full) jet mass spectrum

$$\frac{\rho}{\sigma}\frac{d\sigma}{d\rho} = R'_{\text{full}} \exp(-R_{\text{full}})$$

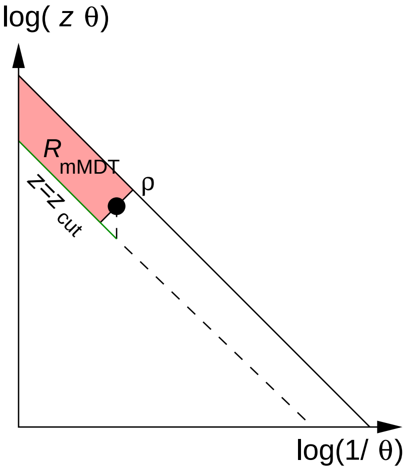
1. veto on larger-mass (Sudakov)

$$R_{\text{full}} \sim \frac{\alpha_s C_R}{2\pi} \log^2(1/\rho)$$

2. emission of given mass

$$R'_{\text{full}} \sim \frac{\alpha_s C_R}{\pi} \log(1/\rho)$$

Anatomy of the phase-space (at Leading Log)



(mMDT) jet mass spectrum

$$\frac{\rho}{\sigma}\frac{d\sigma}{d\rho} = R'_{\text{mMDT}} \exp(-R_{\text{mMDT}})$$

1. veto on larger-mass (Sudakov)

$$R_{\text{mMDT}} \sim \frac{\alpha_s C_R}{\pi} \log(1/\rho) \log(1/z_{\text{cut}})$$

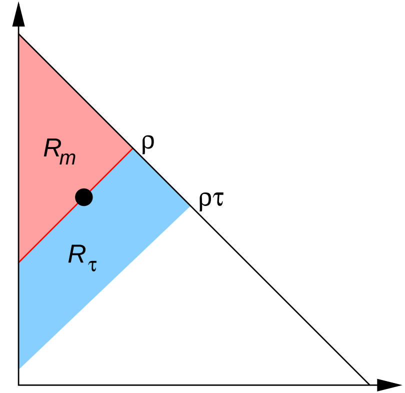
2. emission of given mass

$$R'_{\text{mMDT}} \sim \frac{\alpha_s C_R}{\pi} \log(1/z_{\text{cut}})$$

Smaller $R \longrightarrow$ less bkg suppression
Smaller $R' \longrightarrow$ more bkg suppression

[M.Dasgupta,A.Fregoso,S.Marzani,G.Salam]

Anatomy of the phase-space (at Leading Log)



jet mass with a cut $\tau_{21} < \tau$:

$$\frac{\rho}{\sigma}\frac{d\sigma}{d\rho}\bigg|_{<\tau} = R'_{\text{full}} \exp(-R_{\text{full}} - R_\tau)$$

Extra suppression

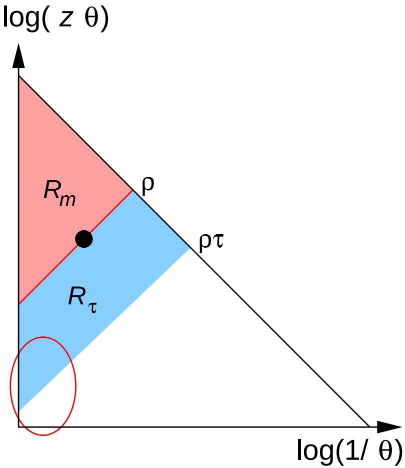
$$R_{\text{full}} \sim \frac{\alpha_s C_R}{2\pi} \log^2(1/\rho)$$

becomes

$$R_{\text{full}} + R_\tau \sim \frac{\alpha_s C_R}{2\pi} \log^2(1/\rho\tau)$$

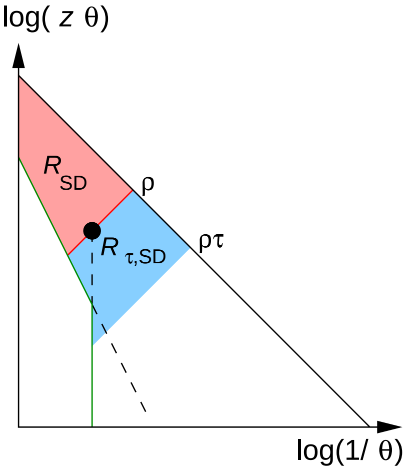
[M.Dasgupta,L.Schunk,,GS]

Anatomy of the phase-space (at Leading Log)



WATH OUT:

sensitivity to soft-large-angle
e.g. UE and pileup (+hadr.)
$\Rightarrow$ poor control



SOLUTION:
"groom" (remove) that region

Can be done by "SoftDrop"

- smaller suppression
- better control

[see Frederic's talk tomorrow]

Main question for this talk:

How can we combine prong-tagging and radiation constraint?

[G.Salam,L.Schunk,GS, arXiv:1612.03917]

Various possible combinations

Take a jet after tagging (mMDT) and impose a cut $\tau_{21} < \tau$

3 options for the computation of τ_{21}

Tagged

$$\tau_{21}^{\text{tagged}} = \frac{\tau_2^{\text{mMDT}}}{\tau_1^{\text{mMDT}}}$$

Full

$$\tau_{21}^{\text{full}} = \frac{\tau_2^{\text{full}}}{\tau_1^{\text{full}}}$$

dichroic

$$\tau_{21}^{\textbf{dichroic}} = \frac{\tau_2^{\textbf{full}}}{\tau_1^{\textbf{mMDT}}}$$

Various possible combinations

Take a jet after tagging (mMDT) and impose a cut $\tau_{21} < \tau$

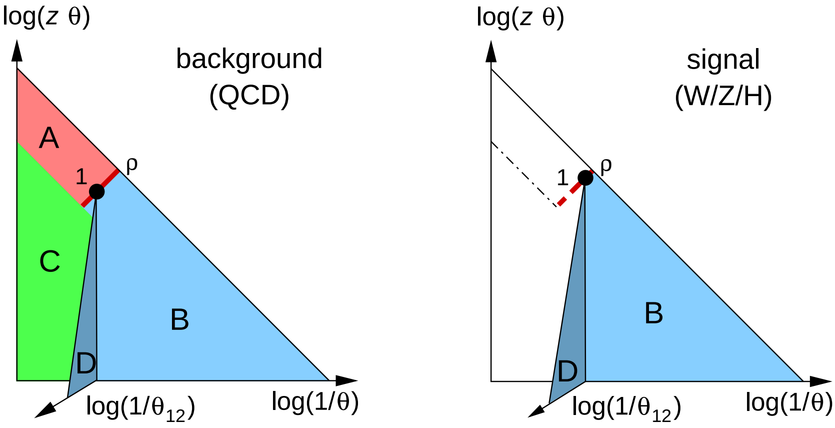
3 options for the computation of τ_{21}

Tagged	Full	dichroic
$\tau_{21}^{\text{tagged}} = \frac{\tau_2^{\text{mMDT}}}{\tau_1^{\text{mMDT}}}$	$\tau_{21}^{\text{full}} = \frac{\tau_2^{\text{full}}}{\tau_1^{\text{full}}}$	$\tau_{21}^{\textbf{dichroic}} = \frac{\tau_2^{\textbf{full}}}{\tau_1^{\textbf{mMDT}}}$

currently used

NEW

4 phase-space regions



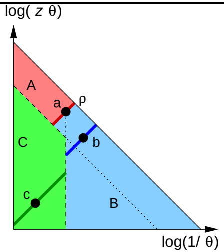
A: in mMDT but at larger mass (vetoed for QCD jets)

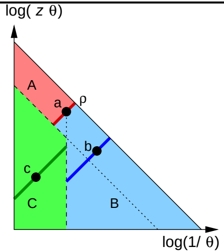
B: inside mMDT, small angles (hard prong)

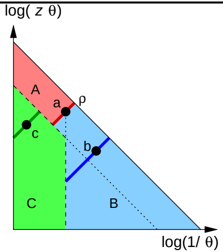
C: outside mMDT, large angles (absent in signal)

D: soft prong: Mostly q/g discrimination; neglected hereafter

	$z_c\theta_c^2 \ll z_b\theta_b^2 \ll z_a\theta_a^2$		$z_b\theta_b^2 \ll z_c\theta_c^2 \ll z_a\theta_a^2$		$z_b\theta_b^2 \ll z_a\theta_a^2 \ll z_c\theta_c^2$	





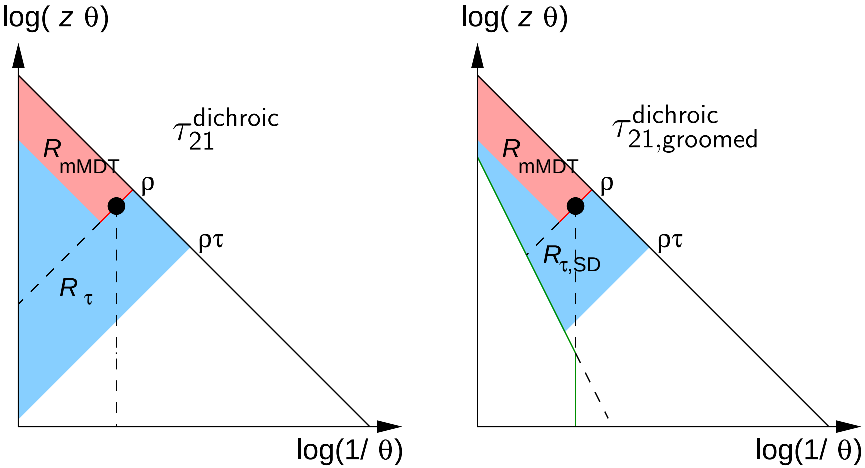


	bkg	sig	bkg	sig	bkg	sig
tagged	**b/a**	b/a	b/a	b/a	b/a	b/a
full	**b/a**	b/a	**c/a**	b/a	a/c	b/a
dichroic	**b/a**	b/a	**c/a**	b/a	**a/a**	b/a

Same for the signal

Background: $\tau_{21}^{\text{dichroic}}$ is the largest
$\Rightarrow$ better discrimination

How does it work?

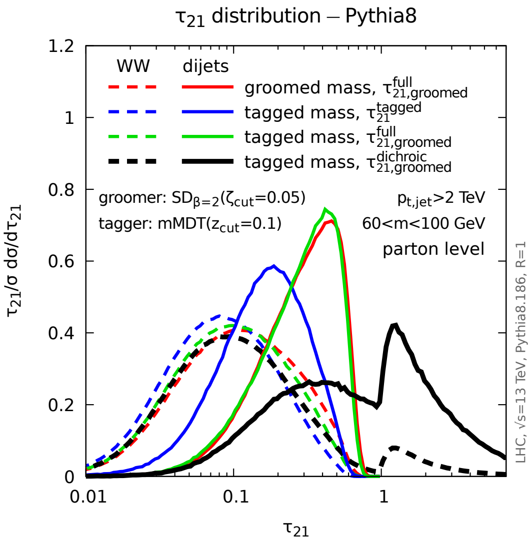


$$\frac{\rho}{\sigma}\frac{d\sigma}{d\rho}\bigg|_{<\tau} = R'_{\text{mMDT}}\,\exp(-R_{\text{mMDT}} - R_\tau)$$

- small pre-factor
- large Sudakov

Monte-Carlo validation

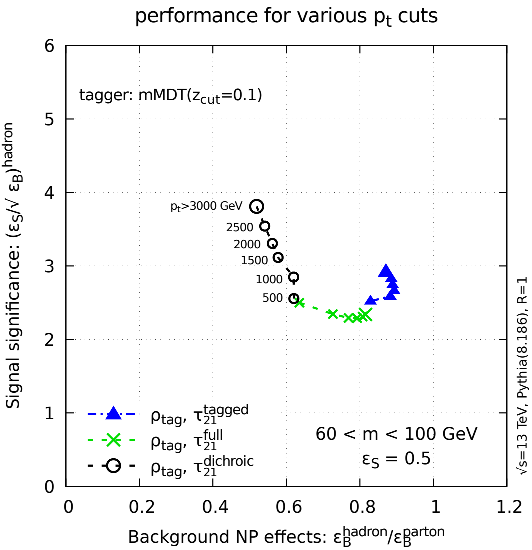
Signal v. Background distributions



- Similar signal distributions
- background larger for dichroic

$\Rightarrow$ dichroic gives the best discrimination

Significance v. NP sensitivity



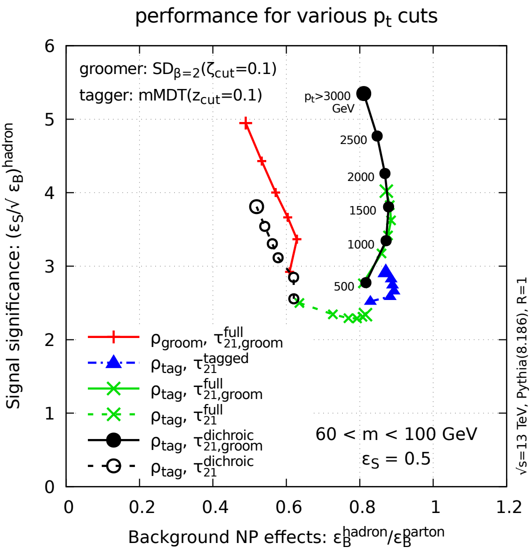
Setup

- fix τ to get ε_S^{hadron}=0.5
- $S/\sqrt{B}$ v. NP effects

dichroic (black)

- improved significance
- non-negligible NP effects

Significance v. NP sensitivity



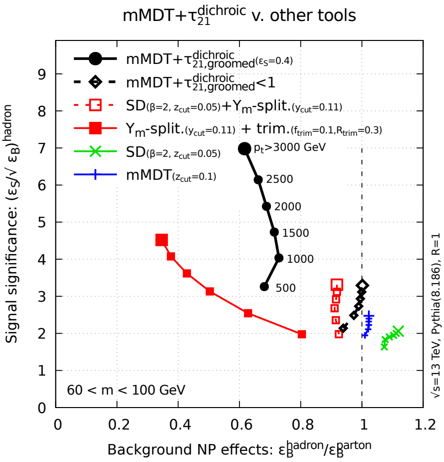
Setup

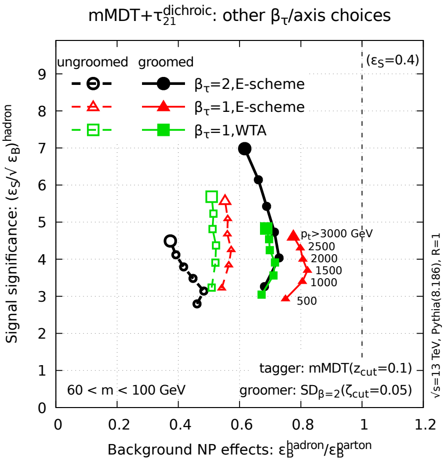
- fix τ to get $\varepsilon_S^{hadron}=0.5$
- $S/\sqrt{B}$ v. NP effects

"groomed" version

- improved significance
- small NP effects

comparison to other methods





Outperforms other tools
(see Mrinal's talk tomorrow)

Preference for $\beta = 2$
(experiments use $\beta = 1$)

Towards a better analytic control

Calculation: idea

Target accuracy:

- $\tau \ll 1$: Include all double logs: $\alpha_s^n \, log^{2n}(1/\text{any of } \rho, \tau)$
- τ finite: Include leading logs of ρ: $\alpha_s^n \log^n(1/\rho) f(\tau)$

Calculation: idea

Target accuracy:

- $\tau \ll 1$: Include all double logs: $\alpha_s^n \, log^{2n}(1/\text{any of } \rho, \tau)$
- τ finite: Include leading logs of ρ: $\alpha_s^n \log^n(1/\rho) f(\tau)$

Details:

- Start with τ_{21} for the full jet
- Essentially works for τ_{21}, $\tau_{21}^{\text{dichroic}}$, $\tau_{21,\text{groomed}}^{\text{dichroic}}$, $\tau_{21}^{(\beta=1)}$, D_2, M_2, τ_{32}
- Ungroomed or groomed variants (almost) straightforward

Calculation: idea

Target accuracy:

- $\tau \ll 1$: Include all double logs: $\alpha_s^n\, log^{2n}(1/\text{any of } \rho, \tau)$
- τ finite: Include leading logs of ρ: $\alpha_s^n \log^n(1/\rho) f(\tau)$

Details:

- Start with τ_{21} for the full jet
- Essentially works for τ_{21}, $\tau_{21}^{\text{dichroic}}$, $\tau_{21,\text{groomed}}^{\text{dichroic}}$, $\tau_{21}^{(\beta=1)}$, D_2, M_2, τ_{32}
- Ungroomed or groomed variants (almost) straightforward

Consider n emissions $(z_1, \theta_1), \dots, (z_n, \theta_n)$
At this accuracy, we have (thanks to $\beta = 2$ and axes choice)

- $\rho = \tau_1 = \sum_{i=1}^{n} z_i \theta_i^2$
- $\tau_2 = \tau_1 - \max\{z_i \theta_i^2\}$ (min or gen-k_t axes)

Calculation: method

Start from multiple-emissions assuming "emission 1 most massive"

$$\frac{\rho\tau}{\sigma}\frac{d^2\sigma}{d\rho\, d\tau} = \lim_{\epsilon\to 0} \exp\left[-\int_{z\theta^2>\epsilon}^{1} \frac{d\theta^2}{\theta^2} P(z)dz \frac{\alpha_s(z\theta)}{2\pi}\right] \sum_{n=2}^{\infty} \frac{1}{(n-1)!}$$

$$\prod_{i=1}^{n} \int_{z_i\theta_i^2>\epsilon}^{1} \frac{d\theta_i^2}{\theta_i^2} P(z_i)dz_i \frac{\alpha_s(z_i\theta_i)}{2\pi} \prod_{i=2}^{n} \Theta(z_i\theta_i^2 < z_1\theta_1^2)$$

$$\rho\delta(\rho - \sum_{i=1}^{n} z_i\theta_i^2)\, \tau\delta(\tau - \sum_{i=2}^{n} z_i\theta_i^2 / \sum_{i=1}^{n} z_i\theta_1^2)$$

- Virtual corrections
- Real emissions phase-space (with "1" most massive)
- Constraints on mass and τ

Calculation: method

Only depends on $\rho_i = z_i\theta_i^2$ (thanks to $\beta = 2$)

$$\frac{\rho\tau}{\sigma}\frac{d^2\sigma}{d\rho\, d\tau} = \lim_{\epsilon\to 0} \exp\left[-\int_\epsilon^1 \frac{d\tilde\rho}{\tilde\rho} R'(\tilde\rho)\right] \sum_{n=2}^{\infty} \frac{1}{(n-1)!}$$

$$\prod_{i=1}^{n}\int_\epsilon^1 \frac{d\rho_i}{\rho_i} R'(\rho_i) \prod_{i=2}^{n} \Theta(\rho_i < \rho_1)$$

$$\rho\delta(\rho - \sum_{i=1}^{n}\rho_i)\, \tau\delta(\tau - \sum_{i=2}^{n}\rho_i / \sum_{i=1}^{n}\rho_1)$$

- $R'(\rho) = \int \frac{d\theta^2}{\theta^2} P(z) dz \frac{\alpha_s(z\theta)}{2\pi}\, \rho\delta(\rho - z\theta^2) \sim \alpha_s \log(1/\rho)$
- Easily written for SoftDrop (use $R = R_{SD}$)

Calculation: method

Only depends on $\rho_i = z_i\theta_i^2$ (thanks to $\beta = 2$)

$$\frac{\rho\tau}{\sigma}\frac{d^2\sigma}{d\rho\, d\tau} = \lim_{\epsilon\to 0} \exp\left[-\int_\epsilon^1 \frac{d\tilde\rho}{\tilde\rho} R'(\tilde\rho)\right] \sum_{n=2}^{\infty} \frac{1}{(n-1)!}$$

$$\prod_{i=1}^{n}\int_\epsilon^1 \frac{d\rho_i}{\rho_i} R'(\rho_i) \prod_{i=2}^{n} \underline{\Theta(\rho_i < (1-\tau)\rho)}$$

$$\rho\delta((1-\tau)\rho - \rho_1)\, \underline{\rho\tau\delta(\rho\tau - \sum_{i=2}^{n}\rho_i)}$$

- Competition between 2 constraints
- Different behaviour for $\tau < 1/2$ and $\tau > 1/2$

Calculation: method

After "CEASAR-like" manipulations:

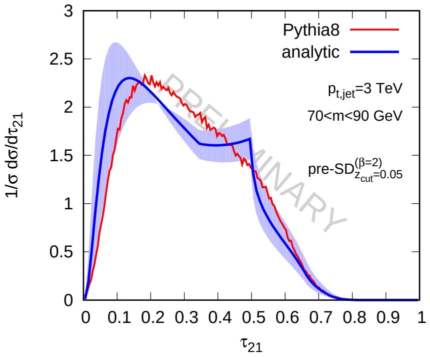
$$\frac{\rho\tau}{\sigma}\frac{d^2\sigma}{d\rho\, d\tau} \overset{\tau<1/2}{=} \frac{e^{-R(\rho\tau)-\gamma_E R'(\rho\tau)}}{\Gamma(R'(\rho\tau))}\frac{R'((1-\tau)\rho)}{1-\tau}$$

$$\overset{\tau>1/2}{=} \frac{e^{-R(\rho(1-\tau))-\gamma_E R'(\rho(1-\tau))}}{\Gamma(R'(\rho(1-\tau)))}\frac{R'((1-\tau)\rho)}{1-\tau} \times \frac{\tau}{1-\tau} f_{\text{ME}}\left(\frac{\tau}{1-\tau}, R'(\rho(1-\tau))\right)$$

Note: $\tau = 1/2$ is the limit at which we need more than 2 emissions and

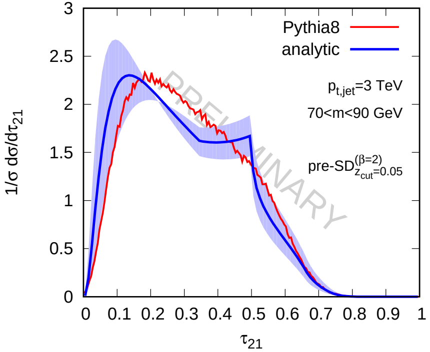
$$\frac{e^{-\gamma_E R'}}{\Gamma(R')} f_{\text{ME}}(x, R') = \lim_{\varepsilon\to 0}\sum_{n=1}^{\infty}\frac{R'^n}{n!}\prod_{i=1}^{n}\int_{\varepsilon}^{1}\frac{dx_i}{x_i}\, e^{-R'\log(1/\varepsilon)}\delta(x-\sum_{i=1}^{n}x_i),$$

Comparison with Monte-Carlo



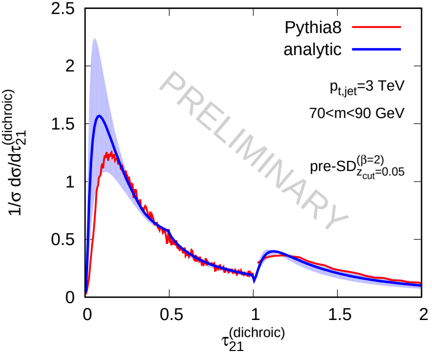
- Uncertainty band: Vary between ρ, $\rho/2$ and 2ρ
- $1 + \mathcal{O}(\alpha_s)$ normalisation
- Good overall description

Comparison with Monte-Carlo



- Uncertainty band:
  Vary between ρ, $\rho/2$ and 2ρ
- $1 + \mathcal{O}(\alpha_s)$ normalisation
- Good overall description
- Kinks
  - 1/2: $\tau > \frac{1}{2}$ requires ≥ 3 emissions
    Smeared by subleading effects
  - 0.34: Start of 2ndary emissions
    Exact position subleading

Comparison with Monte-Carlo



- Uncertainty band:
  Vary between ρ, $\rho/2$ and 2ρ
- $1 + \mathcal{O}(\alpha_s)$ normalisation
- Good overall description
- Kinks
  - 1/2: $\tau > \frac{1}{2}$ requires ≥ 3 emissions
    Smeared by subleading effects
  - 0.34: Start of 2ndary emissions
    Exact position subleading
- Works also with $\tau_{21}^{\text{dichroic}}$

Conclusions

Designing new substructure tools

- Jet substructure starts to be understood from first principles
- New step: from understanding to building new performant tools
- $\tau_{21}^{\text{dichroic}}$ as an example of such new tool

Analytic/resummation side

- same strategy working for a family of observables
- Repetitive appearance of the same objects
- TODO: match to exact α_s^2 (or to $1 \to 3$ collinear branching)
- Assessment of theory uncertainties