Saturation in High-Energy QCD
Scaling laws and phenomenological applications

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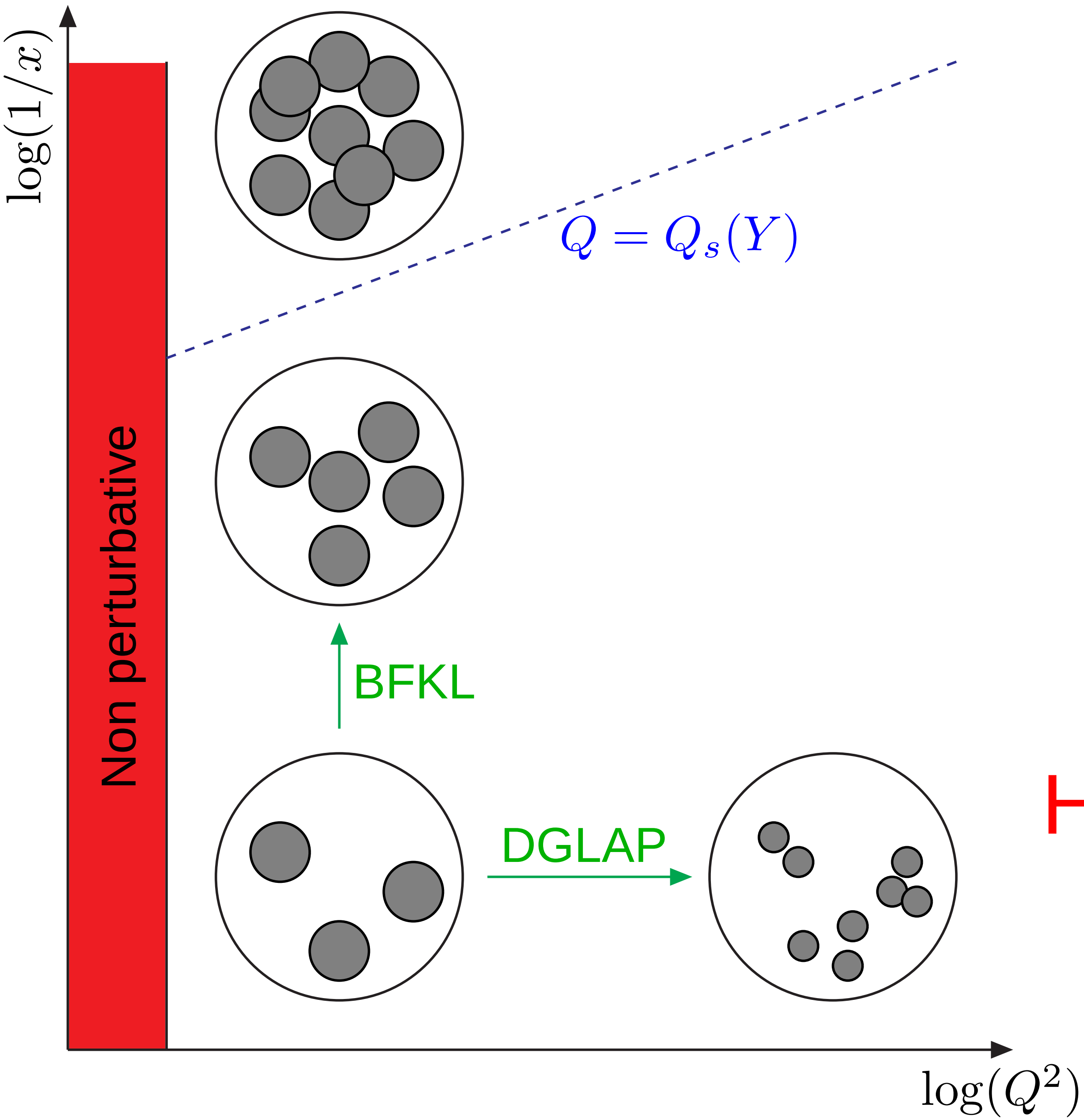
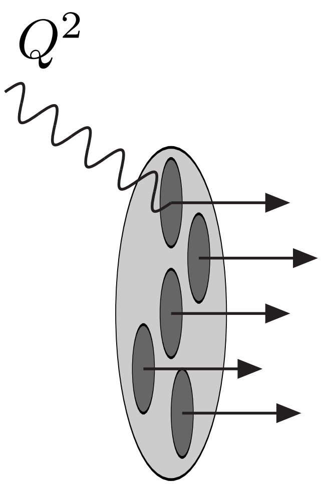
Outline

- General framework
- Perturbative evolution in high-energy QCD:
  - Leading log approx.: BFKL equation
  - Unitarity/Saturation effects: Balitsky/JIMWLK and BK equation
- Asymptotic solutions:
  - saturation $\Rightarrow$ geometric scaling
- Phenomenological consequences
  - The dipole model
  - Geometric scaling for F_2, F_2^D and vector meson production
  - Forward particle production at RHIC

Motivation: why saturation ?





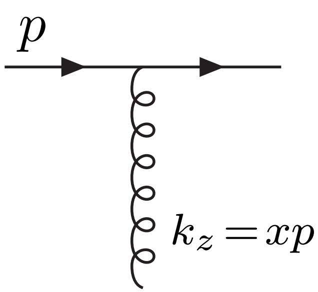
Size $\sim 1/Q$

Energy $\sim Q^2/x$

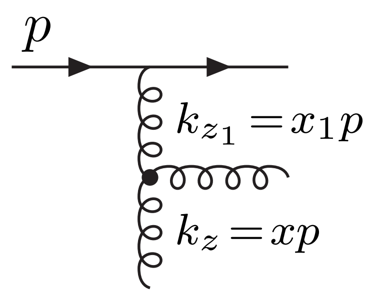
How to describe this in QCD ?

Motivation: why resummation ?

Bremsstrahlung:

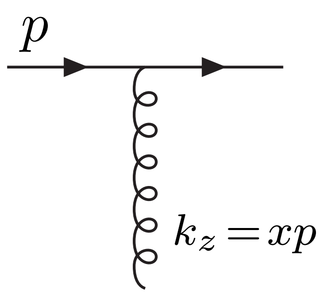


$x \ll 1$

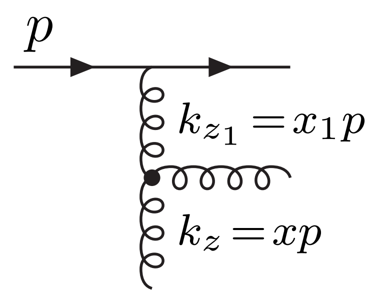


$x \ll x_1 \ll 1$

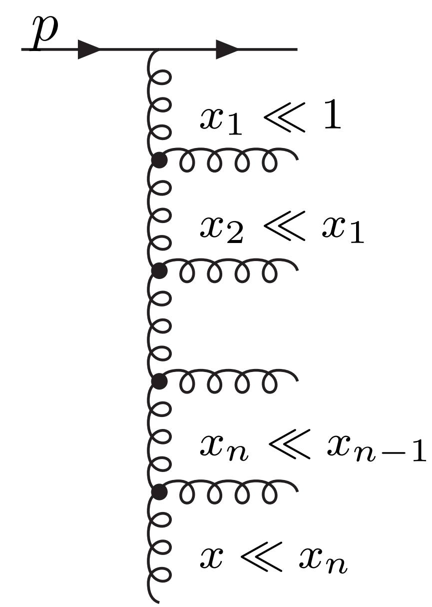
Bremsstrahlung:



$x \ll 1$



$x \ll x_1 \ll 1$



Probability of emission

$$dP \sim \alpha_s \frac{dk_\perp^2}{k_\perp^2} \frac{dx}{x}$$

In the small-x limit

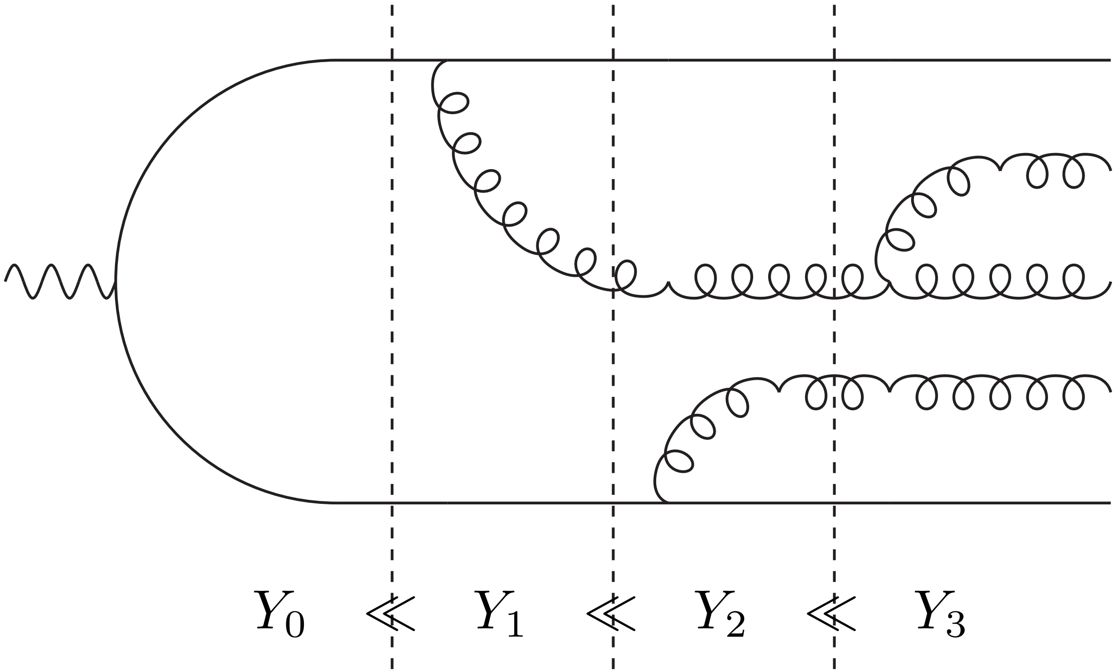
$$\int_x^1 \frac{dx_n}{x_n} \dots \int_{x_2}^1 \frac{dx_1}{x_1} \sim \frac{1}{n!} \alpha_s^n \log^n(1/x)$$

Same order when $\alpha_s \log(1/x) \sim 1 \Rightarrow$ need to be resummed

Perturbative evolution in high-energy QCD

Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$)

[Mueller,93]

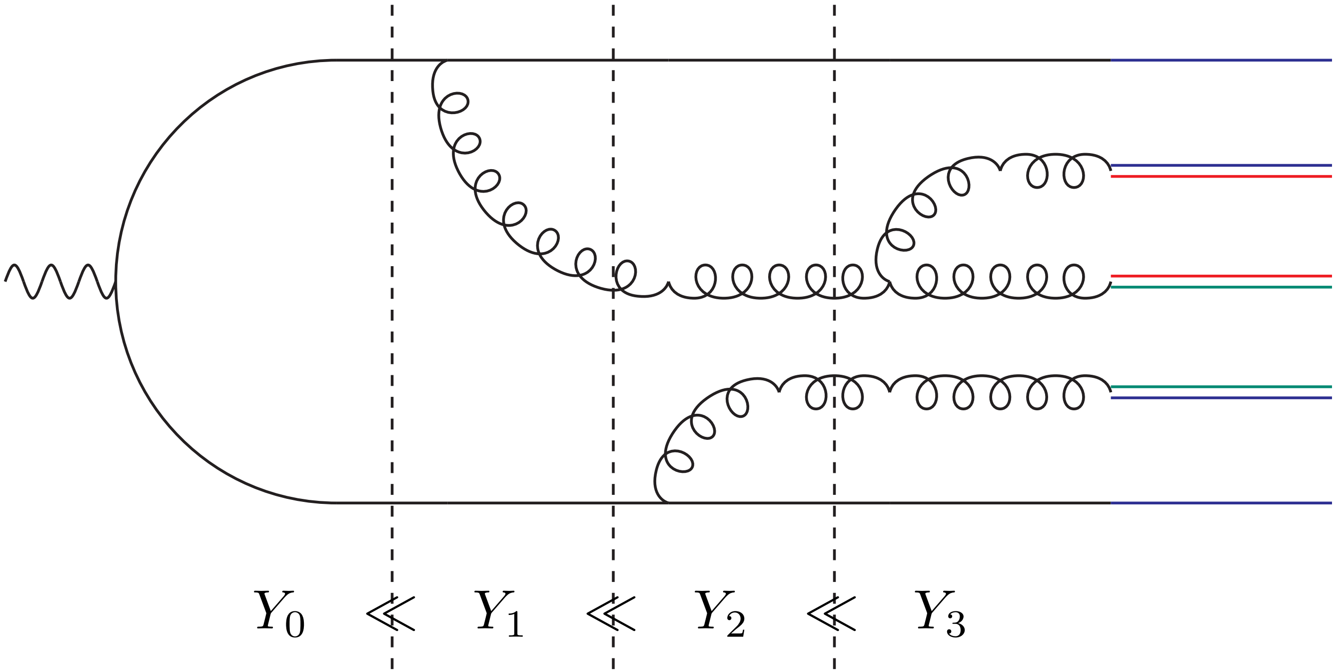


- Probability $\bar{\alpha}K$ of emission
- degree of freedom: transverse position of the gluon

Dipole picture

Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$)

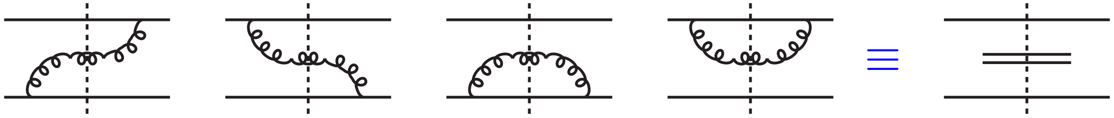
[Mueller,93]



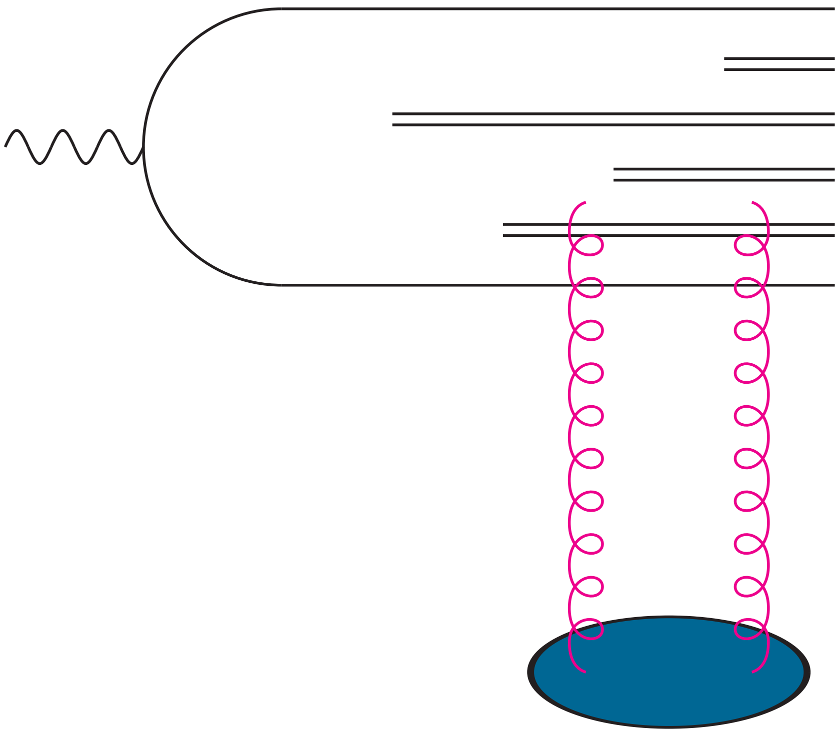
$n(r, Y)$ dipoles

of size r

- Probability $\bar{\alpha}K$ of emission $\equiv$ dipole splitting
- degree of freedom: transverse position of the gluon
- Large-N_c approximation



Projectile-target interaction:

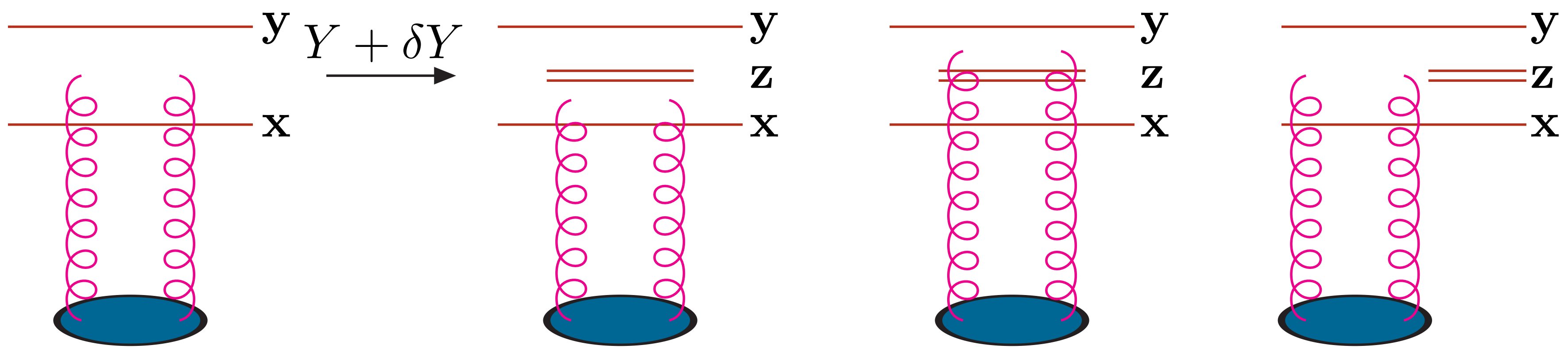


$$T(r,Y) \approx \alpha_s^2 n(r,Y)$$

Count the number of dipoles of a given size

BFKL evolution (2/4)

Consider a small increase in rapidity $\Rightarrow$ splitting



$$\partial_Y T(\mathbf{x},\mathbf{y};Y)$$

$$= \underbrace{\bar{\alpha}\int d^2z \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}}_{\text{Emission proba from pQCD}} \underbrace{[T(\mathbf{x},\mathbf{z};Y)+T(\mathbf{z},\mathbf{y};Y)-T(\mathbf{x},\mathbf{y};Y)]}_{\text{all possible interactions}}$$

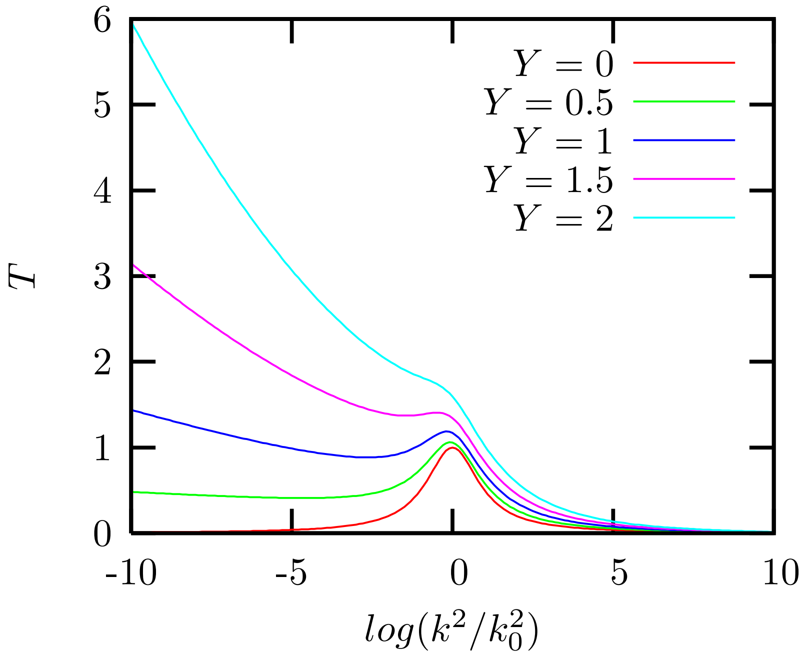
[Balitsky,Fadin,Kuraev,Lipatov,78]

BFKL evolution (4/4)

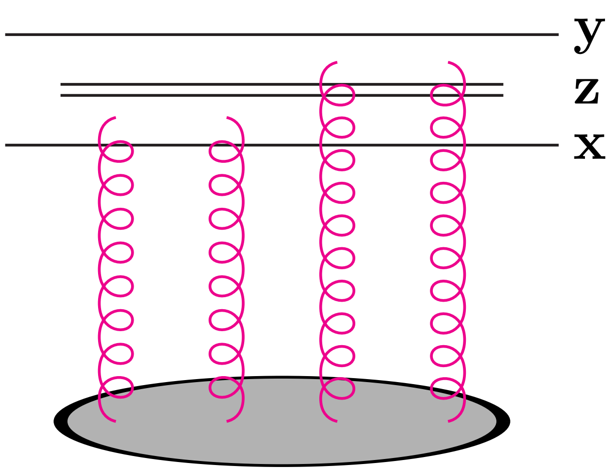
The solution goes like

$$T(Y) \sim e^{\omega Y} \qquad \text{with} \quad \omega = 4\bar{\alpha}\log(2) \approx 0.5$$

- Fast growth of the amplitude
- Intercept value too large
- Violation of the Froissart unitarity:
  $T(Y) \le C\log^2(s) \qquad\qquad T(r,b) \le 1$
- problem of diffusion in the infrared



Saturation effects



Multiple scattering

⋆ Proportional to T^2

⋆ important when $T \approx 1$

$\langle\cdot\rangle \equiv$ average over target field

$$\partial_Y \langle T(\mathbf{x},\mathbf{y};Y)\rangle = \bar{\alpha}\int d^2z\,\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\left[\langle T(\mathbf{x},\mathbf{z};Y)\rangle + \langle T(\mathbf{z},\mathbf{y};Y)\rangle - \langle T(\mathbf{x},\mathbf{y};Y)\rangle - \langle T(\mathbf{x},\mathbf{z};Y)T(\mathbf{z},\mathbf{y};Y)\rangle\right]$$

But

$\partial_Y \langle T(\mathbf{x},\mathbf{y};Y)\rangle$ contains a new object: $\langle T(\mathbf{x},\mathbf{z};Y)T(\mathbf{z},\mathbf{y};Y)\rangle$

Balitsky, Kovchegov and JIMWLK

In general: complete hierarchy [Balitsky, 96]

$$\partial_Y \langle T^k \rangle \longrightarrow \underbrace{\langle T^k \rangle}_{\text{BFKL}}, \underbrace{\langle T^{k+1} \rangle}_{\text{saturation}}$$

- Beyond large-N_c: the hierarchy involves quadrupoles, sextupoles, ...
- Balitsky hierarchy $\equiv$ JIMWKL eq. (Colour Glass Condensate formalism)

Mean field approx.: $\langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle = \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle$

$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar\alpha}{2\pi} \int d^2z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle \right]$$

[Balitsky 96,Kovchegov 99]

Simplest perturbative evolution equation satisfying unitarity constraint

Solutions

The BK equation

b-independent case

Case 1: no impact parameter dependence

$$T_{\mathbf{xy}} \to T\left(\mathbf{r}=\mathbf{x}-\mathbf{y}, \mathbf{b}=\frac{\mathbf{x}+\mathbf{y}}{2}\right) \to T(r)$$

Note:

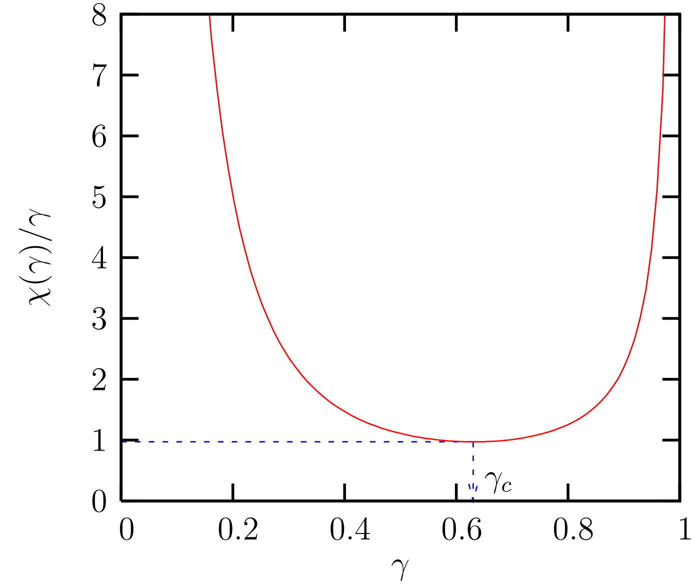
- all arguments work for $T(r)$ or its Fourier transform $\tilde{T}(k)$
- for $\tilde{T}$, the non-linear term is simply $-\tilde{T}^2(k)$

$$L = \log\left(k^2/k_0^2\right) \quad \text{or} \quad \log\left(r_0^2/r^2\right)$$

Evolution mechanism

BK equation: $\partial_Y T = \underbrace{\chi(-\partial_L) T}_{\text{BFKL}} - T^2$

[S.Munier,R.Peschanski,03]



$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

When $T \ll 1$ BFKL works: $\partial_Y T = \chi(-\partial_L) T$

Solution known:

$$T(k) = \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[\chi(\gamma)\bar{\alpha}Y - \gamma L\right]$$

$$= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[-\gamma\left(L - \frac{\chi(\gamma)}{\gamma}\bar{\alpha}Y\right)\right]$$

$\Rightarrow$ Wave of slope γ travels at speed $v = \chi(\gamma)/\gamma$

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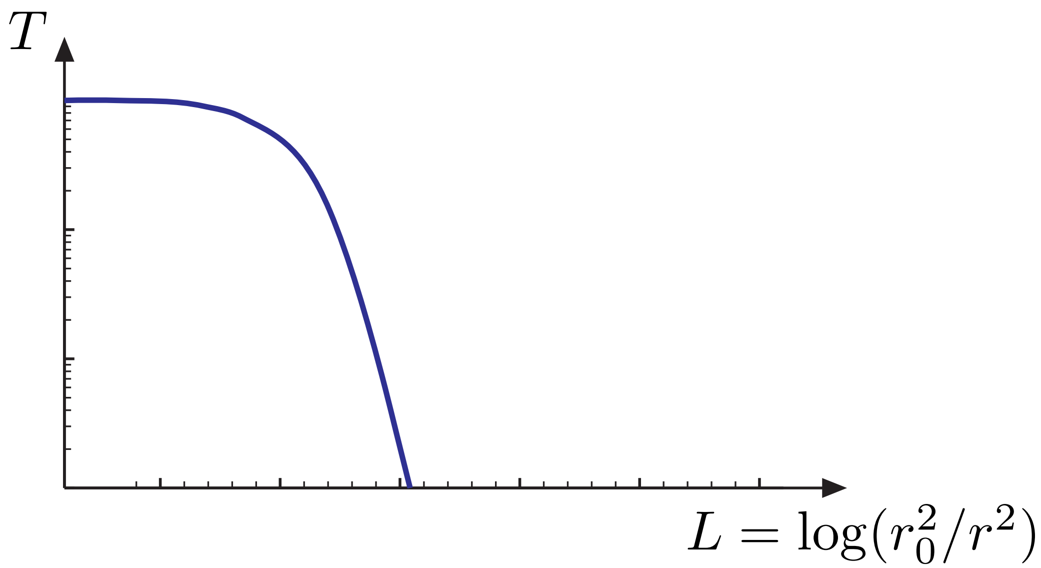

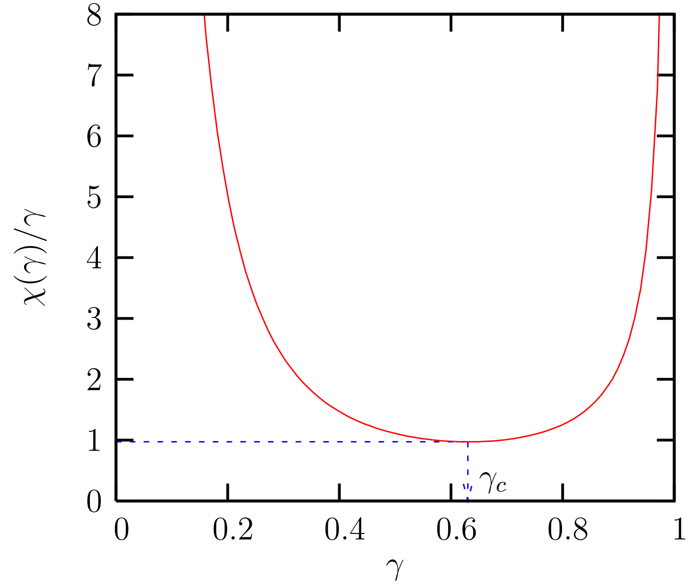

$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

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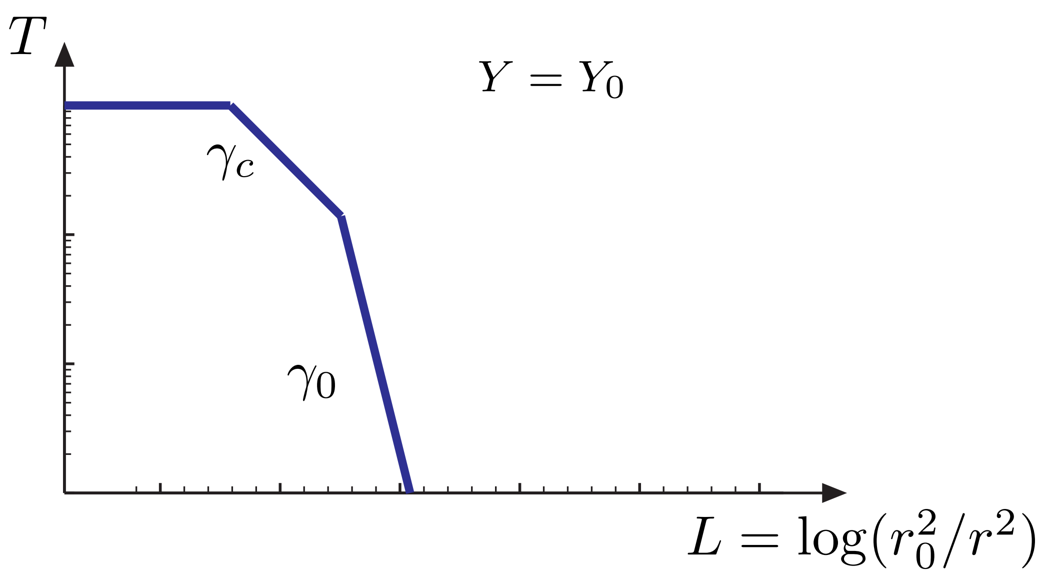

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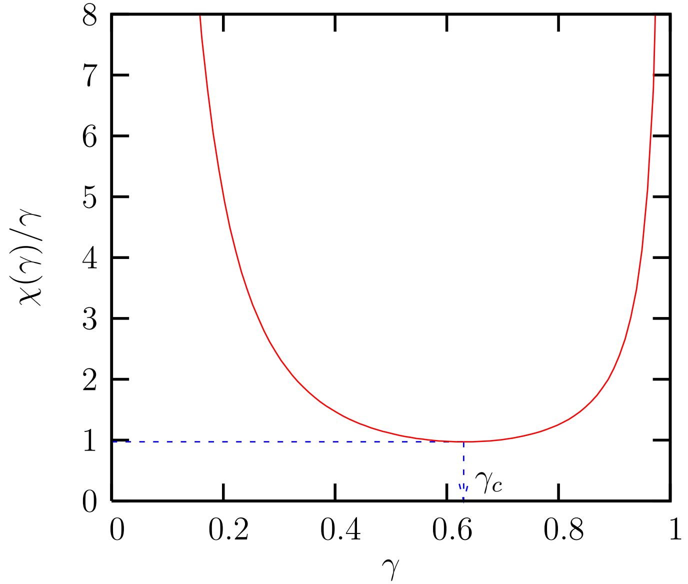

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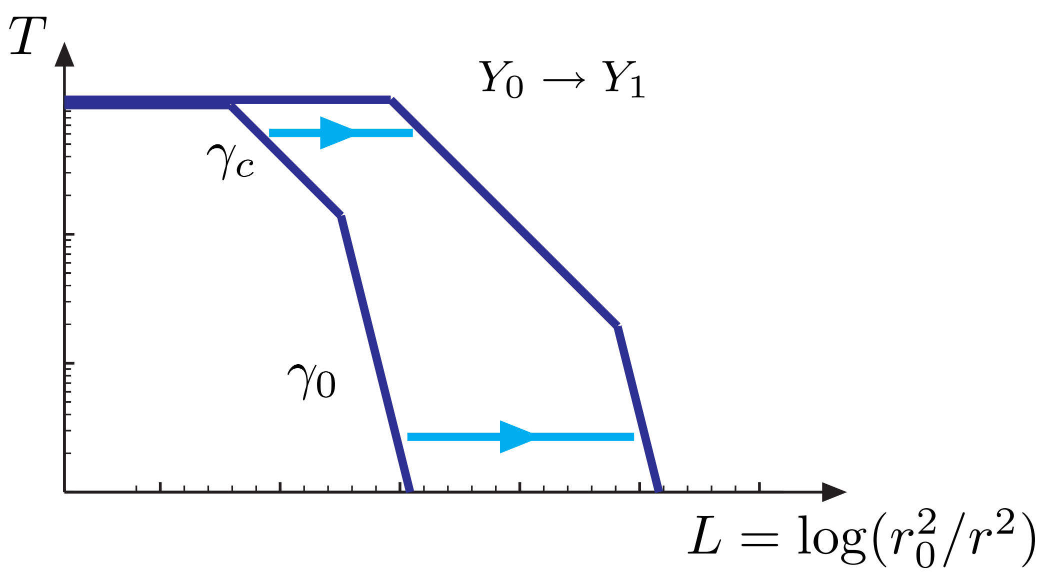

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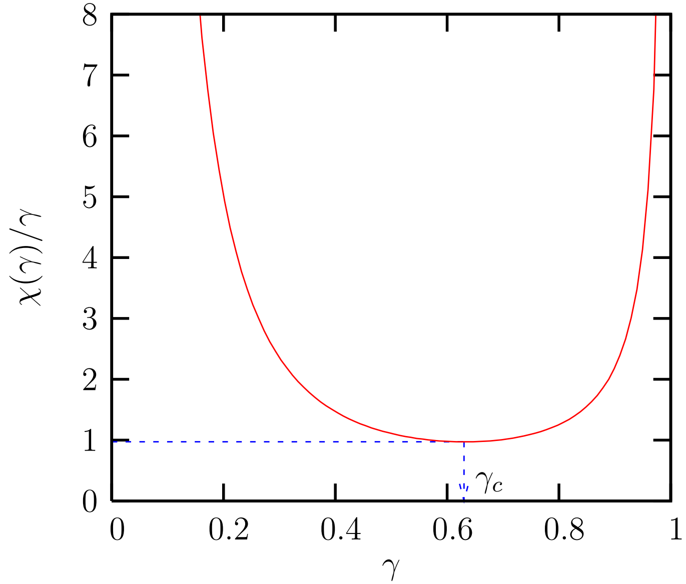

$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

The minimal speed is selected during evoution

Consequence: geometric scaling ($Q_s \equiv$ saturation scale $\equiv$ front position)

$$T(r,Y) = T\left[rQ_s(Y)\right] \quad \text{with} \quad Q_s^2(Y) = v_c\bar{\alpha}Y$$

$$\stackrel{rQ_s \ll 1}{=} \underbrace{\left[r^2Q_s^2(Y)\right]^{\gamma_c}}_{\text{slope } \gamma_c} \underbrace{\exp\left[\frac{\log^2(r^2Q_s^2)}{2\chi''(\gamma_c)\bar{\alpha}Y}\right]}_{\text{scaling window}}$$

$$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar{\alpha}Y}$$

Geometric scaling

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$$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar\alpha Y}$$

- Generic arguments: exponential rise + saturation $\Rightarrow$ select γ_c
- Parameters fixed by linear kernel only
- Saturation effects even though $T \ll 1$

Geometric scaling

Consequence: geometric scaling ($Q_s \equiv$ saturation scale $\equiv$ front position)

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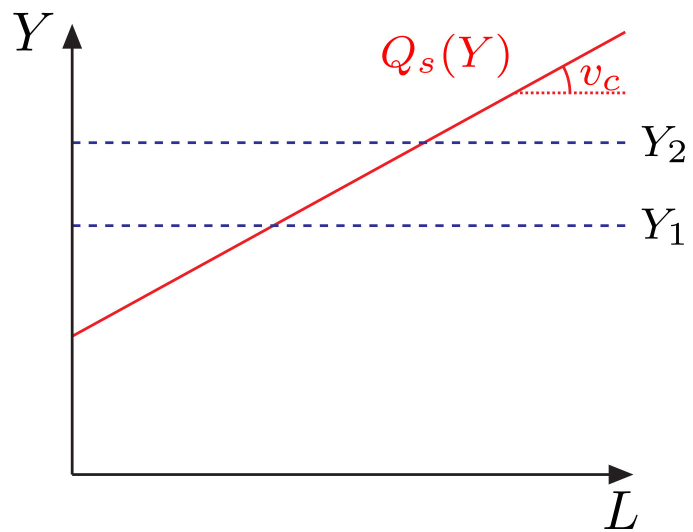
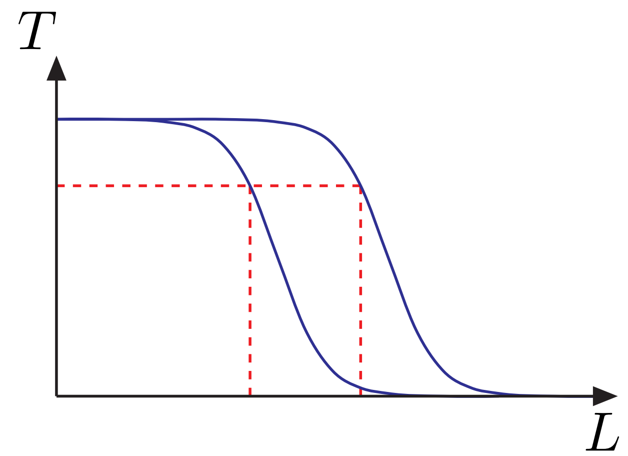
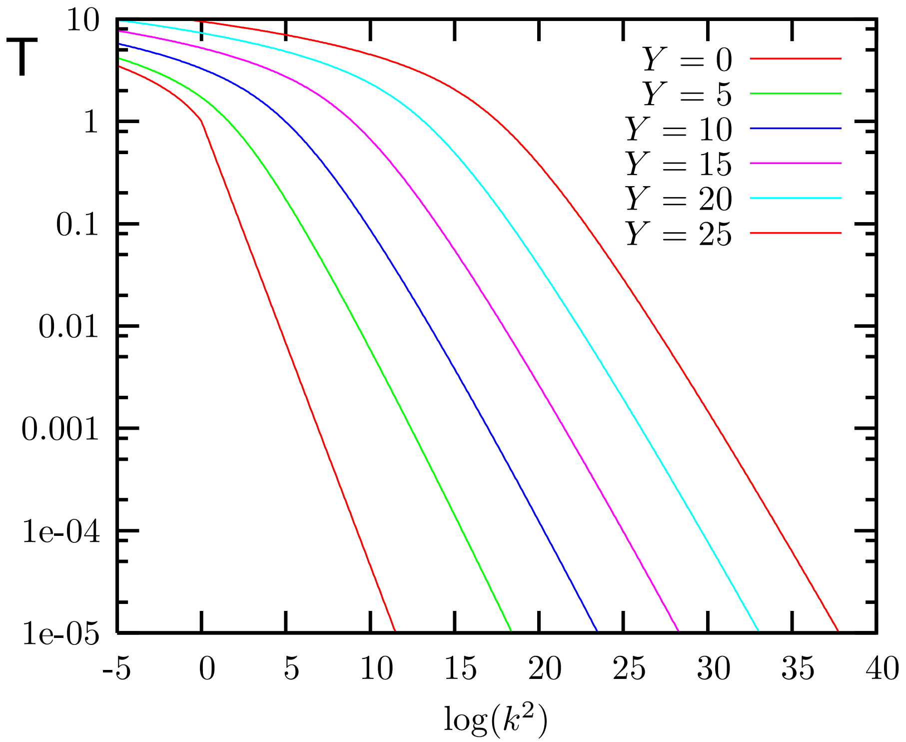
Interpretation: invariance along the saturation line





Geometric scaling

Numerical simulations:



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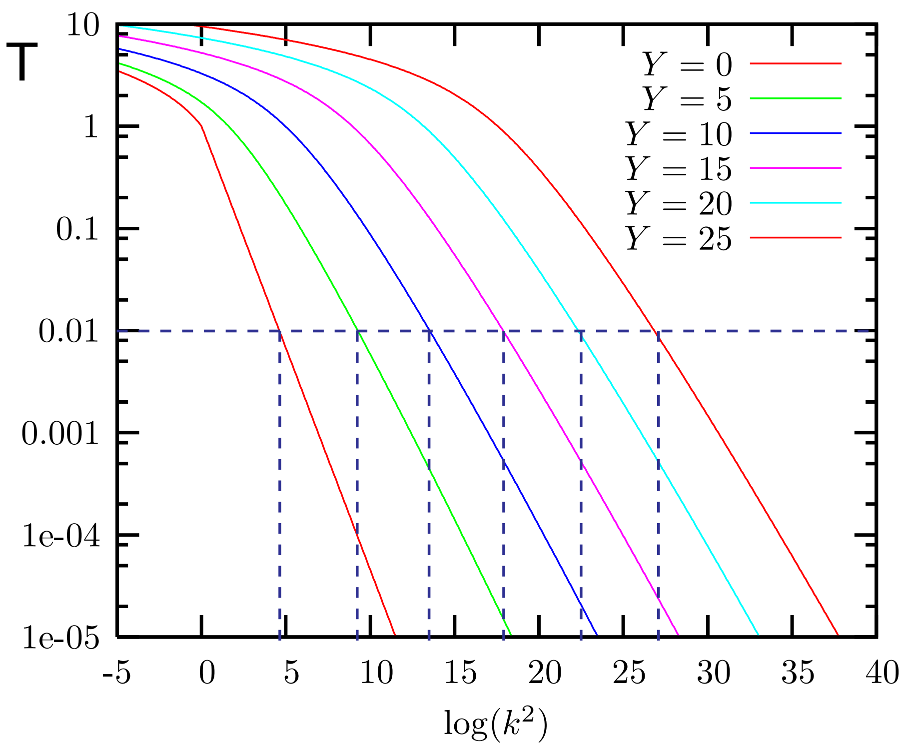
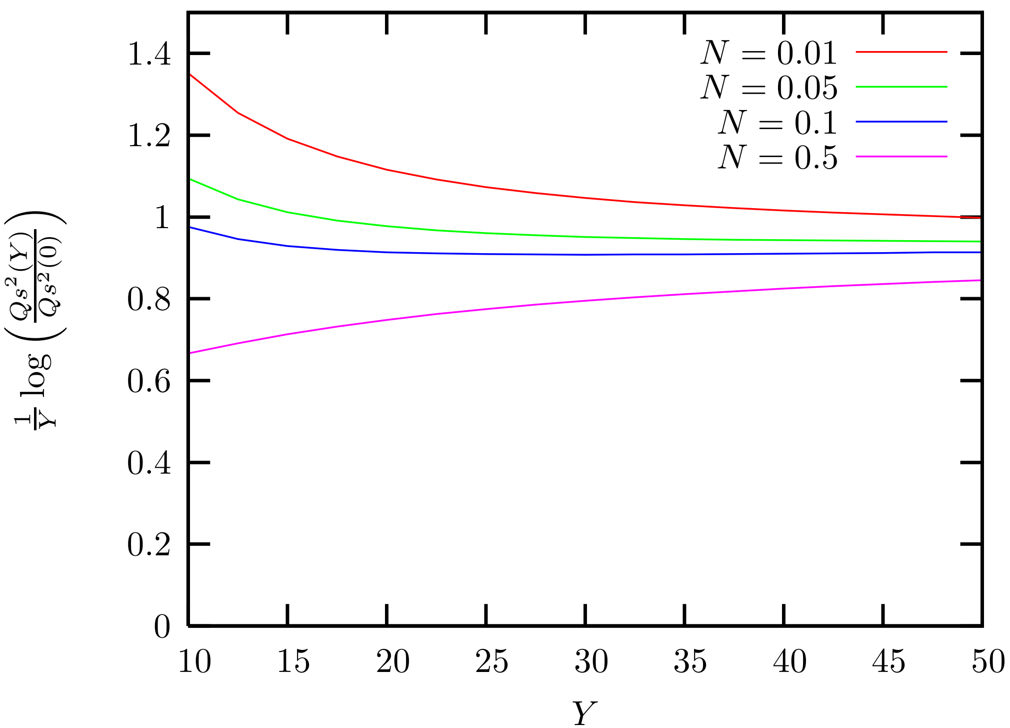




$$T(k,Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \approx \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c}$$

$$Q_s^2(Y) \propto \exp(v_c Y)$$

Case 2: including impact parameter

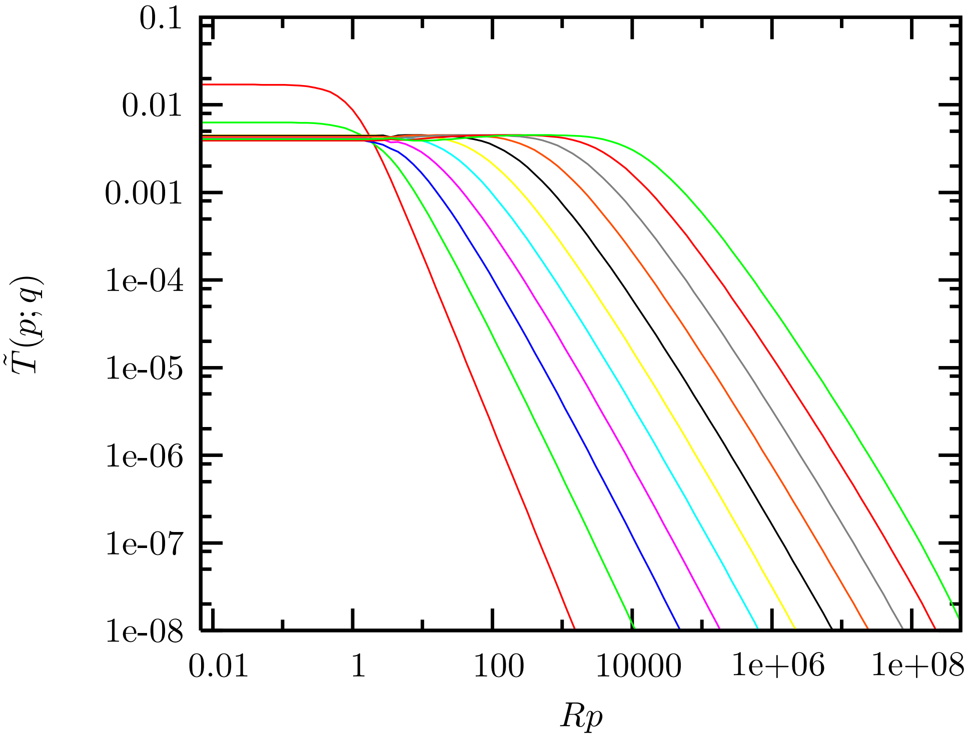
Go to momentum space: use momentum transfer $\mathbf{q}$

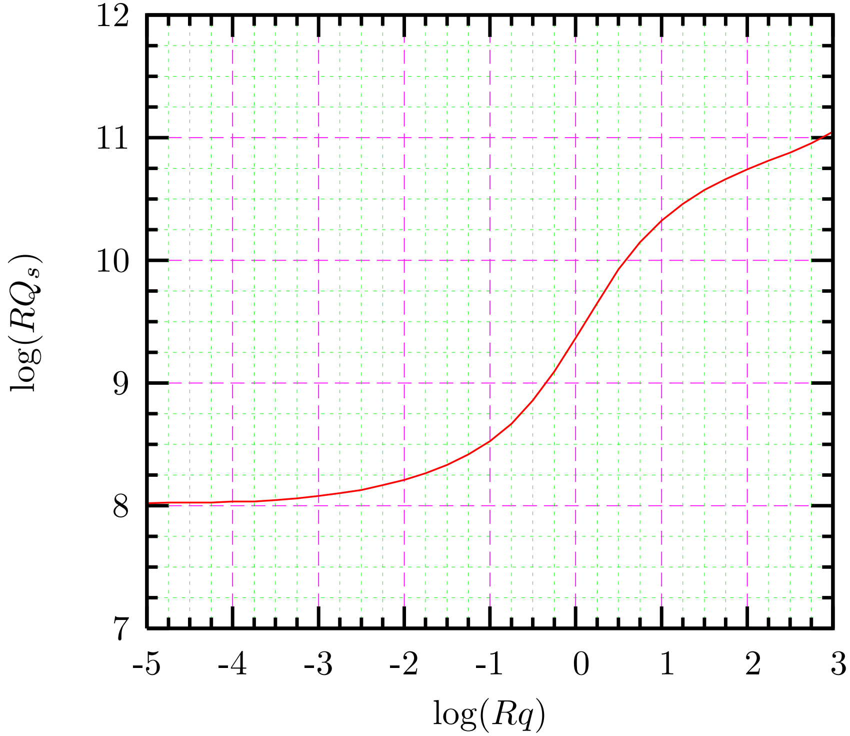
$$\tilde{T}(\mathbf{k},\mathbf{q}) = \int d^2x\, d^2y\, e^{i\mathbf{k}.\mathbf{x}} e^{i(\mathbf{q}-\mathbf{k}).\mathbf{y}} \frac{T(\mathbf{x},\mathbf{y})}{(\mathbf{x}-\mathbf{y})^2}$$

new form of the BK equation

$$\begin{aligned} \partial_Y \tilde{T}(\mathbf{k},\mathbf{q}) &= \frac{\bar{\alpha}}{\pi}\int \frac{d^2k'}{(k-k')^2}\left\{\tilde{T}(\mathbf{k}',\mathbf{q}) - \frac{1}{4}\left[\frac{k^2}{k'^2}+\frac{(q-k)^2}{(q-k')^2}\right]\tilde{T}(\mathbf{k},\mathbf{q})\right\} \\ &- \frac{\bar{\alpha}}{2\pi}\int d^2k'\, \tilde{T}(\mathbf{k},\mathbf{k}')\tilde{T}(\mathbf{k}-\mathbf{k}',\mathbf{q}-\mathbf{k}') \end{aligned}$$

[C.Marquet, R.Peschanski, G.S., 05]





One can prove analytically that:

- traveling wave at large k: BFKL $\Rightarrow$ same γ_c, v_c
- q dependence: $Q_s^2(q, Y) = \min(k_0^2, q^2) \exp(v_c Y)$

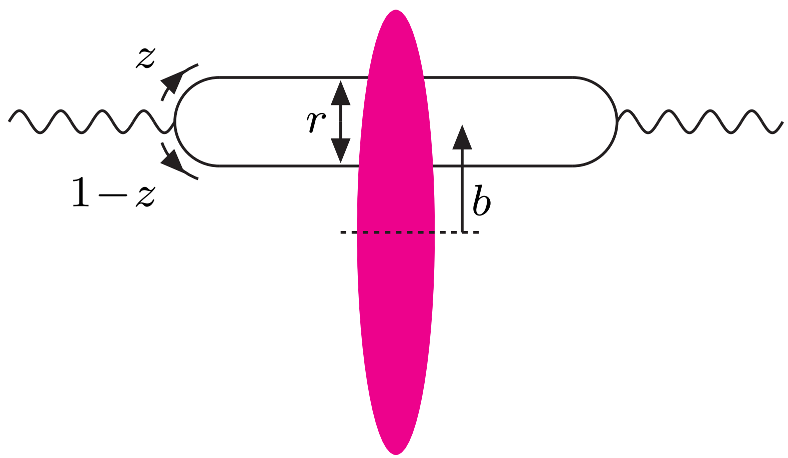
Predicts geometric scaling for t-dependent processes

Phenomenology

F_2 in the dipole picture

Factorisation formula at small x:

$$\frac{\sigma_{L,T}^{\gamma^* p}}{d^2 b} = \int d^2 r \int_0^1 dz \, \left|\Psi_{L,T}(z, r; Q^2)\right|^2 \, T(\mathbf{r}, \mathbf{b}; Y)$$



- $\Psi \equiv$ photon wavefunction $\gamma^* \to q\bar{q}$: QED process
- $T \equiv$ scattering amplitude from high-energy QCD.

$$\int d^2 b \, T(r, b, Y) = 2\pi R_p^2 \, T(r; Y) \qquad F_2 = \frac{Q^2}{4\pi\alpha_e} \left[\sigma_L^{\gamma^* p} + \sigma_T^{\gamma^* p}\right]$$

A guideline: geometric scaling

[A. Stasto, K. Golec-Biernat, J. Kwiecinski, 2001]
[F. Gelis, R. Peschanski, L. Schoeffel, GS, 2006]

Parametrisation of T motivated by one observation:

$$\sigma^{\gamma^* p}(x, Q^2) = \sigma^{\gamma^* p}(\tau)$$

with $\tau = \log(Q^2) - \lambda \log(1/x)$
$= \log(Q^2/Q_s^2)$

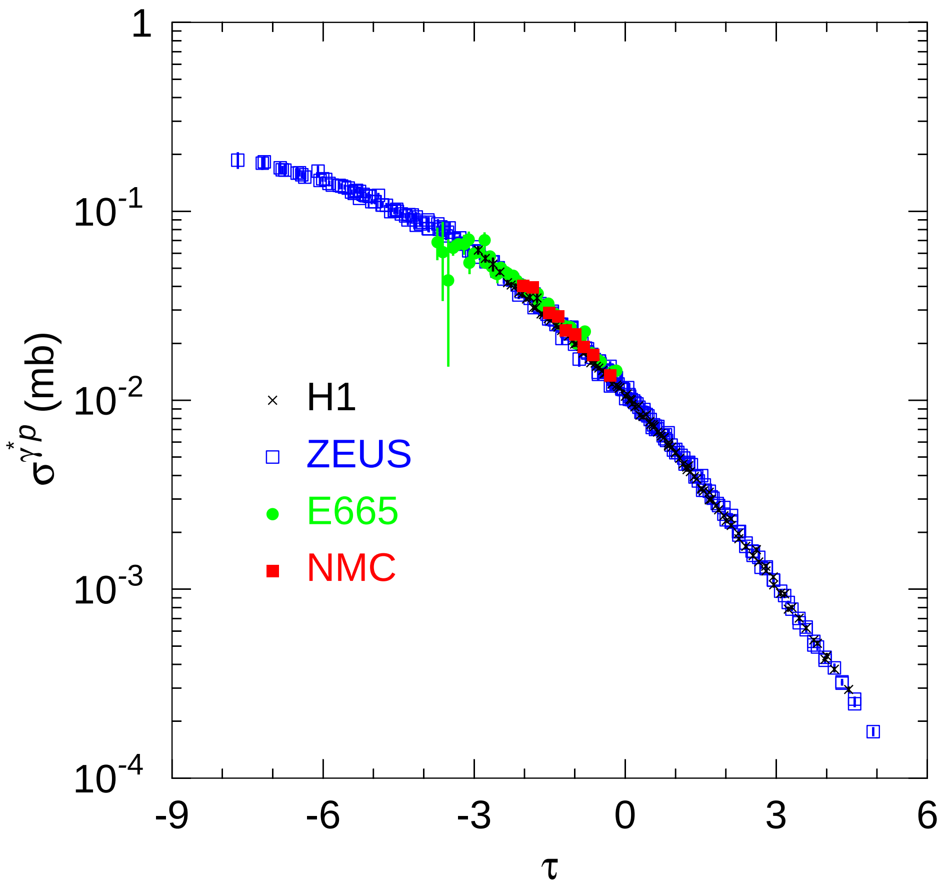
Geometric scaling

Saturation scale:

$$Q_s^2(x) = Q_0^2\, x^{-\lambda}$$

Since $Q \sim 1/r$ this suggests

$$T(r, x) = T(rQ_s)$$



HERA: $Q_s \sim 1$ GeV

Two approaches for T

- Eikonal models
  - $T \sim 1 - \exp(-r^2 Q_s^2)$ (Golec-Biernat,Wusthoff)
  - Add DGLAP effects: $T \sim 1 - \exp(-r^2\, xg)$ (Bartels,Golec-Biernat,Kowalski)
  - Account for heavy quarks (Golec-Biernat,Sapeta)
- Balitsky-Kovchegov-based models
  - Asymptotic solutions of the BK equation (Iancu,Itakura,Munier)
  - Account for heavy quarks (G.S.)

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Heavy quark problem:
Including heavy quarks $\Rightarrow Q_s$ down by a factor ~ 2.

BK equation and geometric scaling

BK equation: ($Y = \log(1/x)$) [Balitsky,Kovchegov]

- $\partial_Y T(r)$ from perturbative QCD in the high energy limit
- Resum BFKL logarithms + non-linear effects (saturation/unitarity)

Solution: ($\rho = \log\left(\frac{4}{r^2 Q_s^2}\right)$; $\bar{\alpha} = \alpha_s N_c/\pi$) [Iancu,Itakura,Mc Lerran] [Munier,Peschanski]

$$T(r;x) \overset{rQ_s \ll 1}{\propto} \exp\left(-\gamma_c \rho - \frac{\rho^2}{2\bar{\alpha}\chi''_c Y}\right)$$

- Sat. scale grows with rapidity: $Q_s^2(Y) = Q_0^2 \exp(\bar{\alpha}\chi'_c Y)$
- γ_c, χ'_c and χ''_c determined from BFKL only

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$$T(r;x) \overset{rQ_s \ll 1}{\propto} \underbrace{\exp\left(-\gamma_c \rho\right)}_{\text{geometric scaling}} \quad \underbrace{\exp\left(-\frac{\rho^2}{2\bar\alpha\chi''_c Y}\right)}_{\text{scaling violations; window width}}$$

- High-energy QCD predicts

  geometric scaling as a consequence of saturation
- Validity in the scaling window

$$\log(1/r^2) \lesssim \log(Q_s^2) + \sqrt{2\bar\alpha\chi''_c Y}$$

i.e. beyond saturation scale

IIM model and heavy quarks

- Parametrisation: $Q_s(x) = (x_0/x)^\lambda$ GeV

$$T(r,Y) = \begin{cases} T_0 \exp\left(\gamma_c \rho - \frac{\rho^2}{2\lambda\kappa Y}\right) & \text{if } rQ_s < 2 \quad \text{(geometric scaling)} \\ 1 - \exp[-a(\rho+b)^2] & \text{if } rQ_s > 2 \quad \text{(dense BK)} \end{cases}$$

with $T_0 = 0.7$ and $\kappa = (\chi''_c/\chi'_c)_{\text{LO BFKL}} = 9.9$ (fixed)

- Fit region: $x < 0.01$ and $Q^2 < 150$ GeV ; H1 and ZEUS

NB: no differences between $Q^2 < 45$ GeV and $Q^2 < 150$

- Results: ($\gamma_c = 0.6275$ is the LO BK result)

model	γ_c	v_c	x_0	R_p	χ^2/n
IIM(no heavy q)	0.6275	0.253	2.67 10^{-5}	3.250	≈0.9

IIM model and heavy quarks

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with $T_0 = 0.7$ and $\kappa = (\chi''_c/\chi'_c)_{\rm LO\,BFKL} = 9.9$ (fixed)

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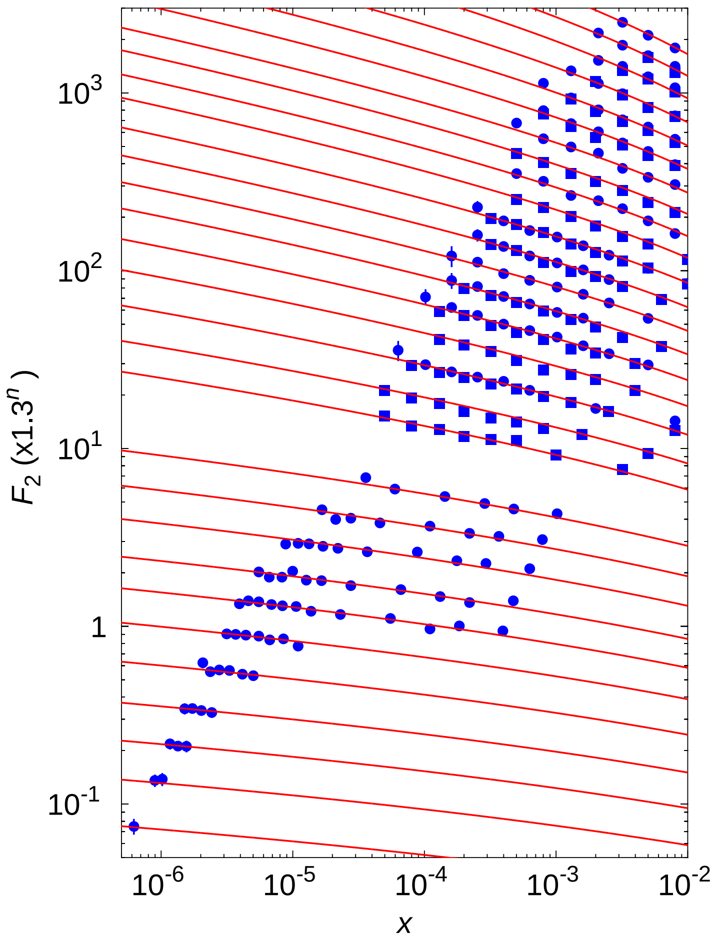
γ_c free	0.7065	0.222	1.19 10^{-5}	3.299	0.963

  - Parameters more stable (w.r.t. changes in $Q^2_{\max}$, masses, ...)
  - $\gamma_c \approx 0.7$ is in better agreement with NLO BFKL predictions

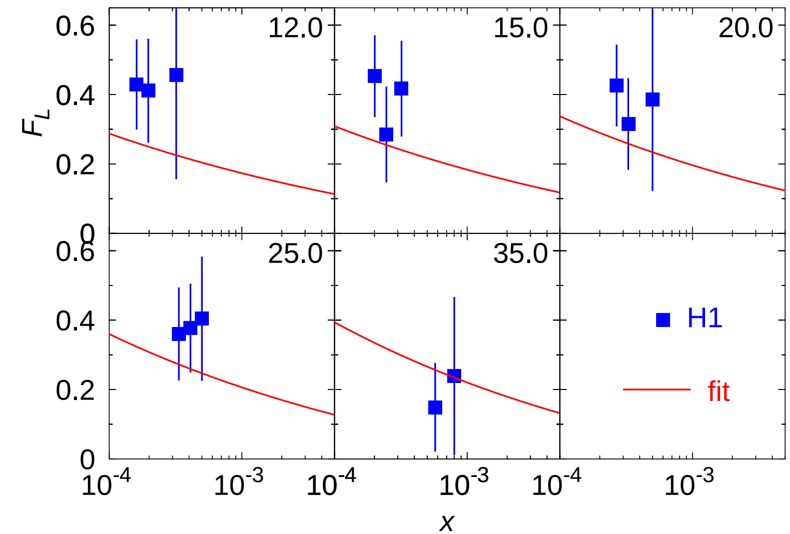
[G.S. 2007]

New fit results

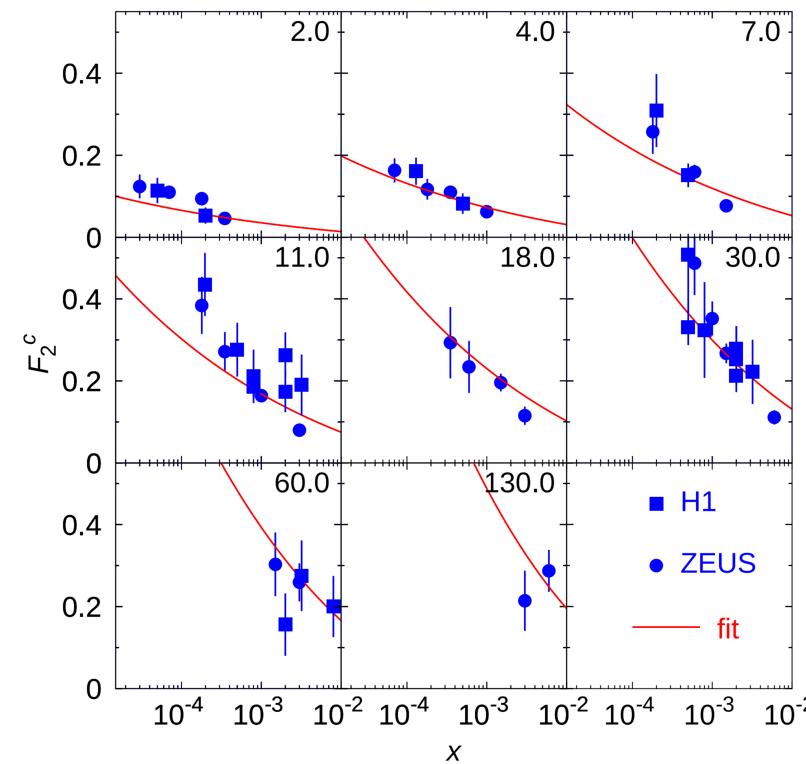
F_2^p



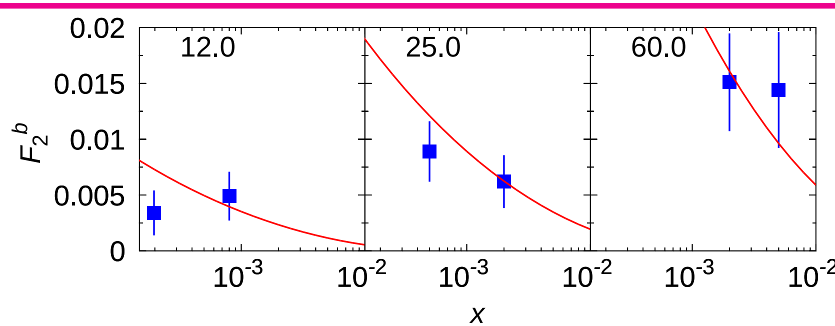
F_L



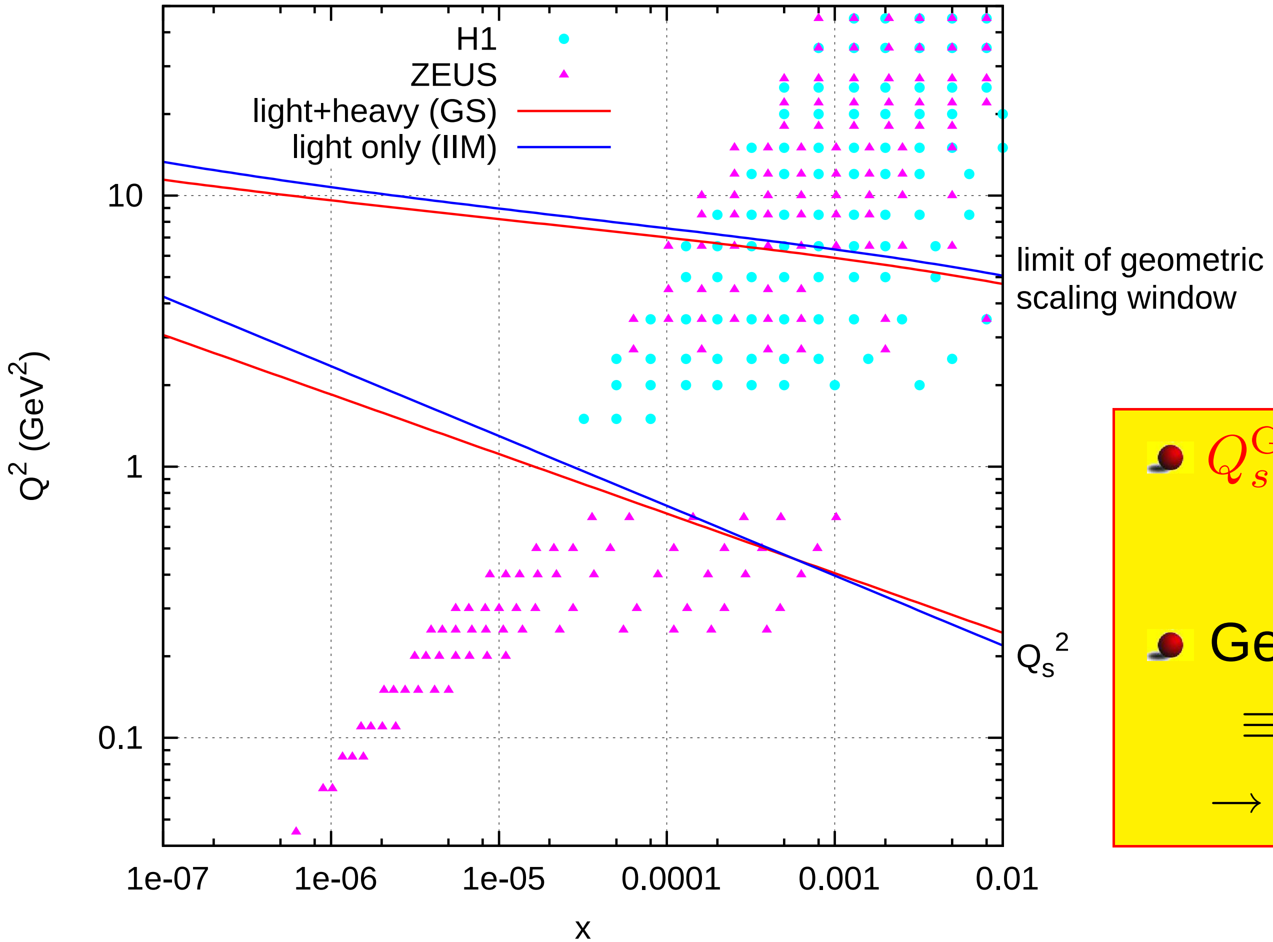
F_2^c



F_2^b



New fit results

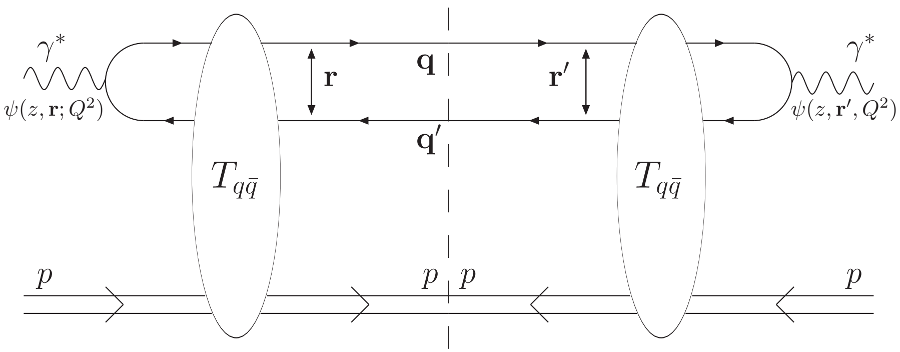


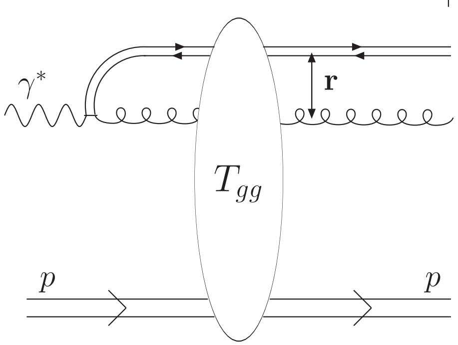
- $Q_s^{\rm GS} \approx Q_s^{\rm IIM}$: NO drop down
- Geometric scaling window
  $\equiv$ we “feel” saturation
  $\rightarrow$ up to $Q^2 \sim 5-7$ GeV^2

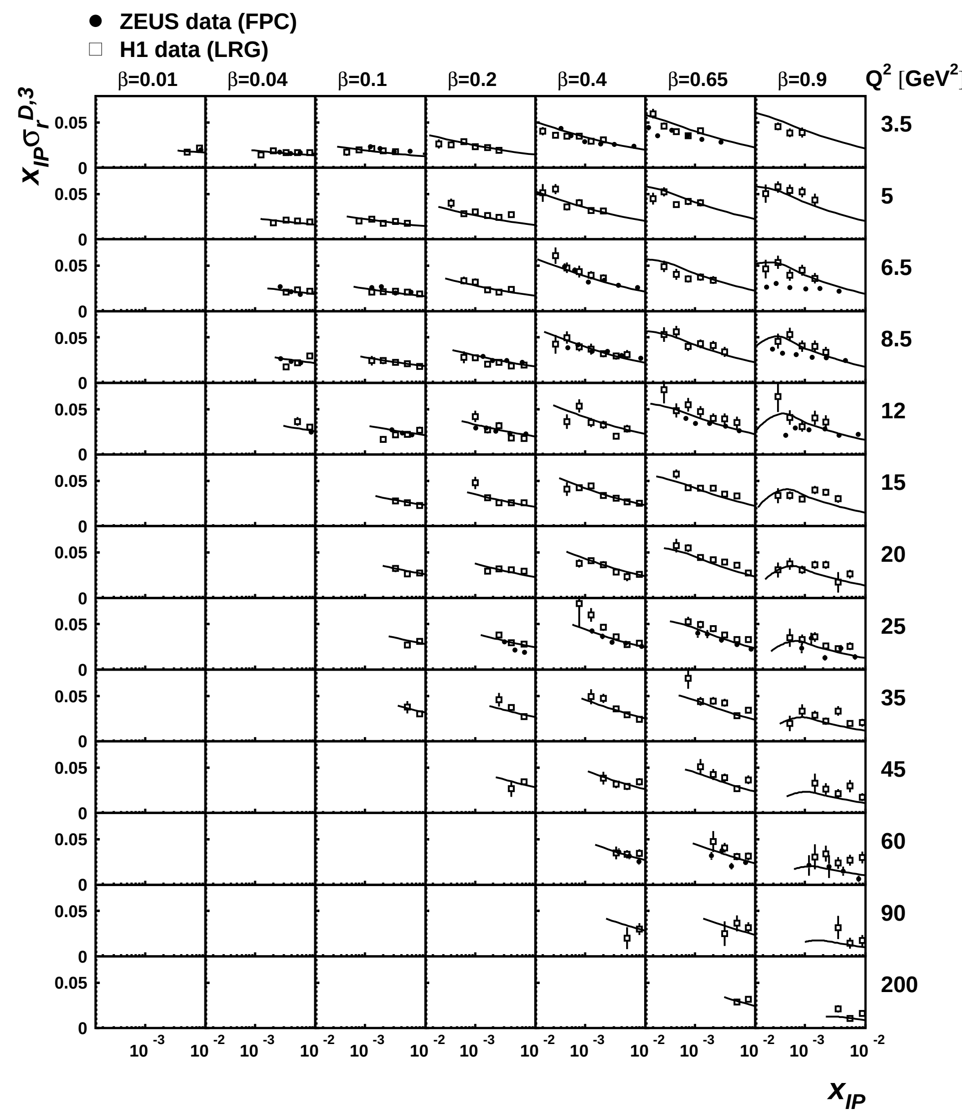
Same kind of factorisation

but more contribs:

$$F_2^D = F_2^{D(q\bar{q})} + F_2^{D(q\bar{q}g)} + \dots$$







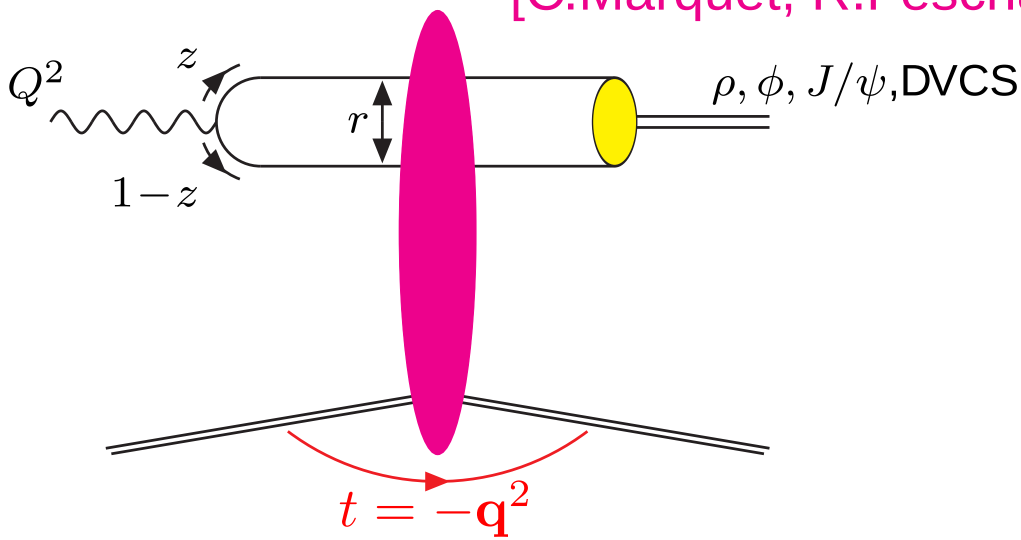
Basically: $F_2^D \propto T^2$

with the same T as for F_2

[Marquet, 07]

Vector mesons and DVCS

[C.Marquet, R.Peschanski, G.S., 07]



- Factorisation formula for

$$\mathcal{A}_{L,T}^{\gamma^* p \to V p} = \Psi^{\gamma^*} \otimes \sigma_{\text{dip}}(\mathbf{r}, \mathbf{q}; Y) \otimes \Psi^{\text{VM}}$$

- t dependence from BK (NB: BK predicts t-dep, not b-dep)

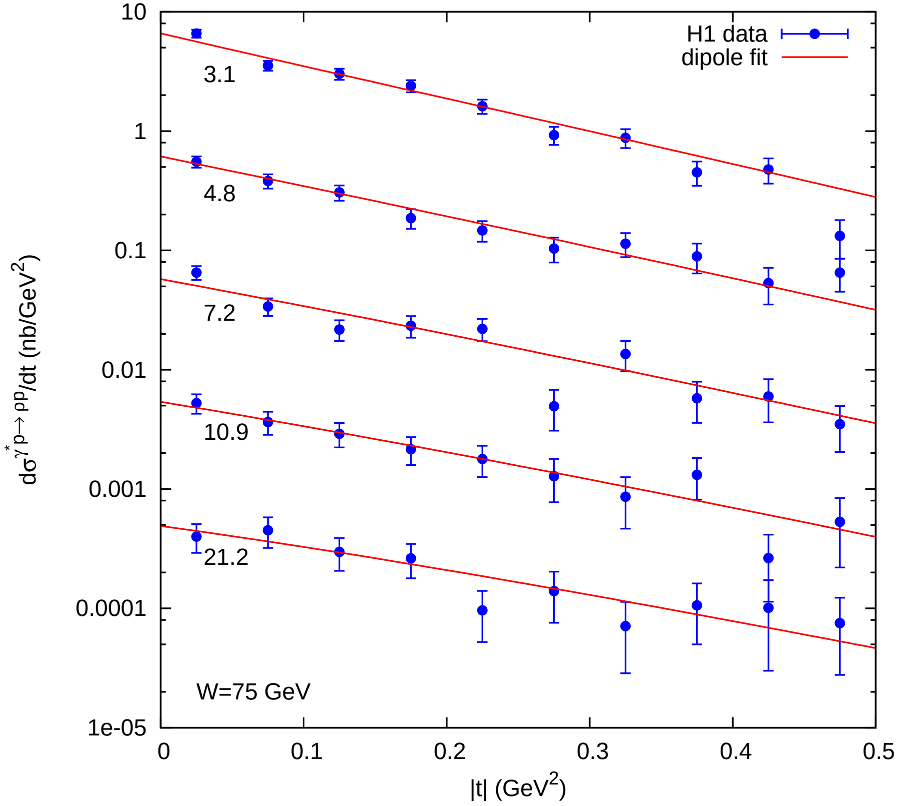
$$\sigma_{\text{dip}}(r, q; Y) = 2\pi R_p^2 e^{-b|t|}\, T_{\text{GS}}\left(r, Q_s^2(q, Y)\right)$$

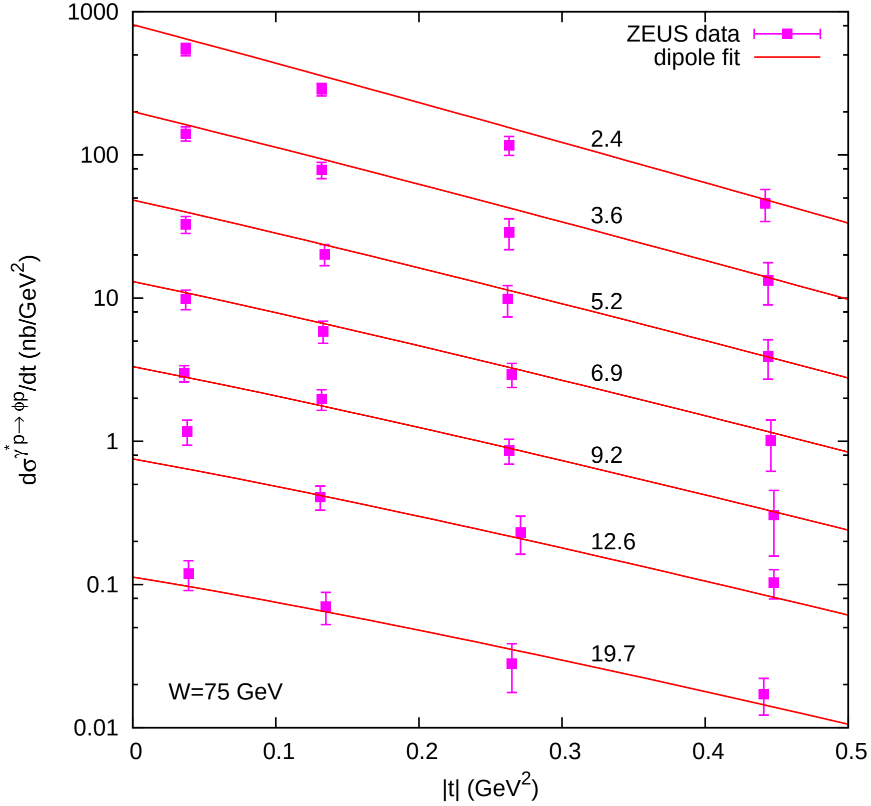
$$Q_s^2 = Q_0^2\, e^{\lambda Y} \qquad \to \qquad Q_s^2 = Q_0^2\, (1 + c|t|)\, e^{\lambda Y}$$

- b, c $\quad \to \quad$ $\frac{d\sigma}{dt}$, σ_{el} (including $\mathcal{R}e\, \mathcal{A}$ and skewedness)

Example: differential cross-section:

$\gamma^* p \to \rho p$ $\gamma^* p \to \phi p$

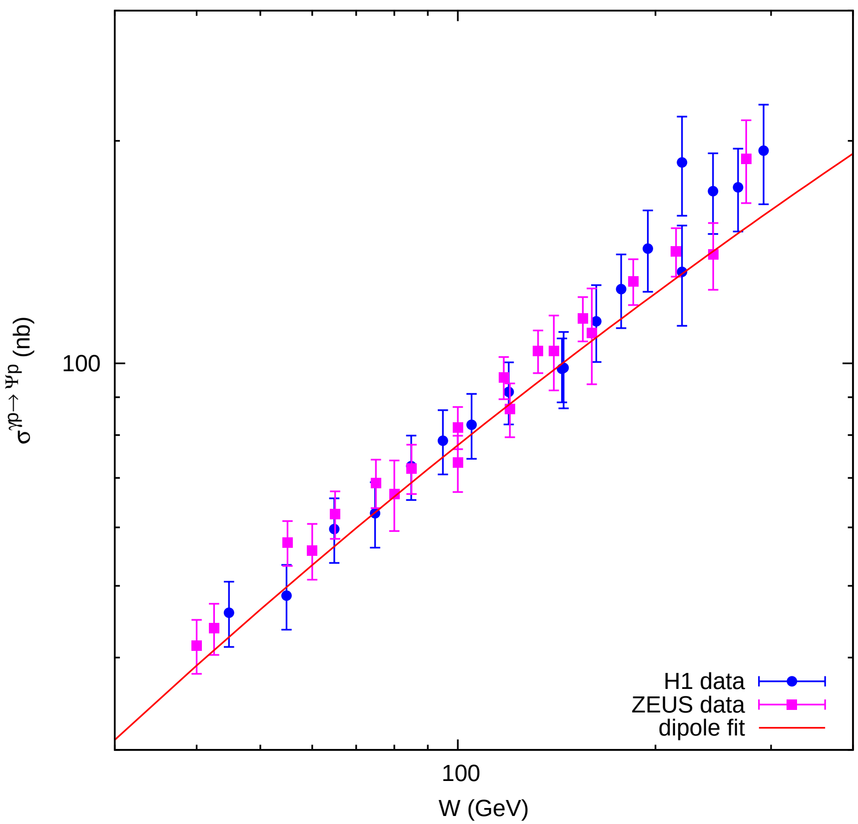


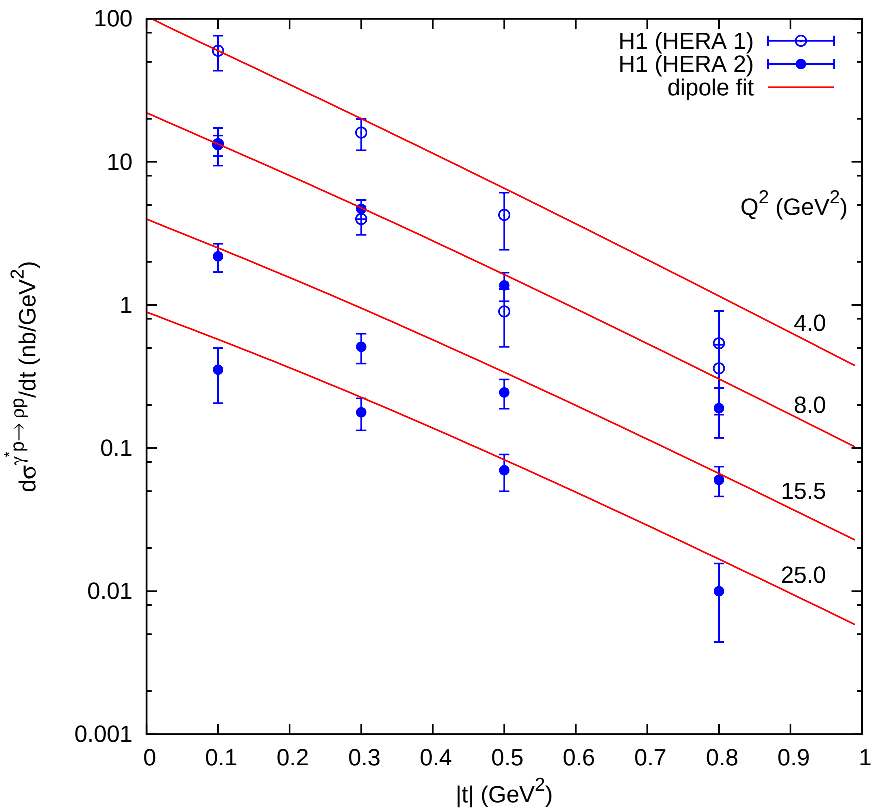


Example: continued

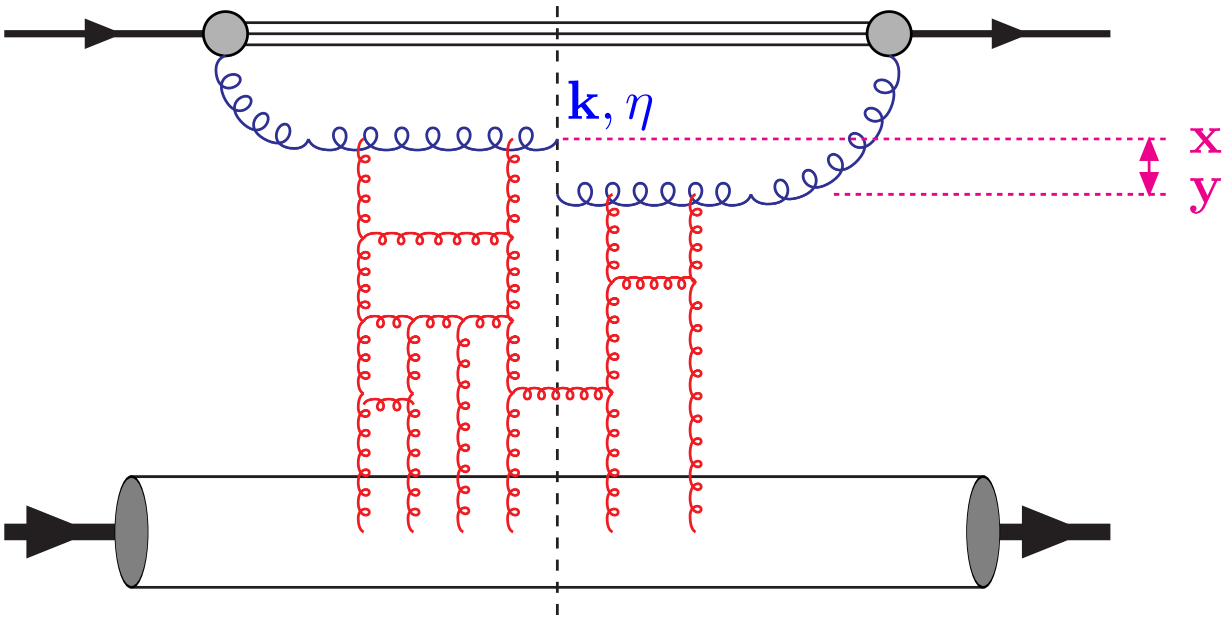
$\gamma p \to J/\Psi p$

Pred. for DVCS





Production of a (forward) gluon, momentum $\mathbf{k}$, rapidity η



Including fragmentation to produce hadrons, this gives:

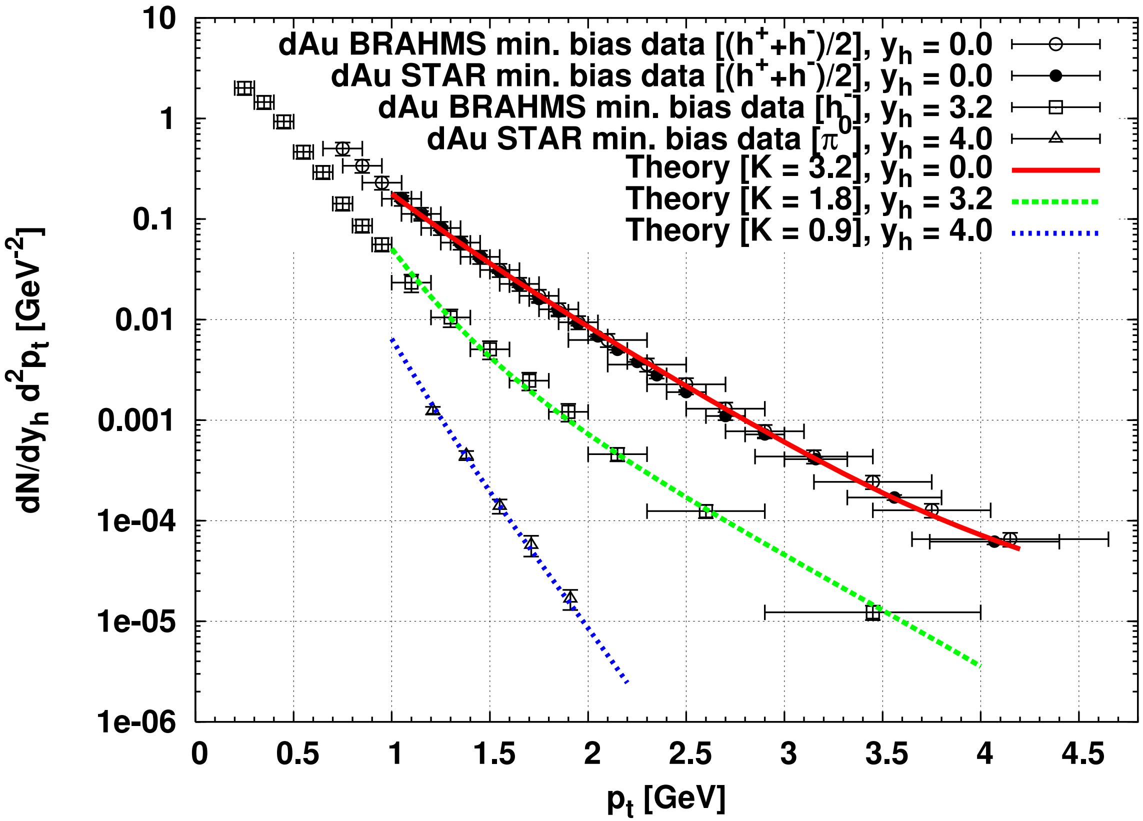
$$\frac{d^3N}{d\eta\, d^2k_t} = \frac{K}{(2\pi)^2}\int_{x_F}^1 d\xi\, \frac{\xi}{x_F} q(\xi,k_t^2) D_{h/q}\left(\frac{x_F}{\xi},k_t^2\right) \tilde{T}\left(\frac{\xi k_t}{x_F}, 2\eta+\log\left(\frac{1}{\xi}\right)\right) + \text{gluon}$$

with $x_F = \frac{k_t}{\sqrt{s}}\exp(-\eta)$ and $\tilde{T}(k_t,y) = \int d^2r\, e^{i\mathbf{k_t}.\mathbf{r}}\, T(r,y)$

Existing descriptions

Various parametrisation available:

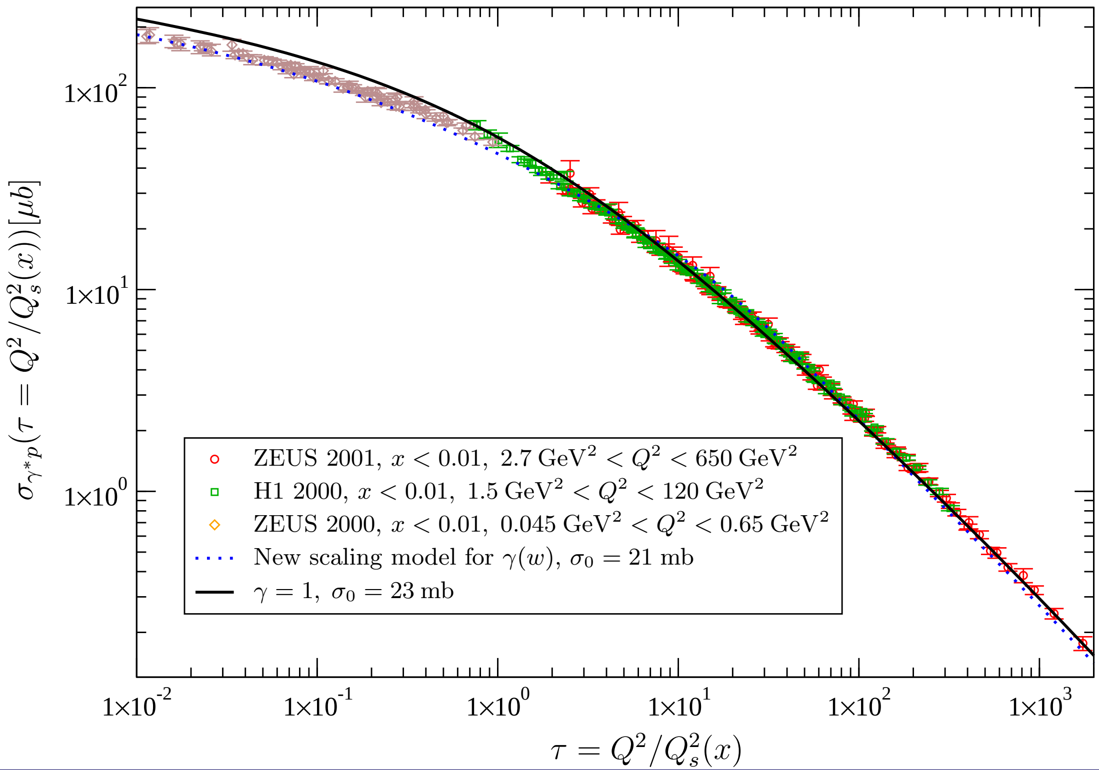
- Kharzeev, Kovchegov, Tuchin (04)
- Dumitru, Hayashigaki and Jalilian-Marian (05)



- Boer, Uterman, Wessels (07)

Comments:

- Generally, start with $T = (r^2\, Q_s^2)^{\gamma(r,Y)}$ and parametrise γ
- Usually simplify the Fourier transform using $r = 1/k_t$
- no (convincing) descriptions of HERA and RHIC at the same time



Conclusions

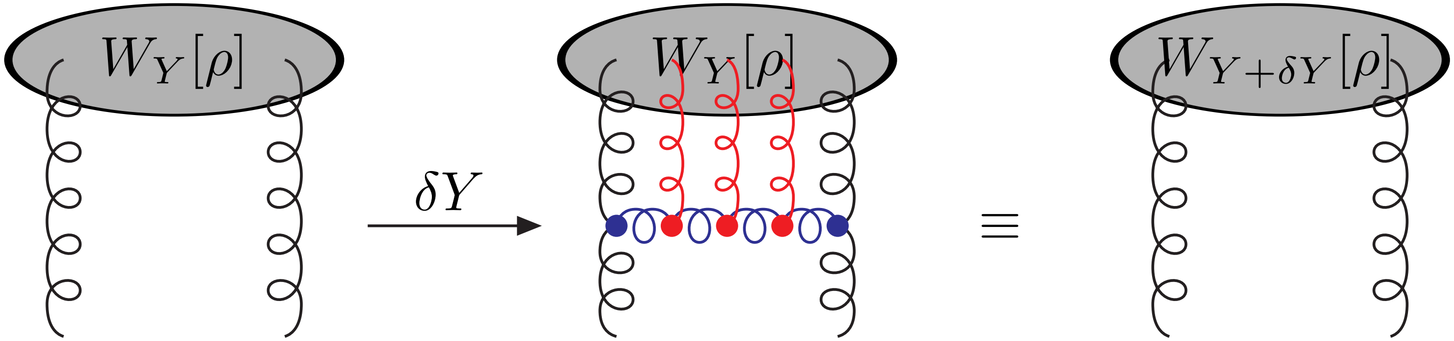
- Part 1: QCD evolution
  - BFKL resums $\alpha_s^n \log^n(1/x)$ to all orders
  - Saturation effects introduce non-linearities (BK, unitarity is restored)
- Part 2: BK solutions
  - Geometric scaling comes as a predictions from the BK equation
  - Deep links with statistical phyics (F-KPP equation)
  - valid for inclusive and t-dependent processes
- Part 3: Phenomenology
  - combined descriptions of inculsive and exclusive DIS measurements
  - Models for particle production at RHIC (probably need a bit more)

Beyond the overview

The JIMWLK equation

Can we relax the large-N_c assumption?

- the Balitsky hierarchy can be derived at arbitrary N_c
- It is equivalent to the JIMWLK equation, derived in the framework of the Colour Glass Condensate



Evolution

$$\partial_Y W_Y[\rho] = \frac{1}{2}\int_{\mathbf{xy}} \frac{\delta}{\delta\rho^a_{\mathbf{x}}} \chi^{ab}_{\mathbf{xy}}[\rho] \frac{\delta}{\delta\rho^a_{\mathbf{y}}} W_Y[\rho]$$

[Jalilan-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner,97-00]

- Not much influence on the results presented here

Is that the end of the story?

Recently, need to modify the *dilute* limit.

[Iancu,Kovner,Lublinski,Marquet,Mueller,Munier,Shoshi,GS,Triantafyllopoulos,Xiao,05-08]

- Part 1: QCD evolution
  - Fluctuations (appear when $n \sim 1$) and pomeron loops (with saturation)
  - New term added to the hierarchy (large-N_c, 2-gluon exchange) $\longrightarrow$ Langevin equation

$$\partial_Y T(k,Y) = \bar\alpha \mathcal{K} \otimes T(k,Y) - \bar\alpha T^2(k,Y) + \bar\alpha \sqrt{\alpha_s^2 T(k,Y)}\nu(k,Y)$$

  - QCD as a reaction-diffusion process

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  - Events with geometric scaling (individually)
  - Q_s varies from event to event: Gaussian($\mu = vY, \sigma^2 = DY$)
  - $DY \ll 1$: geometric scaling, $DY \gg 1$: diffusive scaling

Pomeron loops

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- Part 3: Phenomenology
  - diffusive scaling predicted for F_2, F_2^D, gluon production
  - Will require higher energies