DIS in the dipole picture:
Saturation effects and heavy quarks

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G. Soyez, JHEP 05 (2007) 086 [arXiv:0704.0292]

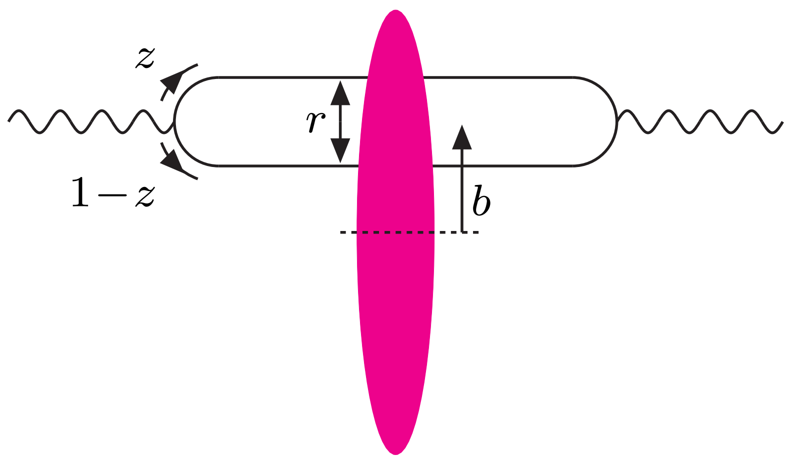
Overview

- Introduction: the dipole picture factorisation
- Underlying ideas: Saturation and geometric scaling
- Two classes of models
- Heavy quarks vs. saturation: how to solve a longstanding issue?
- “Post-motivation”: other processes (F_2^D, VM, DVCM)

F_2 in the dipole picture

Factorisation formula at small x:

$$\frac{\sigma_{L,T}^{\gamma^* p}}{d^2b} = \int d^2r \int_0^1 dz \left|\Psi_{L,T}(z,r;Q^2)\right|^2 T(\mathbf{r},\mathbf{b};Y)$$



- $\Psi \equiv$ photon wavefunction $\gamma^* \to q\bar{q}$: QED process
- $T \equiv$ scattering amplitude from high-energy QCD.

$$\int d^2b\, T(r,b,Y) = 2\pi R_p^2\, T(r;Y) \qquad F_2 = \frac{Q^2}{4\pi\alpha_e}\left[\sigma_L^{\gamma^* p} + \sigma_T^{\gamma^* p}\right]$$

A guideline: geometric scaling

[A. Stasto, K. Golec-Biernat, J. Kwiecinski, 2001]

Parametrisation of T
motivated by one observation:

$$\sigma^{\gamma^* p}(x, Q^2) = \sigma^{\gamma^* p}(\tau)$$

with $\tau = \log(Q^2) - \lambda \log(1/x)$
$= \log(Q^2/Q_s^2)$

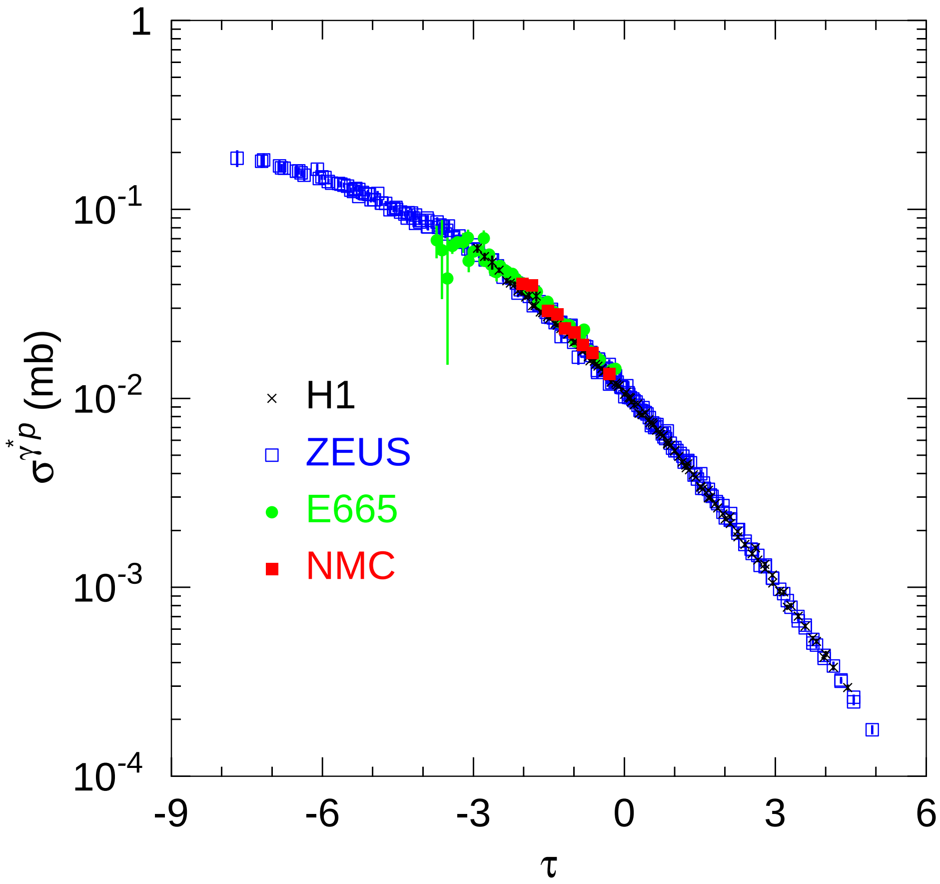
Geometric scaling

Saturation scale:

$$Q_s^2(x) = Q_0^2\, x^{-\lambda}$$

Since $Q \sim 1/r$ this suggests

$$T(r, x) = T(rQ_s)$$



HERA: $Q_s \sim 1$ GeV

Two approaches for T

- Eikonal models
  - $T \sim 1 - \exp(-r^2 Q_s^2)$ (Golec-Biernat,Wusthoff)
  - Add DGLAP effects: $T \sim 1 - \exp(-r^2\, xg)$ (Bartels,Golec-Biernat,Kowalski)
  - Account for heavy quarks (Golec-Biernat,Sapeta)
- Balitsky-Kovchegov-based models
  - Asymptotic solutions of the BK equation (Iancu,Itakura,Munier)
  - Account for heavy quarks (G.S.)

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Heavy quark problem:
Including heavy quarks $\Rightarrow Q_s$ down by a factor ~ 2.

BK equation and geometric scaling

BK equation: ($Y = \log(1/x)$) [Balitsky,Kovchegov]

- $\partial_Y T(r)$ from perturbative QCD in the high energy limit
- Resum BFKL logarithms + non-linear effects (saturation/unitarity)

Solution: ($\rho = \log\left(\frac{4}{r^2 Q_s^2}\right)$; $\bar\alpha = \alpha_s N_c/\pi$) [Iancu,Itakura,Mc Lerran] [Munier,Peschanski]

$$T(r;x) \overset{rQ_s \ll 1}{\propto} \exp\left(-\gamma_c \rho - \frac{\rho^2}{2\bar\alpha \chi''_c Y}\right)$$

- Sat. scale grows with rapidity: $Q_s^2(Y) = Q_0^2 \exp(\bar\alpha \chi'_c Y)$
- γ_c, χ'_c and χ''_c determined from BFKL only

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$$T(r;x) \overset{rQ_s \ll 1}{\propto} \underbrace{\exp\left(-\gamma_c \rho\right)}_{\text{geometric scaling}} \quad \underbrace{\exp\left(-\frac{\rho^2}{2\bar\alpha\chi"_c Y}\right)}_{\text{scaling violations; window width}}$$

- High-energy QCD predicts

  geometric scaling as a consequence of saturation
- Validity in the scaling window

$$\log(1/r^2) \lesssim \log(Q_s^2) + \sqrt{2\bar\alpha\chi"_c Y}$$

i.e. beyond saturation scale

IIM model and heavy quarks

- Parametrisation: $Q_s(x) = (x_0/x)^\lambda$ GeV

$$T(r,Y) = \begin{cases} T_0 \exp\left(\gamma_c\rho - \frac{\rho^2}{2\lambda\kappa Y}\right) & \text{if } rQ_s < 2 \quad \text{(geometric scaling)} \\ 1 - \exp[-a(\rho+b)^2] & \text{if } rQ_s > 2 \quad \text{(dense BK)} \end{cases}$$

with $T_0 = 0.7$ and $\kappa = (\chi"_c/\chi'_c)_{\rm LO\,BFKL} = 9.9$ (fixed)

- Fit region: $x < 0.01$ and $Q^2 < 150$ GeV ; H1 and ZEUS

NB: no differences between $Q^2 < 45$ GeV and $Q^2 < 150$

- Results: ($\gamma_c = 0.6275$ is the LO BK result)

model	γ_c	v_c	x_0	R_p	χ^2/n
IIM(no heavy q)	0.6275	0.253	2.67 10^{-5}	3.250	≈0.9

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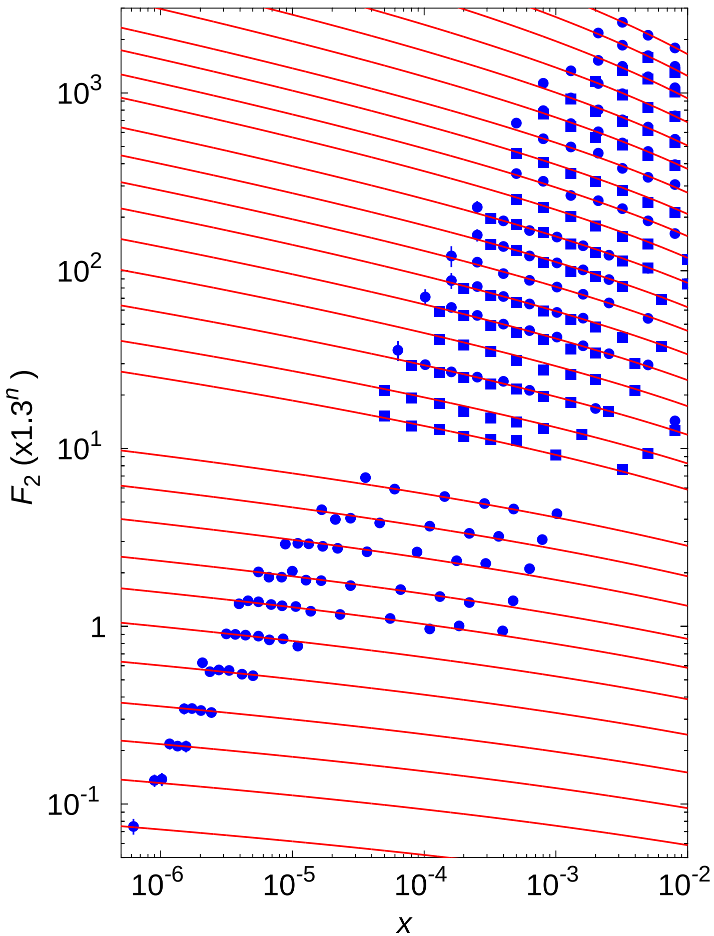
γ_c free	0.7065	0.222	1.19 10^{-5}	3.299	0.963

  - Parameters more stable (w.r.t. changes in $Q^2_{\max}$, masses, ...)
  - $\gamma_c \approx 0.7$ is in better agreement with NLO BFKL predictions

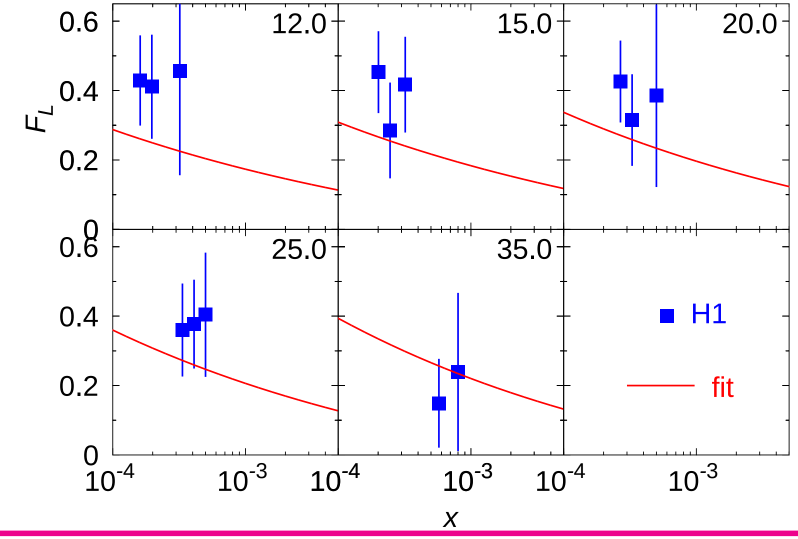
[G.S. 2007]

New fit results

F_2^p



F_L



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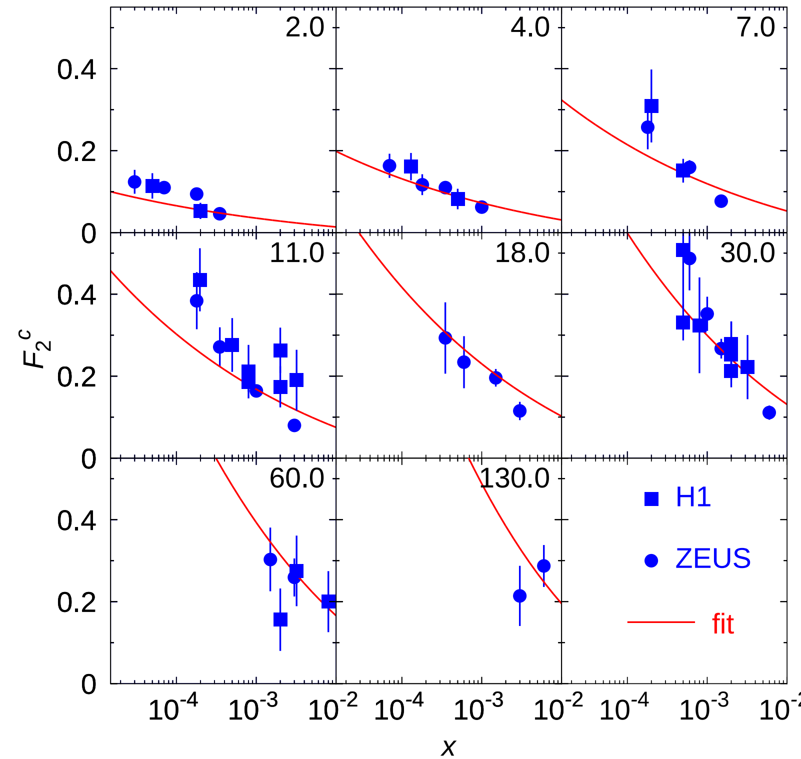
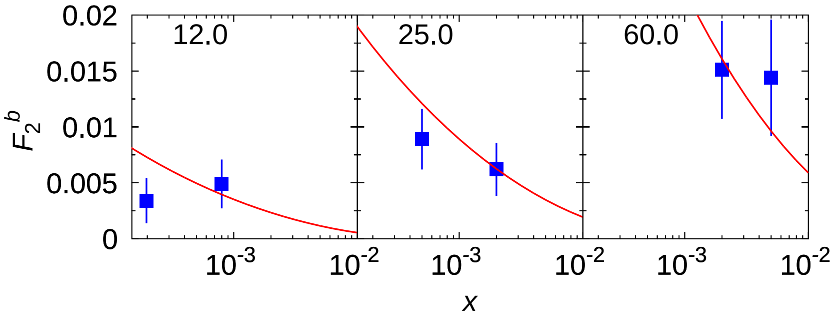


F_2^b



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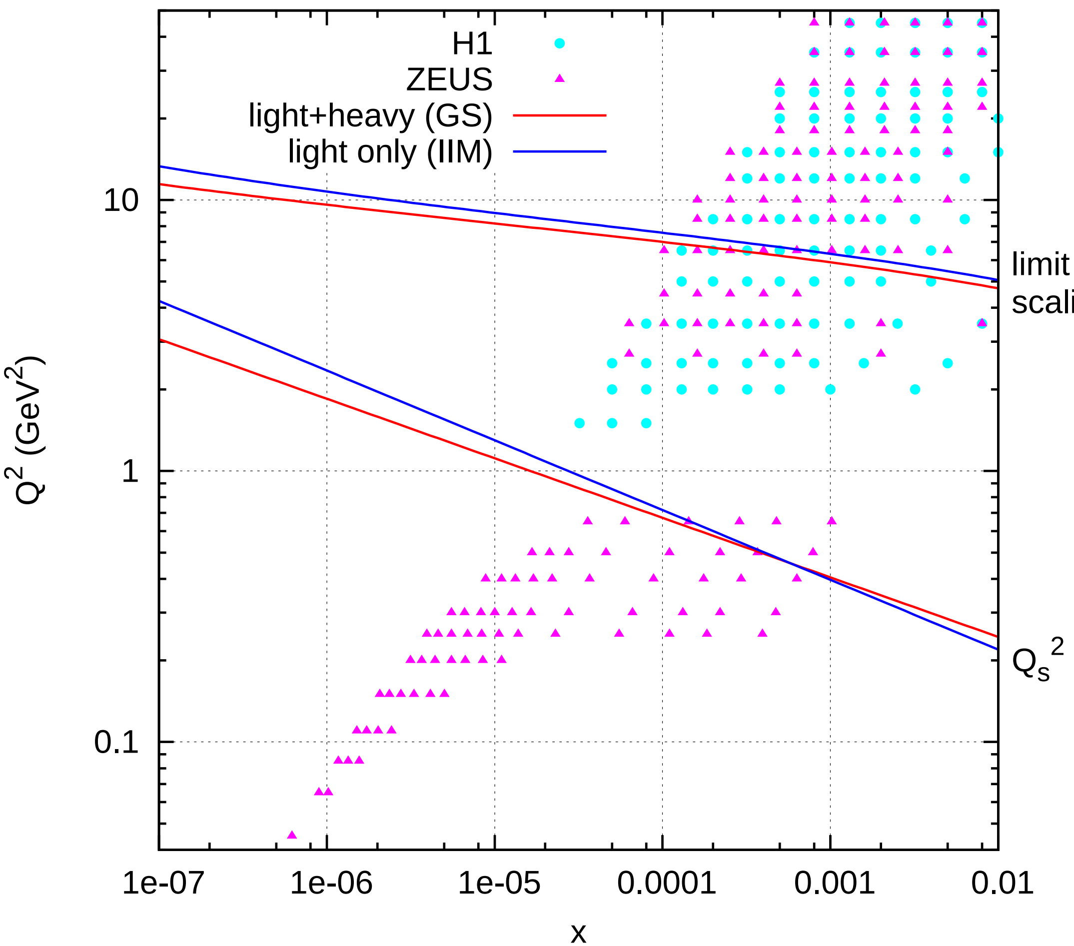
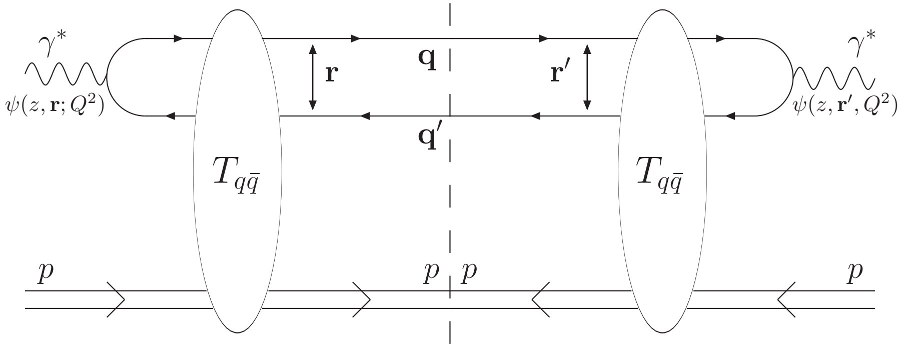


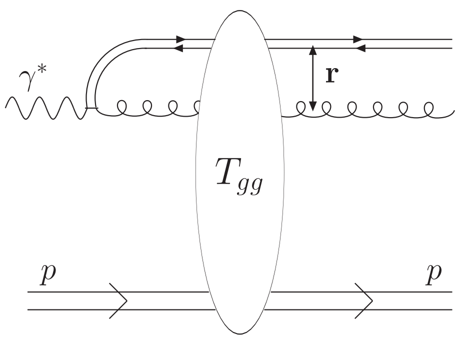
- $Q_s^{\mathrm{GS}} \approx Q_s^{\mathrm{IIM}}$: NO drop down
- Geometric scaling window
  $\equiv$ we “feel” saturation
  $\rightarrow$ up to $Q^2 \sim 5 - 7$ GeV^2

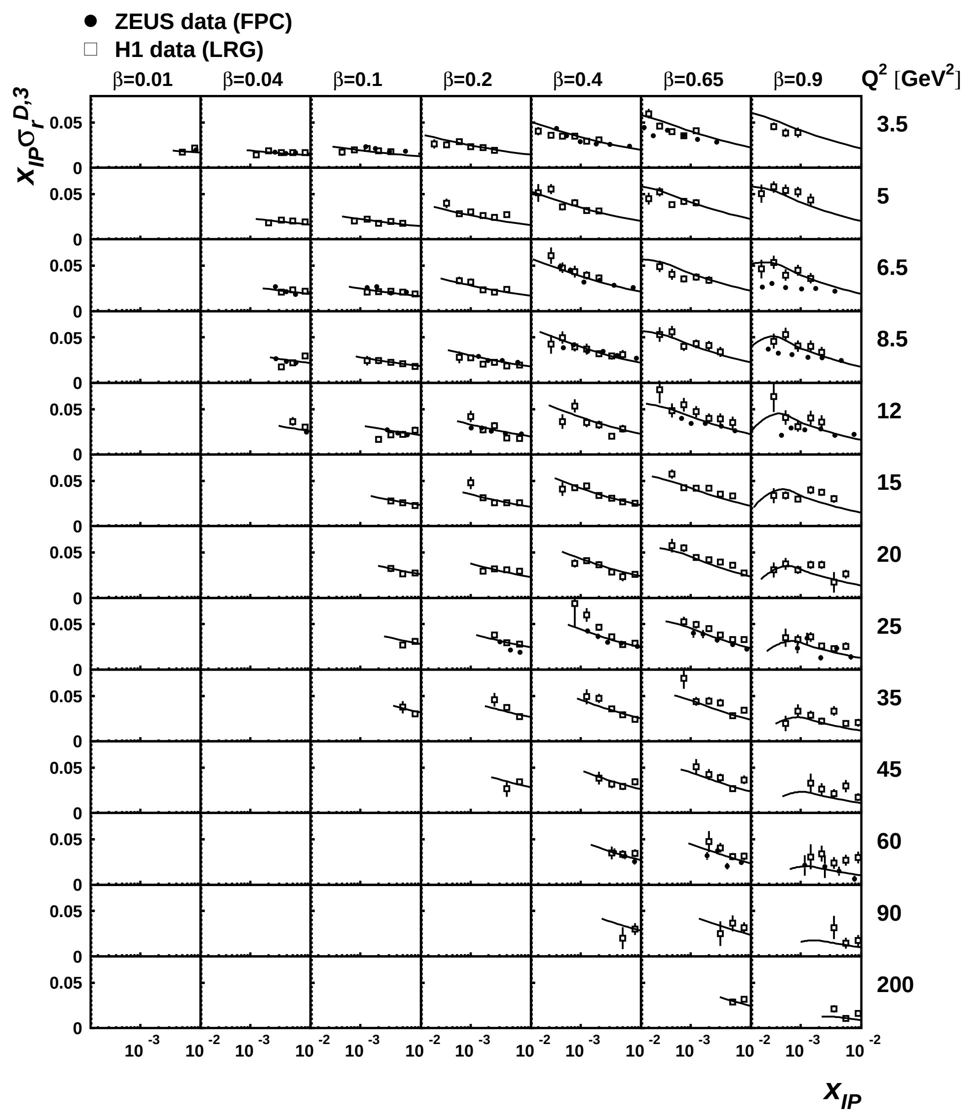
F_2^D

Same kind of factorisation
but more contribs:

$$F_2^D = F_2^{D(q\bar{q})} + F_2^{D(q\bar{q}g)} + \dots$$



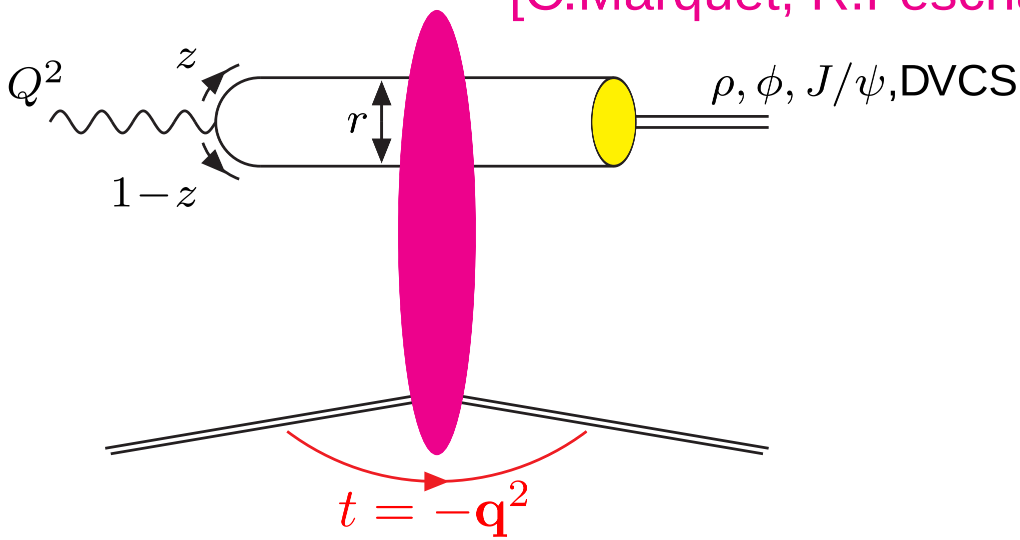




Basically: $F_2^D \propto T^2$
with the same T as for F_2

[Marquet, 07]

[C.Marquet, R.Peschanski, G.S., 07]



- Factorisation formula for;

$$\mathcal{A}_{L,T}^{\gamma^* p \to V p} = \Psi^{\gamma^*} \otimes \sigma_{\text{dip}}(\mathbf{r}, \mathbf{q}; Y) \otimes \Psi^{\text{VM}}$$

- t dependence from BK (NB: BK predicts t-dep, not b-dep)

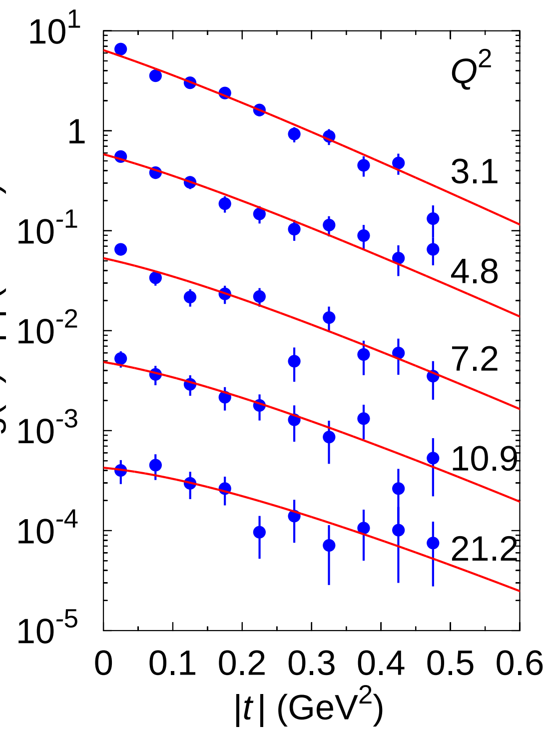
$$\sigma_{\text{dip}}(r, q; Y) = 2\pi R_p^2 e^{-b|t|}\, T_{\text{GS}}\left(r, Q_s^2(q, Y)\right)$$

$$Q_s^2 = Q_0^2\, e^{\lambda Y} \qquad \to \qquad Q_s^2 = Q_0^2\,(1 + c|t|)\, e^{\lambda Y}$$

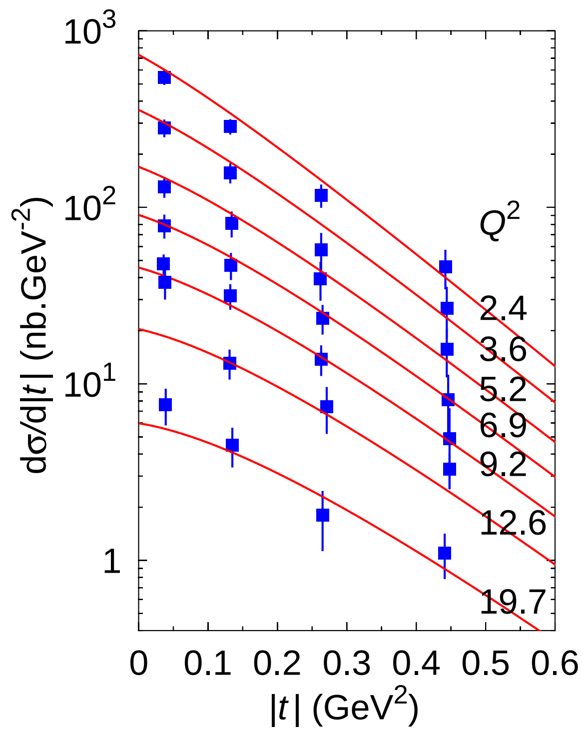
- b, c $\quad\to\quad$ $\frac{d\sigma}{dt}$, σ_{el}

Example: differential cross-section:

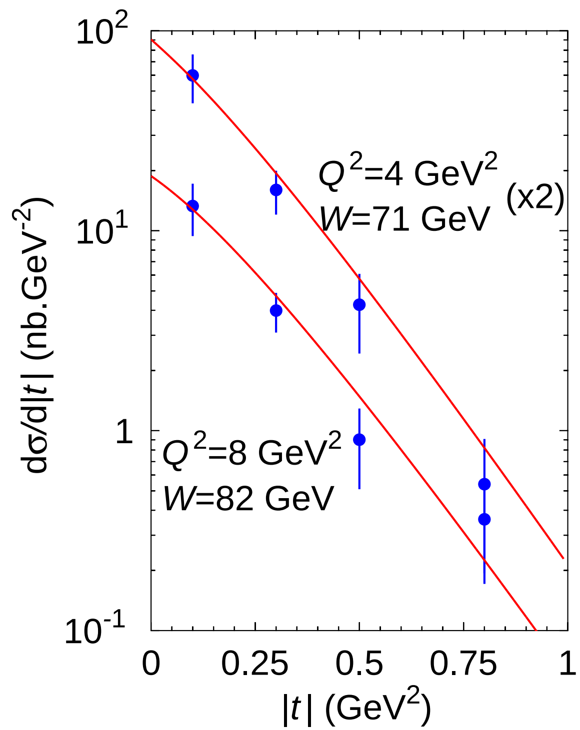
$\gamma^* p \to \rho p$



$\gamma^* p \to \phi p$



pred. for DVCS



Conclusions and perspectives

- Conclusions:
  - Geometric scaling at small x is predicted by BK
  - We can accomodate the IIM model to include heavy quarks and keep $Q_s \sim 1$ GeV
  - The resulting model can be used to describe F_2^D, exclusive VM prod. and DVCS powerful factorisation of the dipole model

- Perspectives:
  - Find a parametrisation for both HERA and RHIC
  - Predictions from NLO BK?
  - Matching with (some kind of) DGLAP evolution at large Q^2