

Saturation in High-Energy QCD

Scaling laws and phenomenological applications

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Based on : [G.S.](#), hep-ph/0504129, Phys. Rev. D72 (2005) 016007

[Y. Hatta](#), [E. Iancu](#), [C. Marquet](#), [G.S.](#), [D. Triantafyllopoulos](#), hep-ph/0601150, Nucl. Phys. A773 (2006) 95

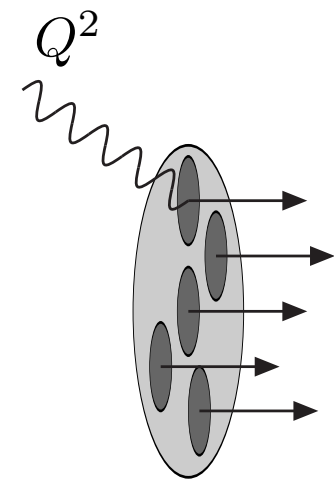
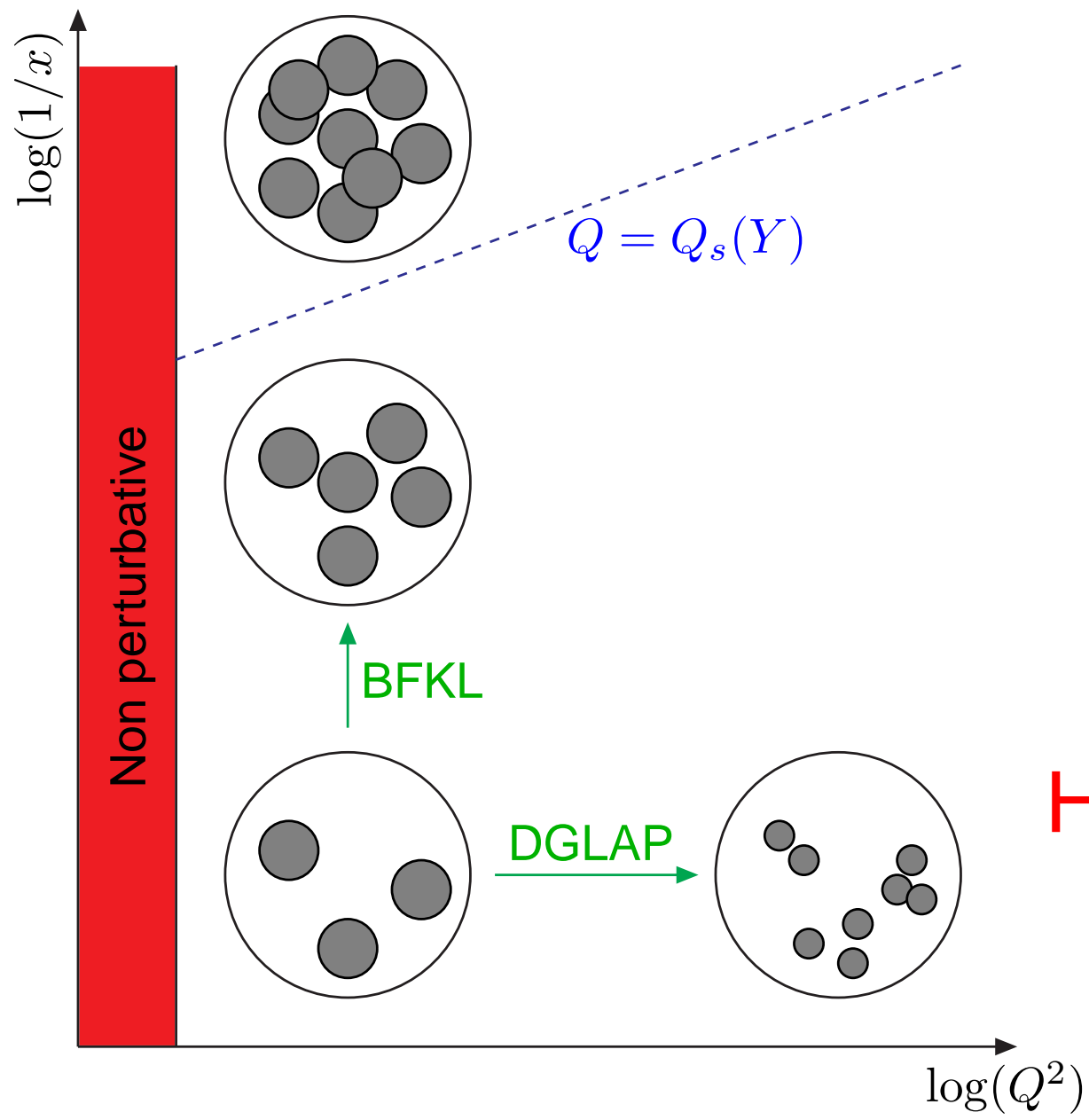
[E. Iancu](#), [C. Marquet](#), [G.S.](#), hep-ph/0605174, to appear in Nucl. Phys. A

[C. Marquet](#), [G.S.](#), [B-W. Xiao](#), hep-ph/0606233, Phys. Lett. B639 (2006) 635

[C. Marquet](#), [R. Peschanski](#), [G.S.](#), in preparation

- Perturbative evolution in high-energy QCD:
 - Leading log approx.: BFKL equation
 - Unitarity/Saturation effects: Balitsky/JIMWLK and BK equation
 - Fluctuation effects: towards a new evolution
- Asymptotic solutions:
 - saturation \Rightarrow geometric scaling
 - fluctuation \Rightarrow Stochastic evolution \Rightarrow Diffusive scaling
- Phenomenological consequences
 - Geometric scaling for F_2 and in vector meson production
 - Diffusive scaling in DIS, diffractive DIS and forward gluon production

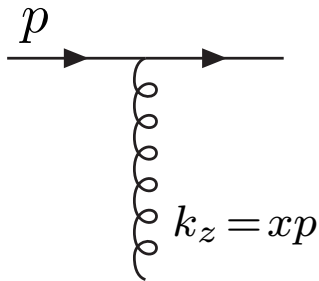
Motivation: why saturation ?



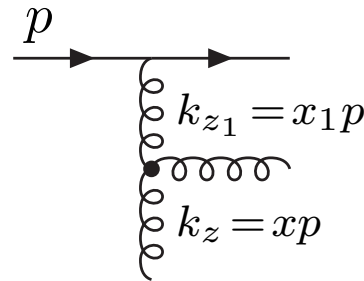
Size $\sim 1/Q$
 Energy $\sim Q^2/x$

How to describe this in QCD ?

Bremsstrahlung:



$$x \ll 1$$



$$x \ll x_1 \ll 1$$

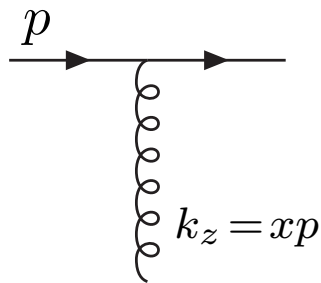
Probability of emission

$$dP \sim \alpha_s \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{dx}{x}$$

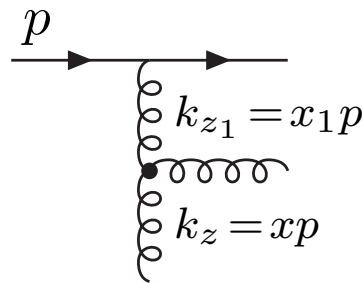
In the small- x limit

$$\int_x^1 \frac{dx_1}{x_1} \sim \alpha_s \log(1/x)$$

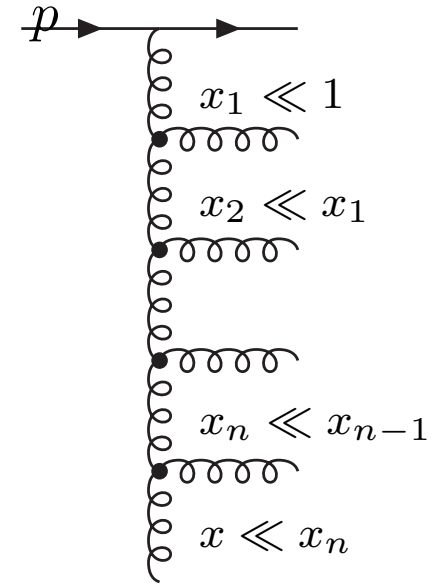
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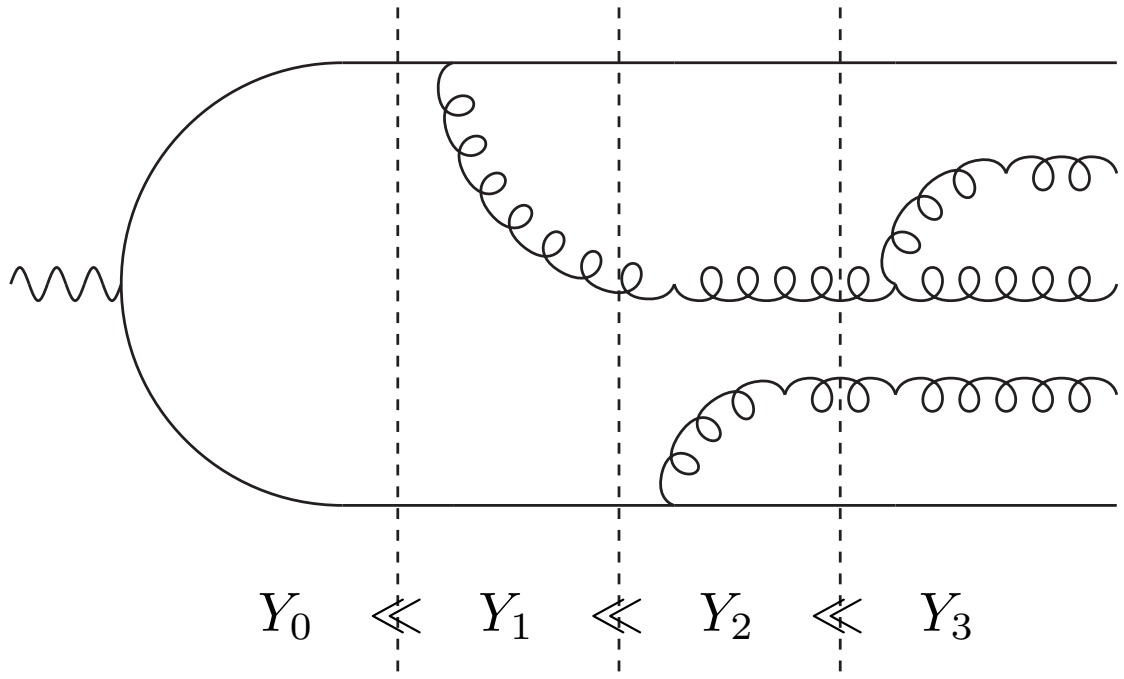
$$\int_x^1 \frac{dx_n}{x_n} \dots \int_{x_2}^1 \frac{dx_1}{x_1} \sim \frac{1}{n!} \alpha_s^n \log^n(1/x)$$

Same order when $\alpha_s \log(1/x) \sim 1$

Perturbative evolution in high-energy QCD

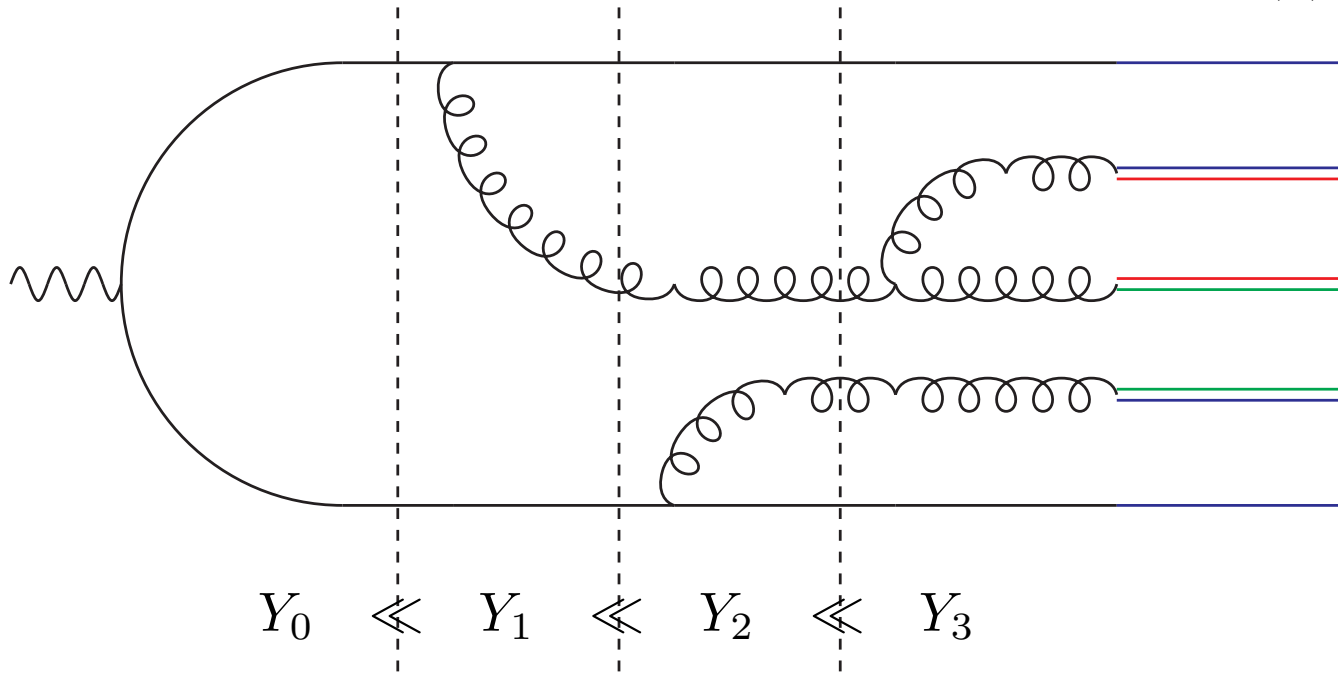
Consider a **fast-moving $q\bar{q}$ dipole** (Rapidity: $Y = \log(s)$)

[Mueller,93]



- Probability $\bar{\alpha}K$ of emission
- Independent emissions in coordinate space (transverse plane)

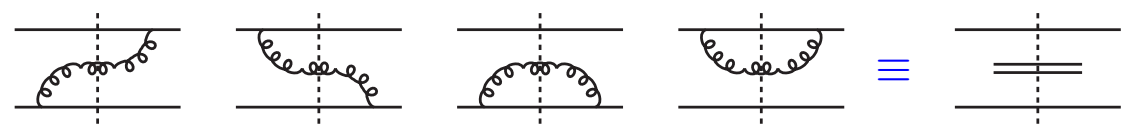
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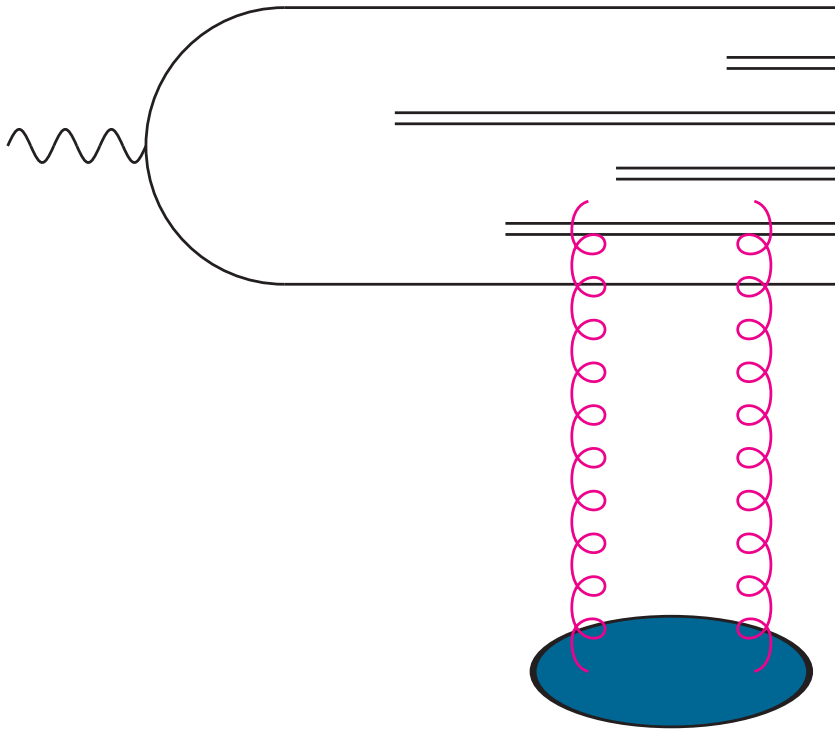
[Mueller,93]

$n(r, Y)$ dipoles
of size r

- Probability $\bar{\alpha}K$ of emission
- Independent emissions in coordinate space (transverse plane)
- Large- N_c approximation



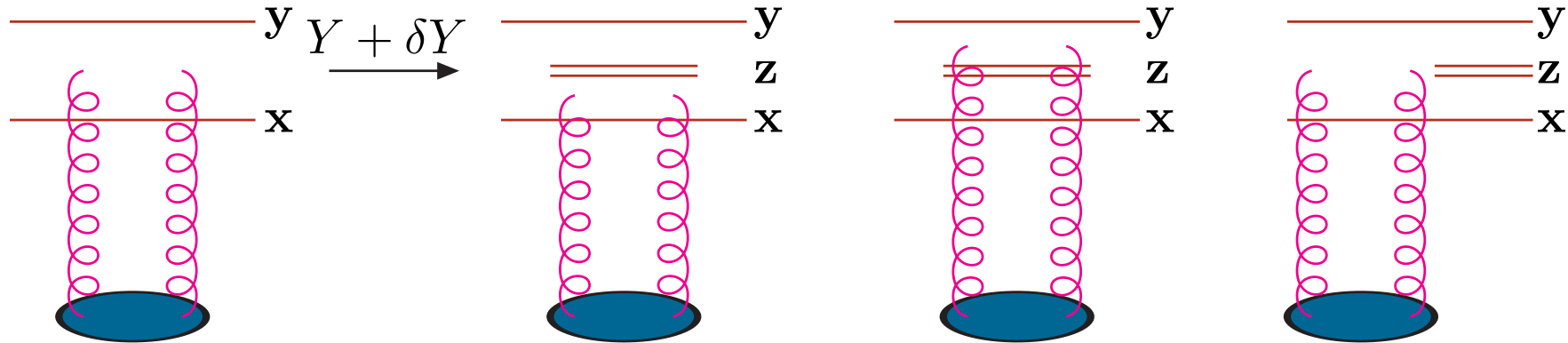
How to observe this system ?



$$T(r, Y) \approx \alpha_s^2 n(r, Y)$$

Count the number of dipoles of a given size

Consider a small increase in rapidity \Rightarrow **splitting**



$$\partial_Y T(\mathbf{x}, \mathbf{y}; Y)$$

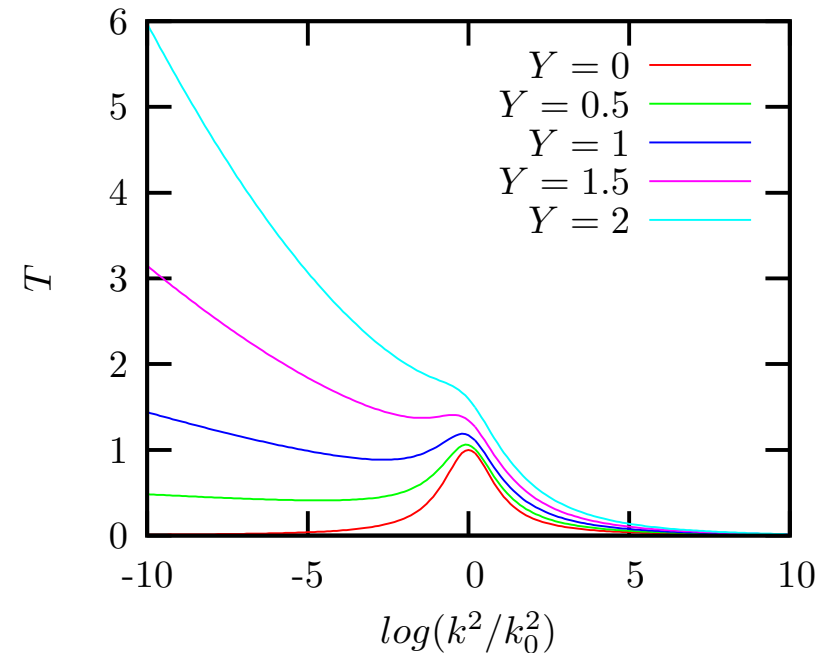
$$= \bar{\alpha} \int d^2 z \underbrace{\frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2}}_{\text{Emission proba from pQCD}} \underbrace{[T(\mathbf{x}, \mathbf{z}; Y) + T(\mathbf{z}, \mathbf{y}; Y) - T(\mathbf{x}, \mathbf{y}; Y)]}_{\text{all possible interactions}}$$

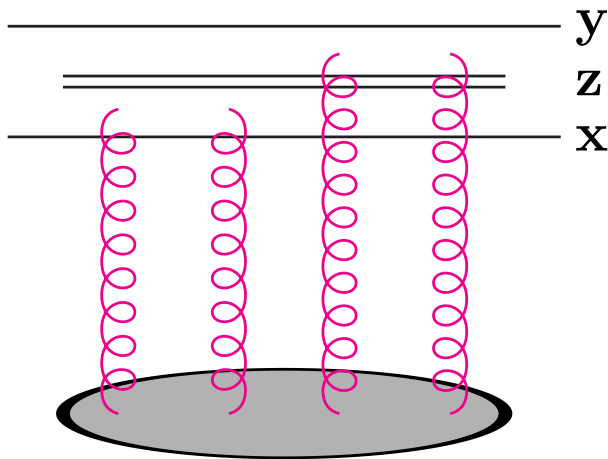
[Balitsky, Fadin, Kuraev, Lipatov, 78]

The solution goes like

$$T(Y) \sim e^{\omega Y} \quad \text{with} \quad \omega = 4\bar{\alpha} \log(2) \approx 0.5$$

- Fast growth of the amplitude
- Intercept value too large
- Violation of the Froissart unitarity:
 $T(Y) \leq C \log^2(s) \quad T(r, b) \leq 1$
- problem of diffusion in the infrared





Multiple scattering

- ★ Proportional to T^2
- ★ important when $T \approx 1$

$\langle \cdot \rangle \equiv$ average over target field

$$\begin{aligned} & \partial_Y \langle T(\mathbf{x}, \mathbf{y}; Y) \rangle \\ &= \bar{\alpha} \int d^2 z \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2} [\langle T(\mathbf{x}, \mathbf{z}; Y) \rangle + \langle T(\mathbf{z}, \mathbf{y}; Y) \rangle - \langle T(\mathbf{x}, \mathbf{y}; Y) \rangle \\ & \quad - \langle T(\mathbf{x}, \mathbf{z}; Y) T(\mathbf{z}, \mathbf{y}; Y) \rangle] \end{aligned}$$

But

$\partial_Y \langle T(\mathbf{x}, \mathbf{y}; Y) \rangle$ contains a new object: $\langle T(\mathbf{x}, \mathbf{z}; Y) T(\mathbf{z}, \mathbf{y}; Y) \rangle$

In general: complete hierarchy

[Balitsky, 96]

$$\partial_Y \langle T^k \rangle \longrightarrow \underbrace{\langle T^k \rangle}_{\text{BFKL}}, \underbrace{\langle T^{k+1} \rangle}_{\text{saturation}}$$

- Beyond large- N_c : the hierarchy involves quadrupoles, sextupoles, ...
- Balitsky hierarchy \equiv JIMWLK eq. (Colour Glass Condensate formalism)

Mean field approx.: $\langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle = \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle$

$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int d^2 z \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2} [\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle]$$

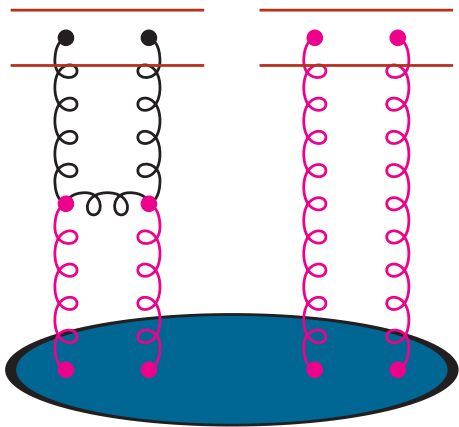
[Balitsky 96, Kovchegov 99]

Simplest perturbative evolution equation satisfying unitarity constraint

Consider evolution of $\langle T^{(2)} \rangle$

[E. Iancu, D. Triantafyllopoulos, 05]

Also A. Mueller, S. Munier, A. Shoshi, S. Wong



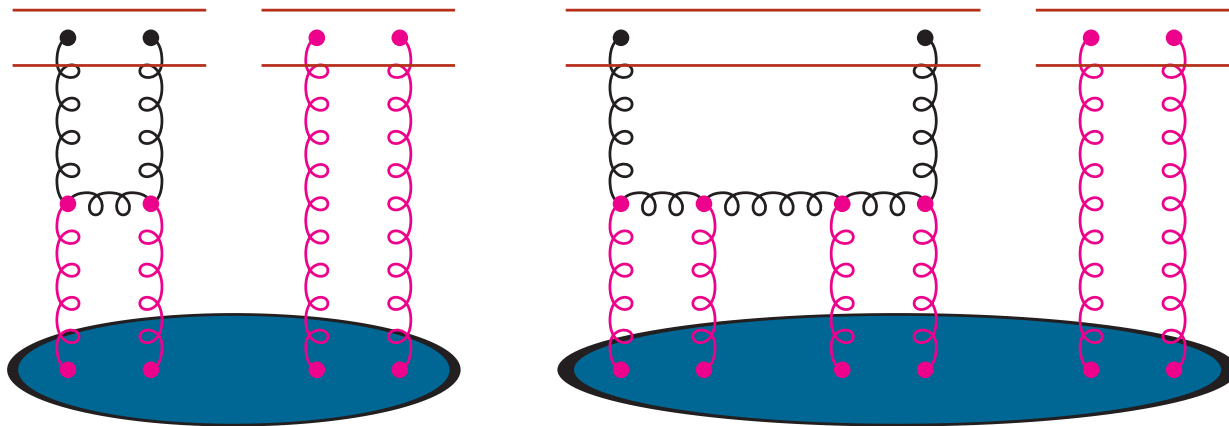
● Usual BFKL ladder

$$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$$

Consider evolution of $\langle T^{(2)} \rangle$

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- Usual BFKL ladder
- fan diagram \longrightarrow saturation effects

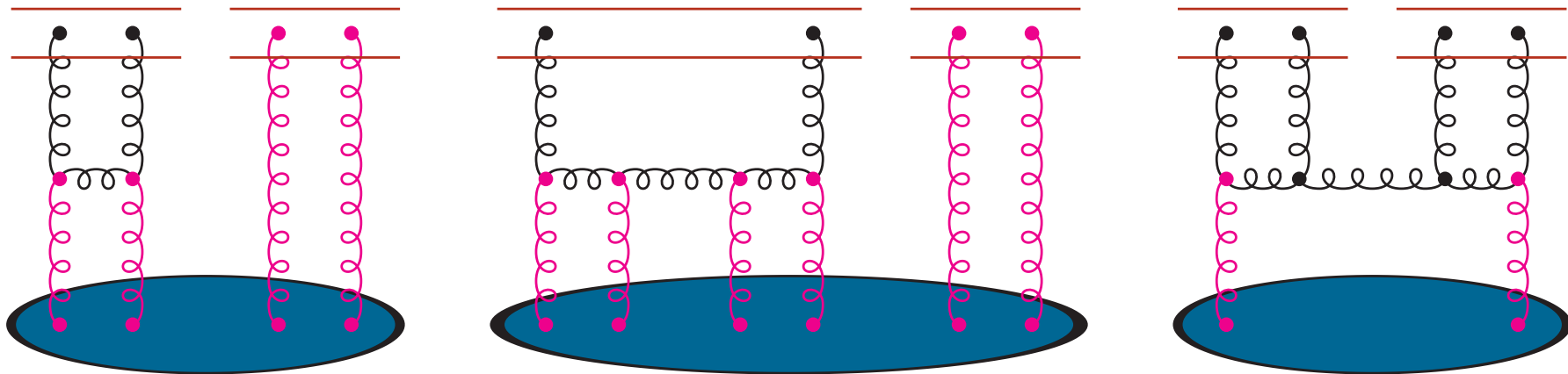
$$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$$

$$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(3)} \rangle$$

Consider evolution of $\langle T^{(2)} \rangle$

[E. Iancu, D. Triantafyllopoulos, 05]

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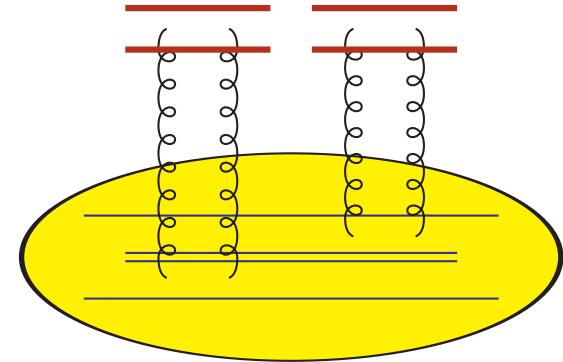
- Usual BFKL ladder
- fan diagram \longrightarrow saturation effects
- splitting \longrightarrow gluon-number fluctuations
 \longrightarrow pomeron loops

$$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$$

$$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(3)} \rangle$$

$$\partial_Y \langle T^{(2)} \rangle \propto \langle T \rangle$$

⇒ complicated hierarchy



$$\begin{aligned}
 & \partial_Y \langle T^{(2)}(\mathbf{x}_1, \mathbf{y}_1; \mathbf{x}_2, \mathbf{y}_2) \rangle \\
 &= \frac{\bar{\alpha}}{2\pi} \int d^2z \frac{(\mathbf{x}_2 - \mathbf{y}_2)^2}{(\mathbf{x}_2 - \mathbf{z})^2 (\mathbf{z} - \mathbf{y}_2)^2} \left[\langle T^{(2)}(\mathbf{x}_1, \mathbf{y}_1; \mathbf{x}_2, \mathbf{z}) \rangle + \langle T^{(2)}(\mathbf{x}_1, \mathbf{y}_1; \mathbf{z}, \mathbf{y}_2) \rangle \right. \\
 & \quad \left. - \langle T^{(2)}(\mathbf{x}_1, \mathbf{y}_1; \mathbf{x}_2, \mathbf{y}_2) \rangle - \langle T^{(3)}(\mathbf{x}_1, \mathbf{y}_1; \mathbf{x}_2, \mathbf{z}; \mathbf{z}, \mathbf{y}_2) \rangle + (1 \leftrightarrow 2) \right] \\
 &+ \frac{\bar{\alpha}}{2\pi} \left(\frac{\alpha_s}{2\pi} \right)^2 \int_{\mathbf{u}\mathbf{v}\mathbf{z}} \mathcal{M}_{\mathbf{u}\mathbf{v}\mathbf{z}} \mathcal{A}_0(\mathbf{x}_1 \mathbf{y}_1 | \mathbf{u}\mathbf{z}) \mathcal{A}_0(\mathbf{x}_2 \mathbf{y}_2 | \mathbf{z}\mathbf{v}) \nabla_{\mathbf{u}}^2 \nabla_{\mathbf{v}}^2 \langle T^{(1)}(\mathbf{u}, \mathbf{v}) \rangle
 \end{aligned}$$

- **Saturation:** important when $T^{(2)} \sim T^{(1)} \sim 1$ i.e. **near unitarity**
- **Fluctuations:** important when $T^{(2)} \sim \alpha_s^2 T^{(1)}$ or $T \sim \alpha_s^2$ i.e. **dilute regime**
- **Langevin formulation:** fluctuation = noise

Solutions

The BK equation

Case 1: no impact parameter dependence

$$T_{\mathbf{xy}} \rightarrow T \left(\mathbf{r} = \mathbf{x} - \mathbf{y}, \mathbf{b} = \frac{\mathbf{x} + \mathbf{y}}{2} \right) \rightarrow T(r)$$

Note:

- all arguments work for $T(r)$ or its Fourier transform $\tilde{T}(k)$
- for \tilde{T} , the non-linear term is simply $-\tilde{T}^2(k)$

BK equation: $\partial_Y T = \underbrace{\chi(-\partial_L) T}_{\text{BFKL}} - T^2$

When $T \ll 1$ BFKL works: $\partial_Y T = \chi(-\partial_L) T$

Solution known:

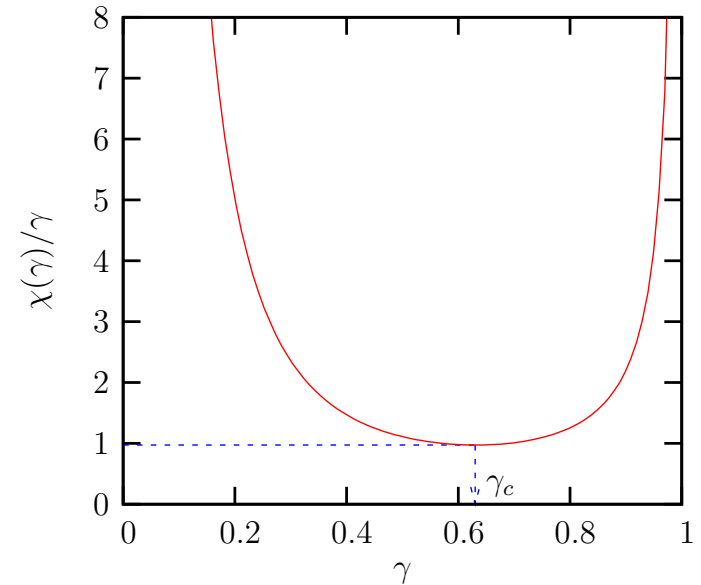
$$T(k) = \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp[\chi(\gamma)\bar{\alpha}Y - \gamma L]$$

$$= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[-\gamma\left(L - \frac{\chi(\gamma)}{\gamma}\bar{\alpha}Y\right)\right]$$

⇒ Wave of slope γ travels at speed $v = \chi(\gamma)/\gamma$

$$Y = Y_0$$

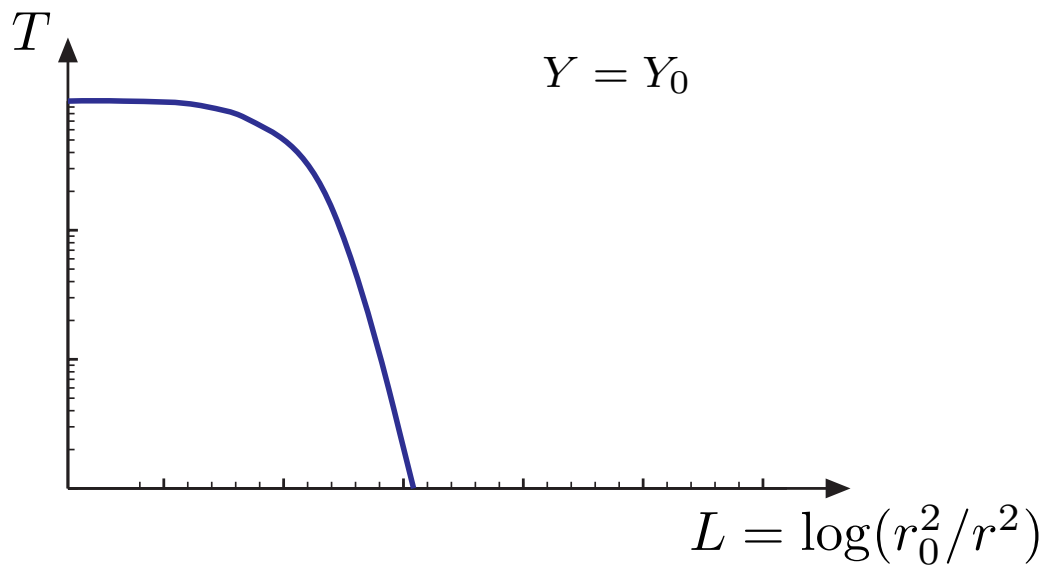
[S.Munier,R.Peschanski,03]



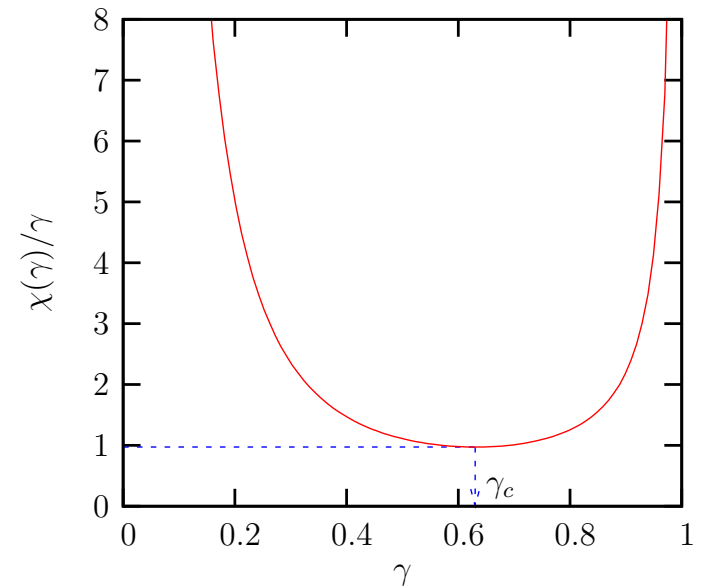
$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

BK equation: $\partial_Y T = \underbrace{\chi(-\partial_L) T}_{\text{BFKL}} - T^2$

\Rightarrow Wave of slope γ travels at speed $v = \chi(\gamma)/\gamma$



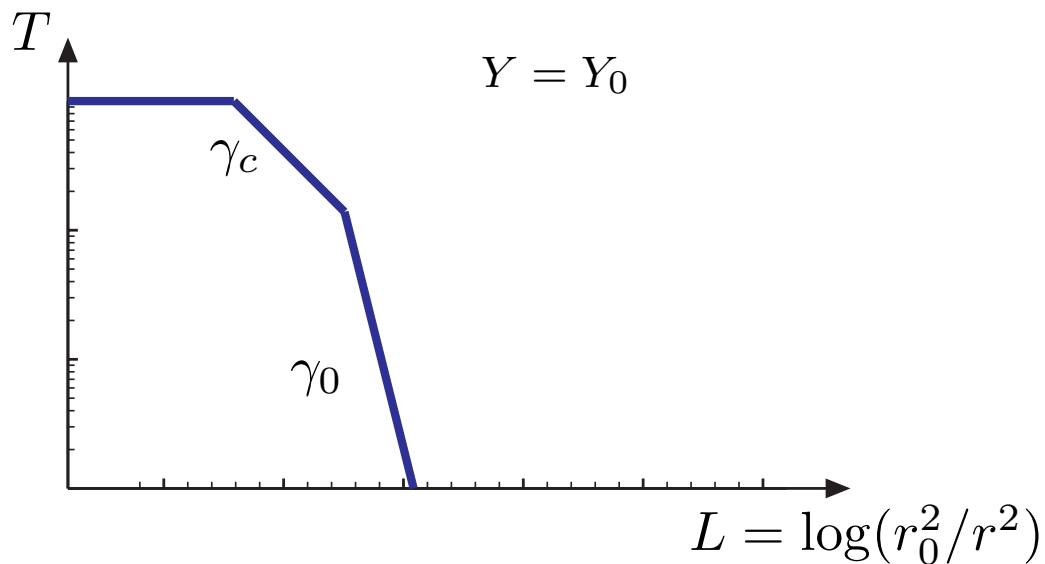
[S.Munier,R.Peschanski,03]



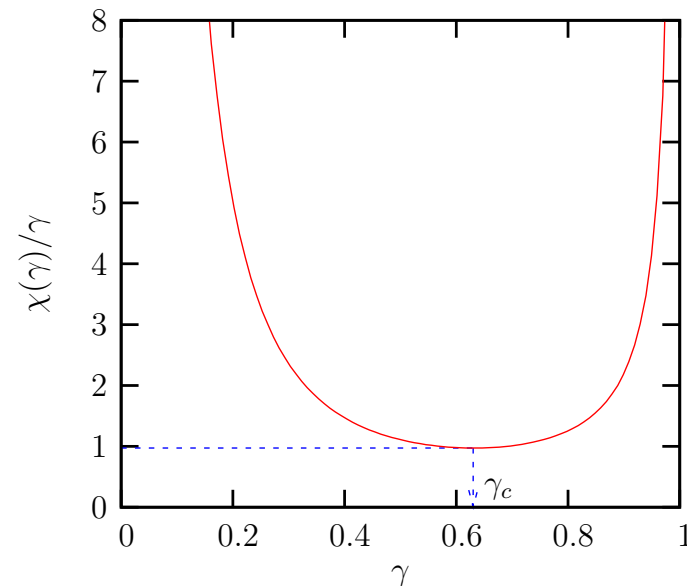
$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

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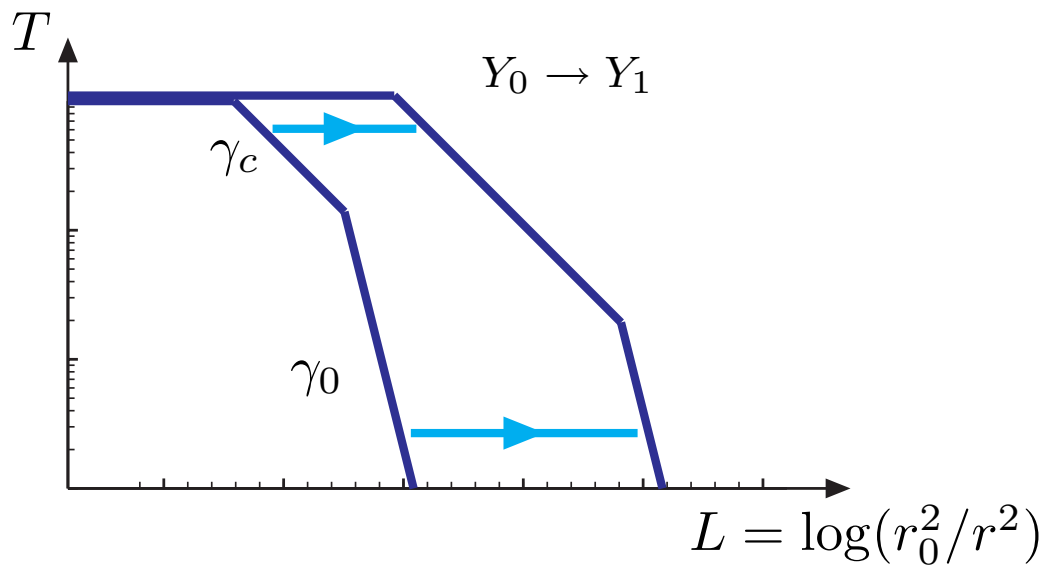
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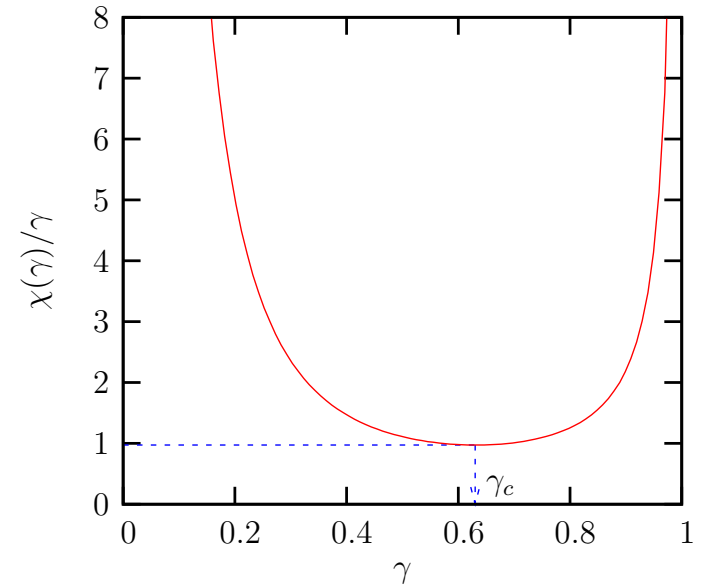
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⇒ Wave of slope γ travels at speed $v = \chi(\gamma)/\gamma$



[S.Munier,R.Peschanski,03]



$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

The minimal speed is selected during evolution

Consequence: **geometric scaling** ($Q_s \equiv$ saturation scale \equiv front position)

$$T(r, Y) = T[rQ_s(Y)] \quad \text{with} \quad Q_s^2(Y) = v_c \bar{\alpha} Y$$

$$rQ_s \underset{=}{\ll} 1 \quad \underbrace{[r^2 Q_s^2(Y)]^{\gamma_c}}_{\text{slope } \gamma_c} \quad \underbrace{\exp \left[\frac{\log^2(r^2 Q_s^2)}{2\chi''(\gamma_c) \bar{\alpha} Y} \right]}_{\text{scaling window}}$$

$$|\log(r^2 Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c) \bar{\alpha} Y}$$

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$$|\log(r^2 Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c) \bar{\alpha} Y}$$

- Generic arguments: exponential rise + saturation \Rightarrow select γ_c
- Parameters fixed by linear kernel only
- Saturation effects even though $T \ll 1$

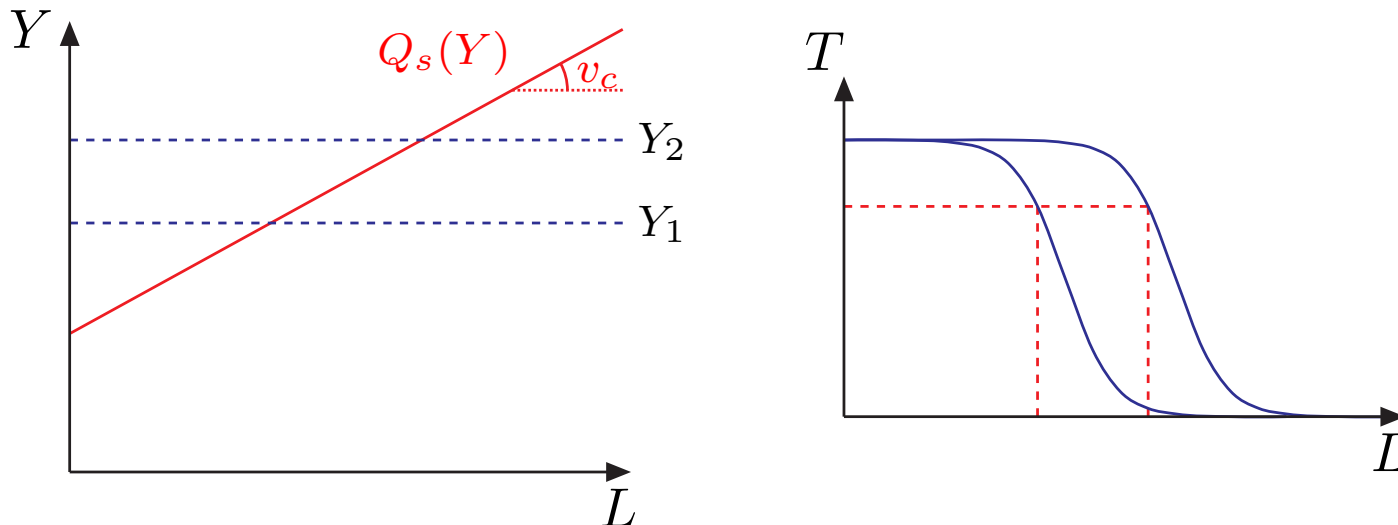
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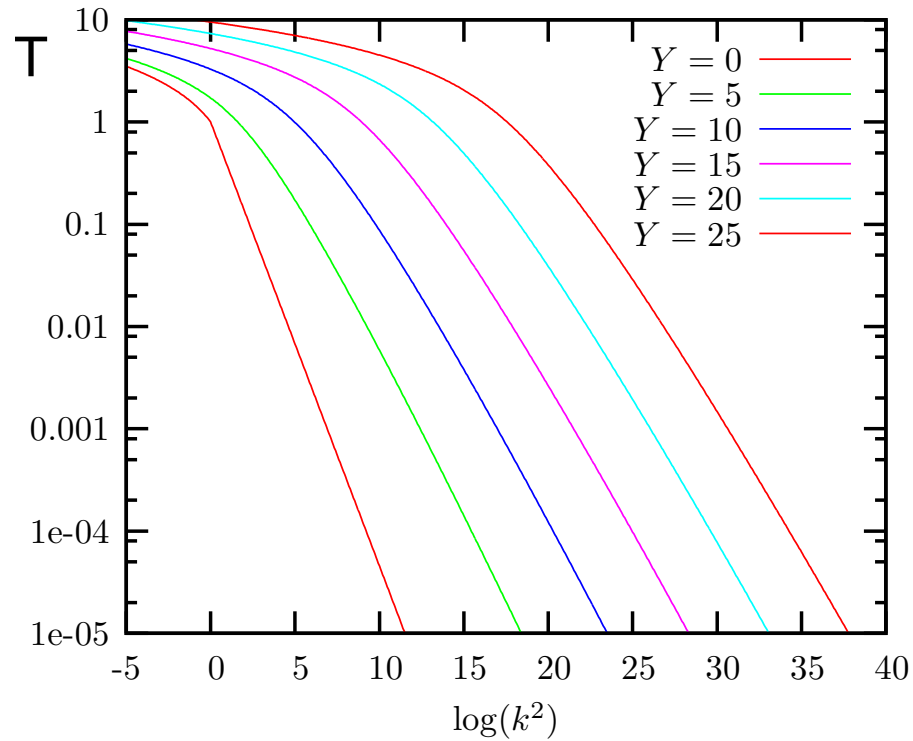
$$\stackrel{rQ_s \ll 1}{=} \underbrace{[r^2 Q_s^2(Y)]^{\gamma_c}}_{\text{slope } \gamma_c} \underbrace{\exp \left[\frac{\log^2(r^2 Q_s^2)}{2\chi''(\gamma_c) \bar{\alpha} Y} \right]}_{\text{scaling window}}$$

$$|\log(r^2 Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c) \bar{\alpha} Y}$$

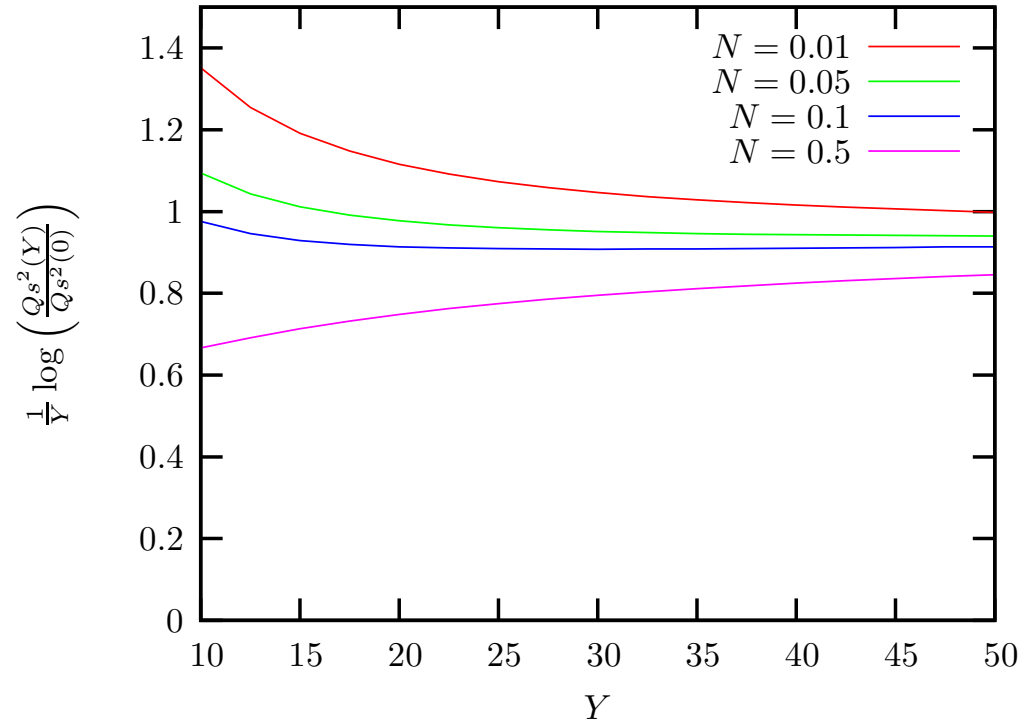
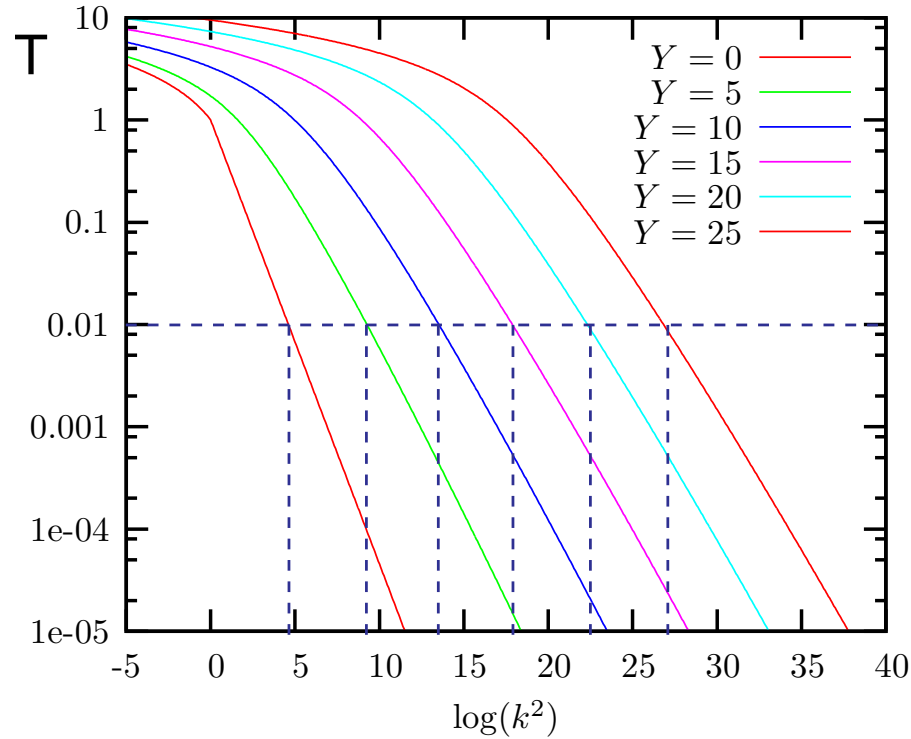
Interpretation: **invariance along the saturation line**



Numerical simulations:



Numerical simulations:



$$T(k, Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \approx \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c}$$

$$Q_s^2(Y) \propto \exp(v_c Y)$$

Case 2: including impact parameter

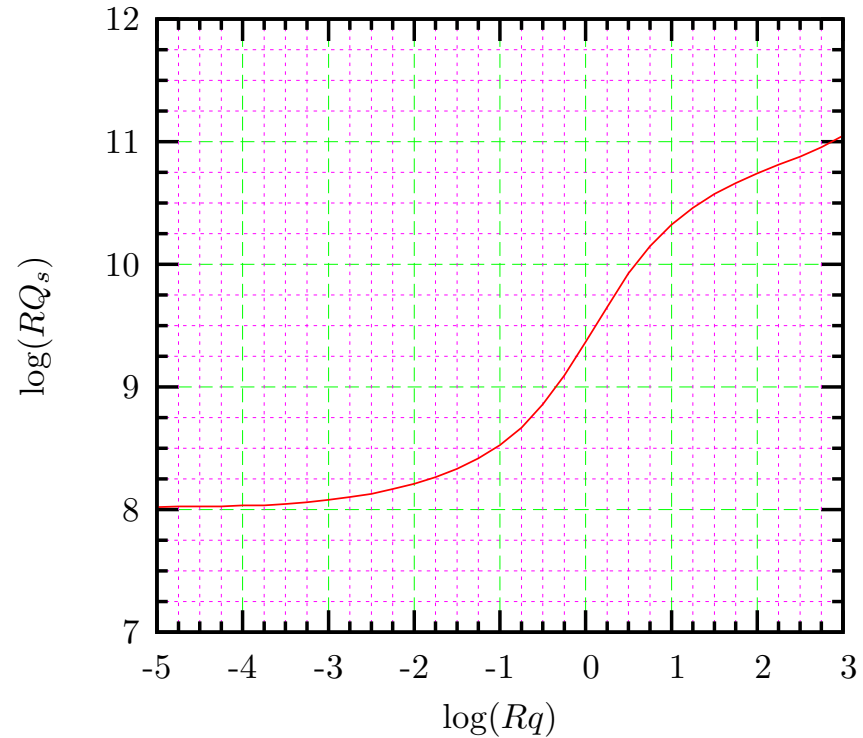
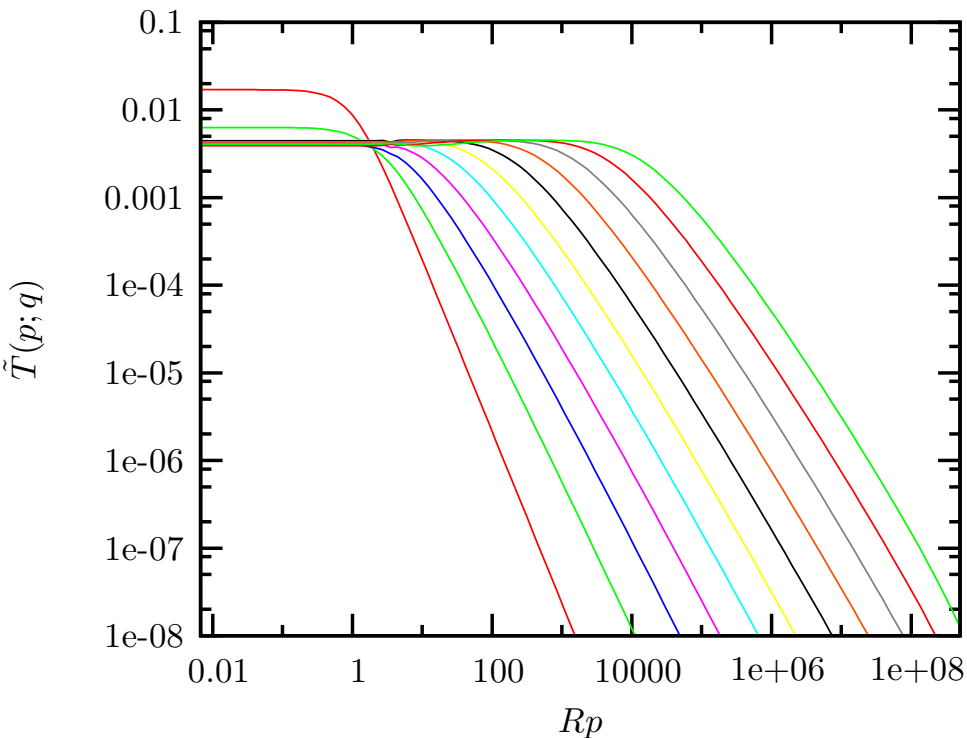
Go to momentum space: use momentum transfer \mathbf{q}

$$\tilde{T}(\mathbf{k}, \mathbf{q}) = \int d^2x d^2y e^{i\mathbf{k}\cdot\mathbf{x}} e^{i(\mathbf{q}-\mathbf{k})\cdot\mathbf{y}} \frac{T(\mathbf{x}, \mathbf{y})}{(\mathbf{x} - \mathbf{y})^2}$$

new form of the BK equation

$$\begin{aligned} \partial_Y \tilde{T}(\mathbf{k}, \mathbf{q}) &= \frac{\bar{\alpha}}{\pi} \int \frac{d^2k'}{(k - k')^2} \left\{ \tilde{T}(\mathbf{k}', \mathbf{q}) - \frac{1}{4} \left[\frac{k^2}{k'^2} + \frac{(q - k)^2}{(q - k')^2} \right] \tilde{T}(\mathbf{k}, \mathbf{q}) \right\} \\ &- \frac{\bar{\alpha}}{2\pi} \int d^2k' \tilde{T}(\mathbf{k}, \mathbf{k}') \tilde{T}(\mathbf{k} - \mathbf{k}', \mathbf{q} - \mathbf{k}') \end{aligned}$$

[C.Marquet, R.Peschanski, G.S., 05]



One can prove **analytically** that:

- traveling wave at large k : BFKL \Rightarrow **same** γ_c, v_c
- q dependence: $Q_s^2(q, Y) = \min(k_0^2, q^2) \exp(v_c Y)$

Predicts geometric scaling for t -dependent processes

Solutions

Fluctuation effects

no b -dependence + coarse-graining (local fluctuations) \longrightarrow Langevin equation

$$\partial_Y T(k, Y) = \bar{\alpha} K_{\text{BFKL}} \otimes T(k, Y) - \bar{\alpha} T^2(k, Y) + \bar{\alpha} \sqrt{\kappa \alpha_s^2 T(k, Y)} \nu(k, Y)$$

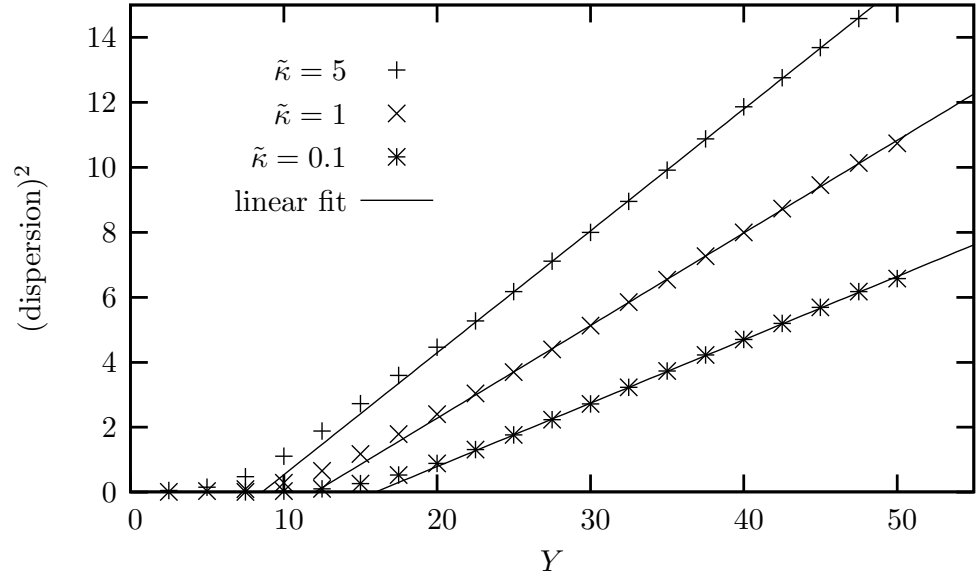
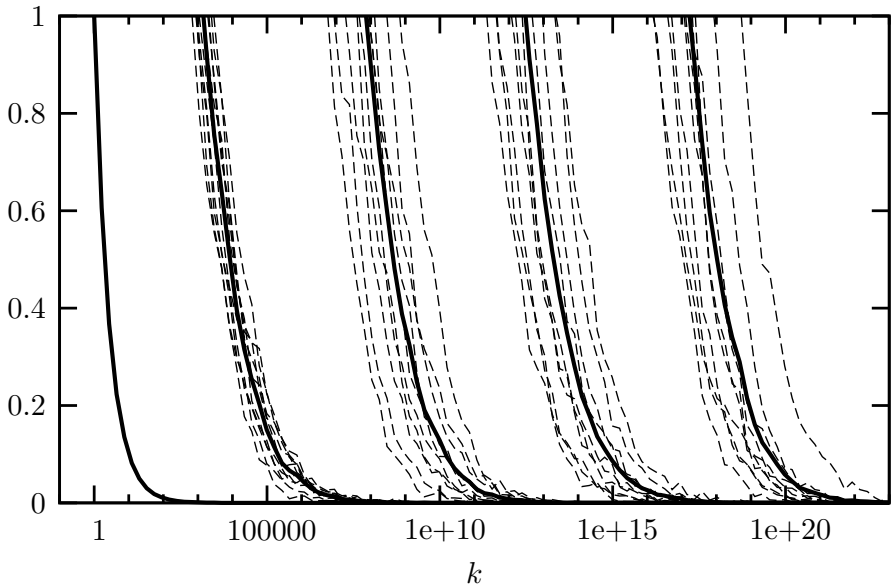
with a Gaussian white noise $\langle \nu(k, Y) \rangle = 0$

$$\langle \nu(k, Y) \nu(k', Y') \rangle = \frac{1}{\bar{\alpha}} \delta(Y - Y') k \delta(k - k')$$

Remarks:

- noise \equiv fluct. target field \Rightarrow Different events \equiv different target fields
- stochasticity as seen in detectors
- observables obtained by averaging over events

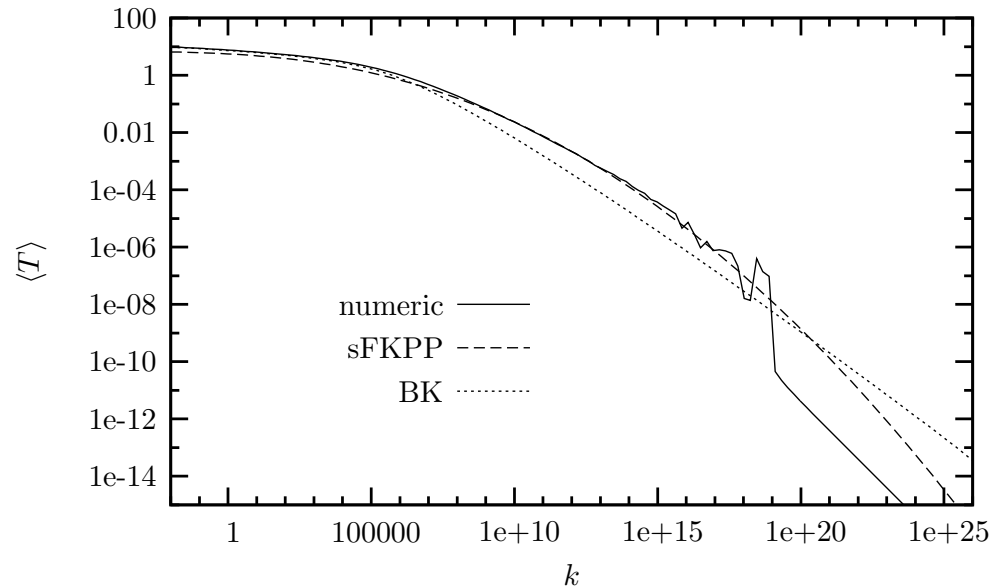
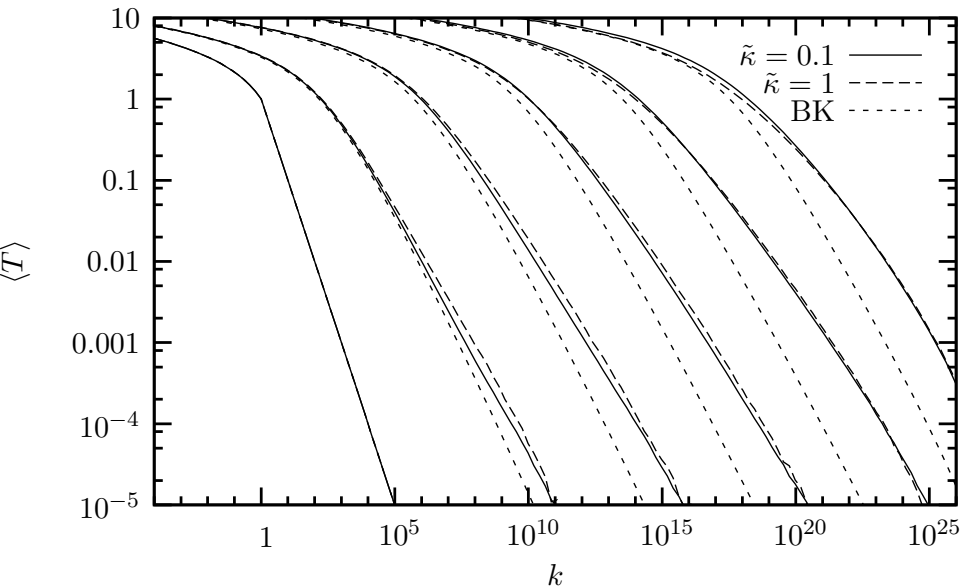
[G.S., 05]



- Traveling wave/Geometric scaling for each event
- Dispersion of the events

$$\Delta \log[Q_s^2(Y)] \approx \sqrt{D_{\text{diff}} \bar{\alpha} Y} \quad \text{with} \quad D_{\text{diff}} \underset{\alpha_s^2 \kappa \rightarrow 0}{\sim} \frac{1}{|\log^3(\alpha_s^2 \kappa)|}$$

[G.S., 05]



- Clear effect of fluctuations: dispersion \Rightarrow spreading
- Violations of geometric scaling
- Agrees with predictions from statistical mechanics (sFKPP)

Evolution with saturation & fluctuations \equiv

- superposition of unitary front (with geometric scaling)
- + dispersion of their position i.e. stochastic Q_s (geom. scaling violations)

$$\langle T(r, Y) \rangle = \int d\rho_s T_{\text{event}}(\rho - \rho_s) P(\rho_s)$$

with $\rho = \log(1/r^2)$, $\rho_s = \log(Q_s^2)$

Evolution with saturation & fluctuations \equiv

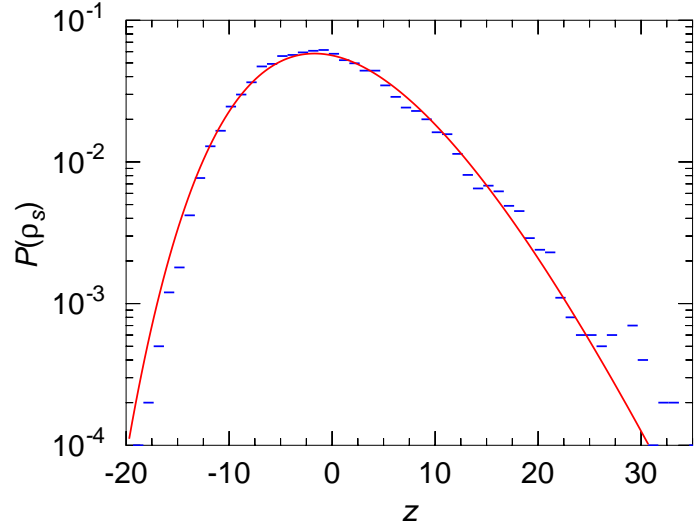
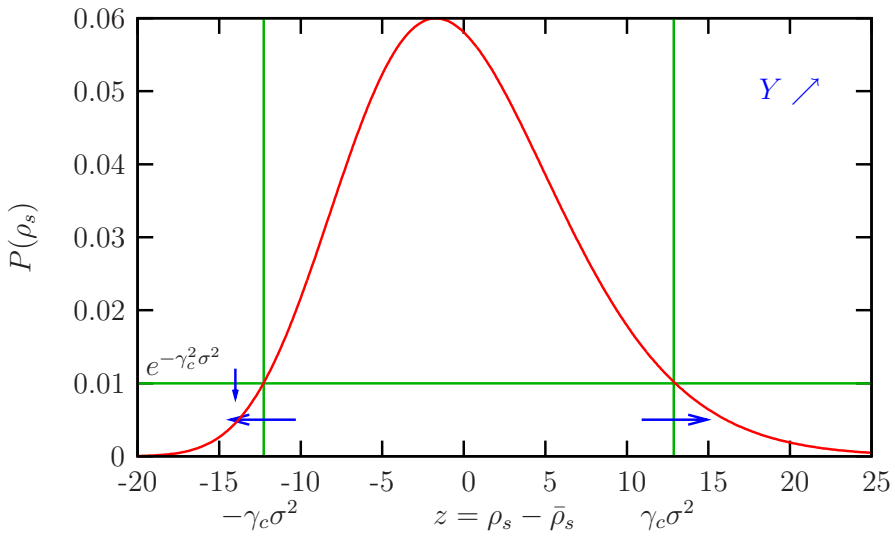
- superposition of unitary front (with geometric scaling)
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$$\langle T(r, Y) \rangle = \int d\rho_s T_{\text{event}}(\rho - \rho_s) \frac{1}{\sqrt{\pi}\sigma} \exp\left(-\frac{(\rho_s - \bar{\rho}_s)^2}{\sigma^2}\right)$$

with $\rho = \log(1/r^2)$, $\rho_s = \log(Q_s^2)$

[C.Marquet, G.S., B.Xiao, 06]

$P(\rho_s)$ can be taken as Gaussian: mean $\bar{\rho}_s \sim \lambda Y$, dispersion $\sigma^2 \sim DY$



Evolution with saturation & fluctuations \equiv

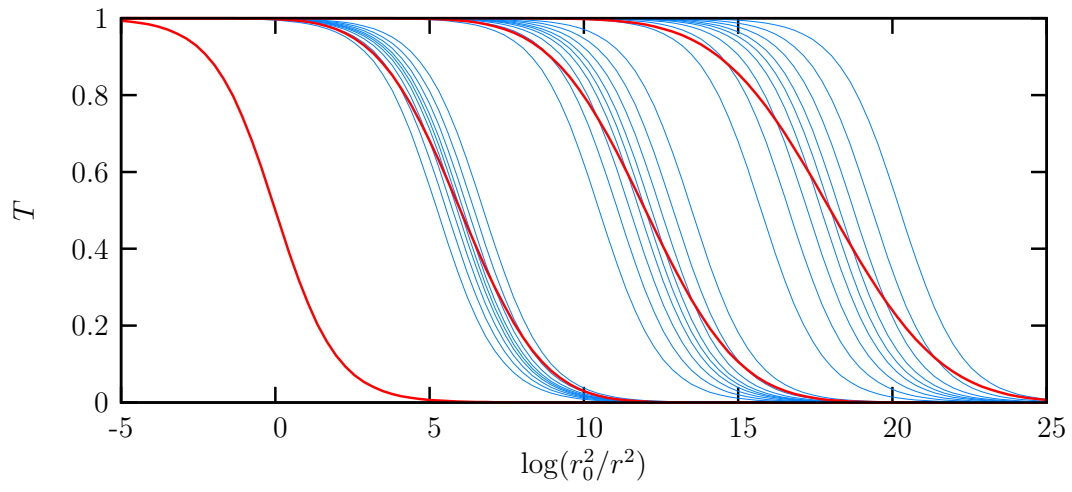
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with $\rho = \log(1/r^2)$, $\rho_s = \log(Q_s^2)$

$$T_{\text{event}}(\rho - \rho_s) = \begin{cases} 1 & r > Q_s & \text{saturation} \\ (r^2 Q_s^2)^\gamma & r < Q_s & \text{geometric scaling} \end{cases}$$

[E. Iancu, Y. Hatta, C. Marquet, G.S., D. Triantafyllopoulos, 06]

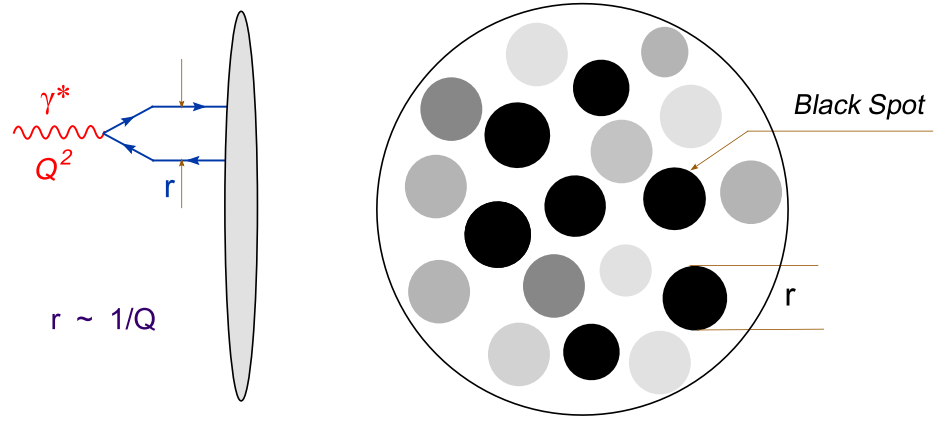


dispersion $\sim DY$

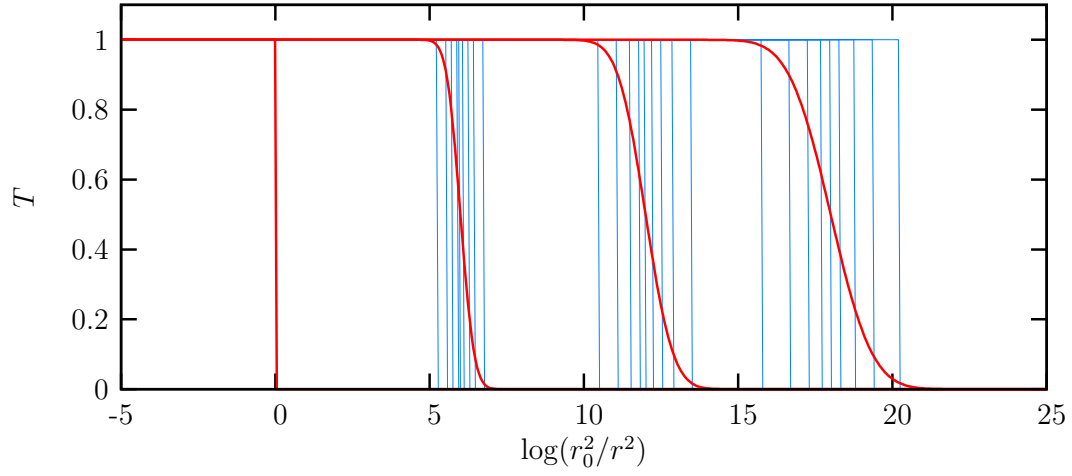
Case 1: Y not too large \Rightarrow small dispersion \Rightarrow Mean field picture $\langle T \rangle \approx T_{\text{event}}$
 \Rightarrow **geometric scaling**:

$$\langle T \rangle = f [\log(k^2/Q_s^2)]$$

$$\langle T^{(k)} \rangle = \langle T \rangle^k$$



[E. Iancu, Y. Hatta, C. Marquet, G.S., D. Triantafyllopoulos, 06]

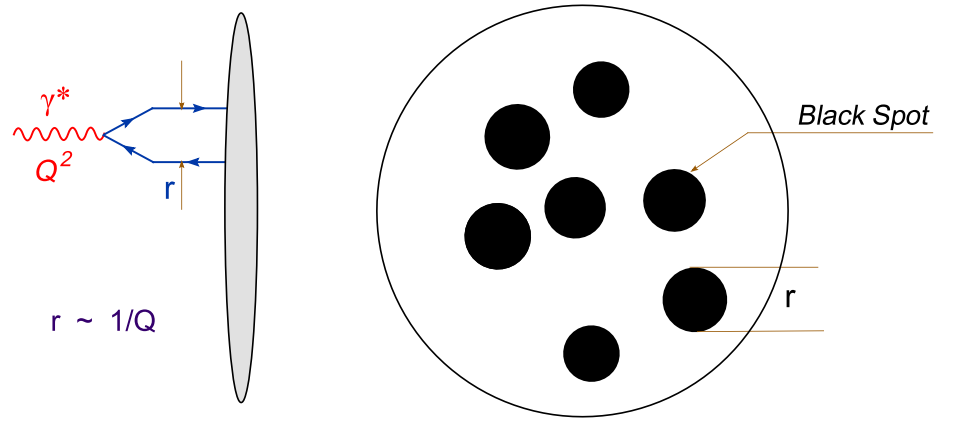


dispersion $\sim DY$

Case 2: Y higher energy \Rightarrow dominated by dispersion $\Rightarrow T = 0$ or $T = 1$
 \Rightarrow **diffusive scaling:**

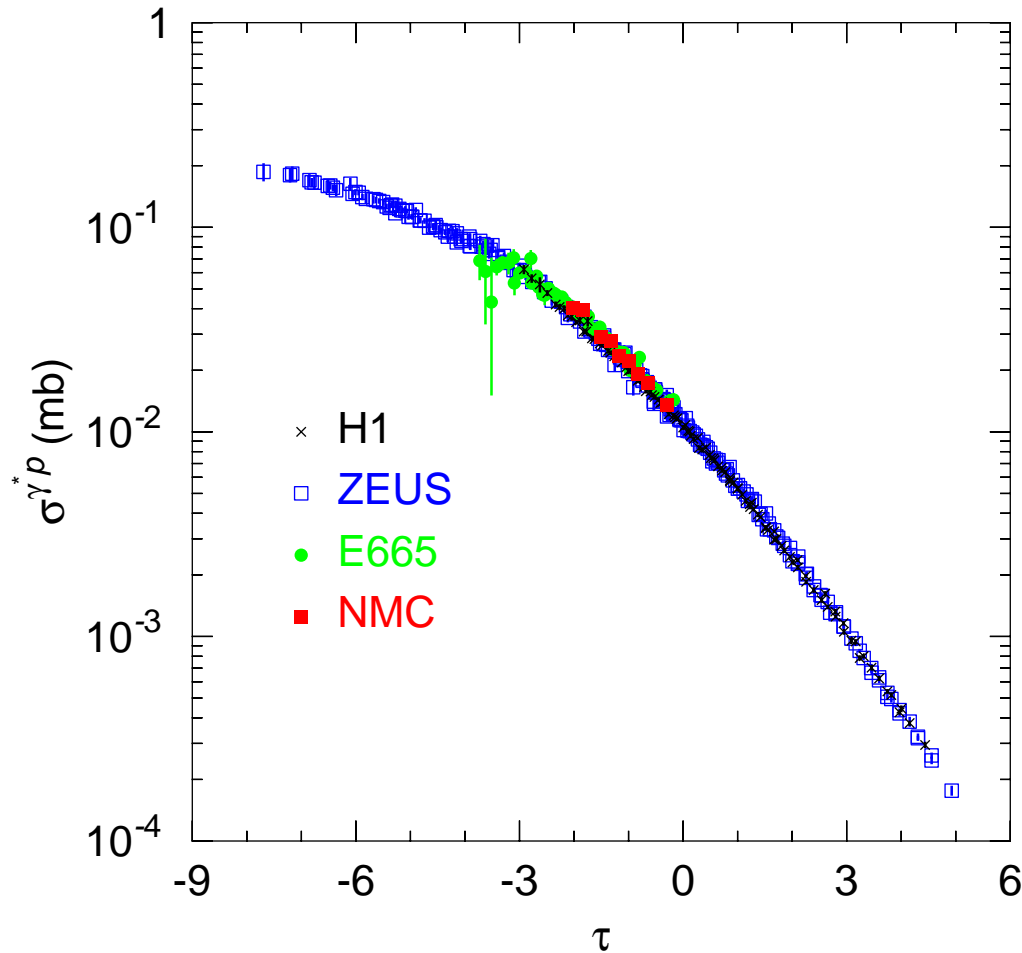
$$\langle T \rangle = f \left[\log(k^2 / Q_s^2) / \sqrt{DY} \right]$$

$$\langle T^{(k)} \rangle = \langle T \rangle$$



Phenomenology

Geometric scaling in F_2



[A.Stasto, K.Golec-Biernat, 01]

$$\sigma^{\gamma^* p}(Q^2, x) = \sigma(\tau)$$

$$\tau = \log(Q^2) - \lambda Y$$

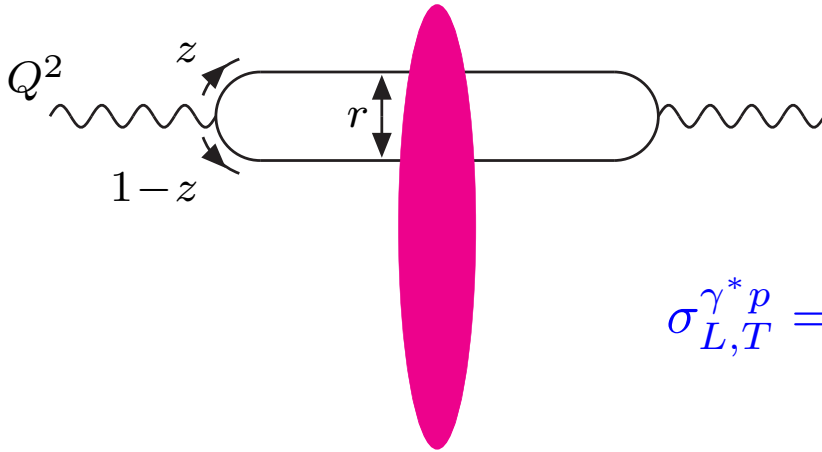
$$\lambda \approx 0.32$$

[F.Gelis, R.Peschanski, L.Schoeffel, G.S., to appear]

$$\tau = \log(Q^2) - \lambda Y \text{ or } \tau = \log(Q^2) - \lambda\sqrt{Y}$$

Factorisation formula:

[E.Iancu, K.Itakura, S.Munier, 03]



$$\sigma_{L,T}^{\gamma^*p} = \int d^2r \int_0^1 dz |\Psi_{L,T}(z, r; Q^2)|^2 2\pi R_p^2 T(\mathbf{r}; Y)$$

- $\Psi_{L,T} \equiv$ photon wavefunction from QED
- dipole amplitude: scaling variable $\tau = \log(r^2 Q_s^2/4)$

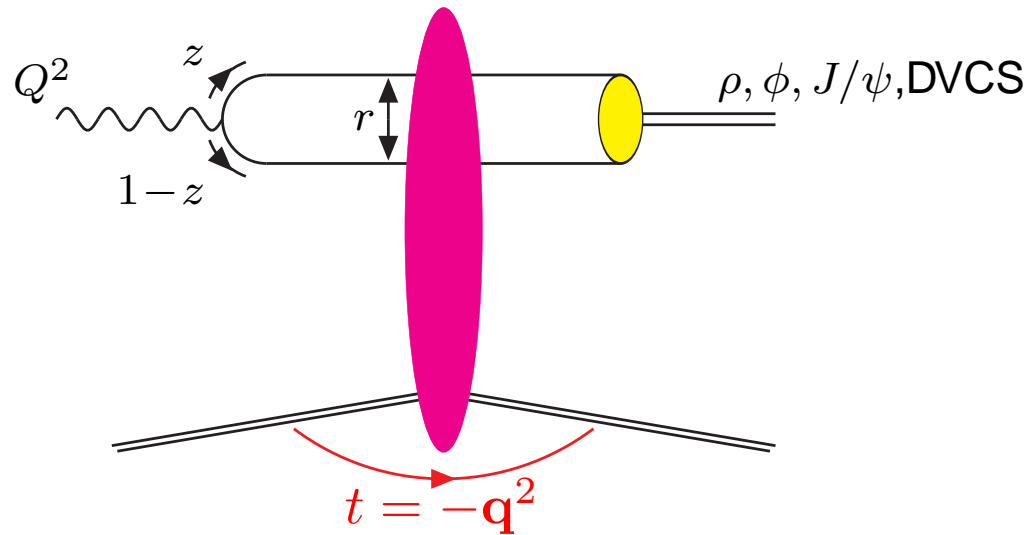
$$T(r; Y) = \begin{cases} T_0 \exp\left(\gamma_c \tau - \frac{\tau^2}{2\bar{\alpha}\chi_c'' Y}\right) & \text{if } rQ_s < 2 & \text{(travelling wave)} \\ 1 - \exp[-a(\tau + b)^2] & \text{if } rQ_s > 2 & \text{(McLerran-Venugopalan)} \end{cases}$$

$Q_s^2(Y) = k_0^2 \exp(\lambda Y) \Rightarrow \lambda \approx 0.25$, in agreement with NLO BFKL predictions.

Phenomenology

Geometric scaling in vector-meson production

[C.Marquet, R.Peschanski, G.S., to appear]



Factorisation formula:

$$\mathcal{A}_{L,T}^{\gamma^* p \rightarrow V p} = i \int d^2 r \int_0^1 dz \Psi_{L,T}(z, r; Q^2) \Psi_{L,T}^*(z, r, q; M_V^2) e^{i z \mathbf{q} \cdot \mathbf{r}} \sigma_{\text{dip}}(\mathbf{r}, \mathbf{q}; Y)$$

$\rightarrow \frac{d\sigma}{dt}, \sigma_{\text{el}}$ for $\rho, \phi, J/\psi, \text{DVCS}$

- photon wavefunction: from QED
Vector-mesons wavefunction: Boosted-Gaussian model
- dipole amplitude:

$$\sigma_{\text{dip}}(r, q; Y) = 2\pi R_p^2 e^{-b|t|} T_{\text{IM}}(r, Q_s^2(q, Y))$$

- Normalisation: only one slope b (no Q^2 dependence)
- T -matrix: t -dependent saturation scale from theoretical predictions:

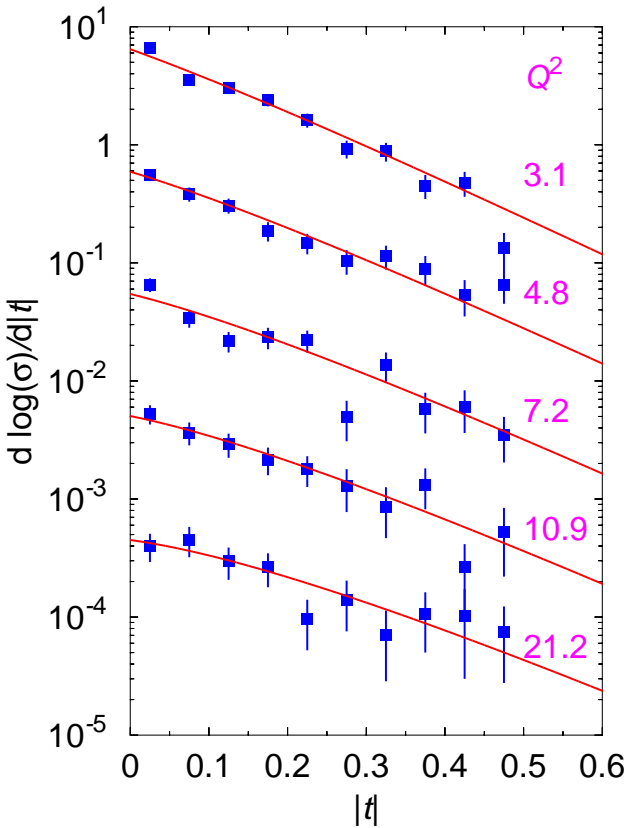
$$Q_s^2 = k_0^2 e^{\lambda Y} \quad \rightarrow \quad Q_s^2 = k_0^2 (1 + c|t|) e^{\lambda Y}$$

Hence:

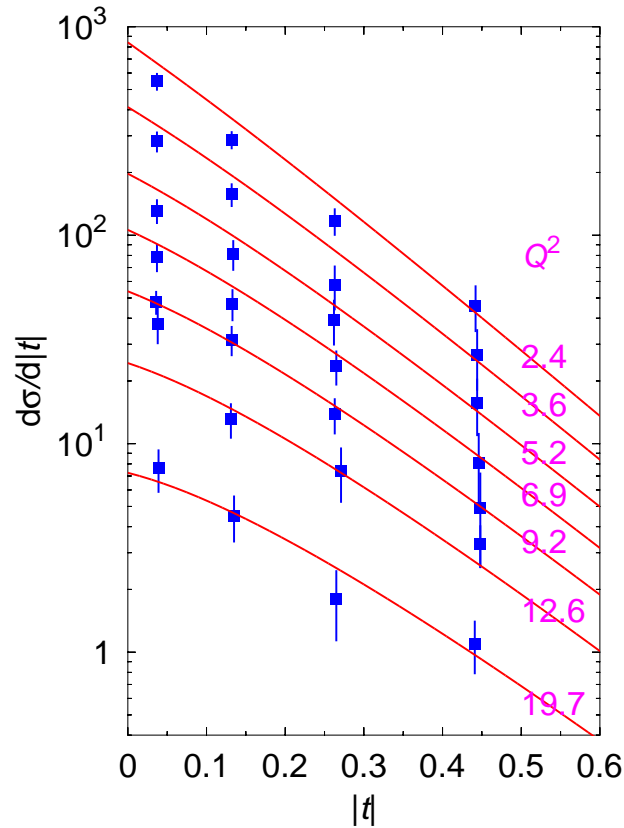
$$b, c \quad \rightarrow \quad \left. \frac{d\sigma}{dt}, \sigma_{\text{el}} \right|_{\rho, \phi} \quad (216 \text{ data})$$

Example: differential cross-section:

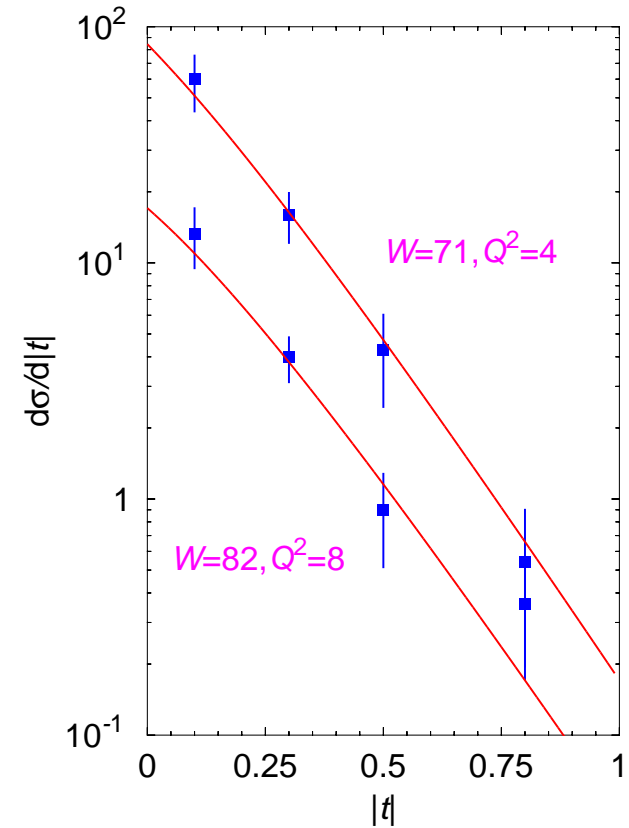
$$\gamma^* p \rightarrow \rho p$$



$$\gamma^* p \rightarrow \phi p$$



pred. for DVCS



Future phenomenology

Diffusive scaling at HERA and LHC

[Y.Hatta,E.Iancu,C.Marquet,G.S.,D.Triantafyllopoulos,06]

We have seen that, at high-energy,

$$\langle T(r, Y) \rangle = T \left(\frac{\log(r\bar{Q}_s)}{\sqrt{Y}} \right) = \frac{1}{2} \operatorname{erfc} \left(\frac{\log^2(r^2\bar{Q}_s^2)}{\sigma^2} \right)$$

with

$$\bar{Q}_s^2(Y) = k_0^2 e^{\lambda Y} \quad \text{and} \quad \sigma^2 = DY$$

Note: λ and D (or Q_s and σ^2) taken as parameters

Consequences on

- DIS and diffractive DIS (DDIS)
- gluon/forward jet production

- Total cross-section

$$\sigma_{\text{DIS}} = \int dr |\Psi(r, Y; Q^2)|^2 \langle T(r, Y) \rangle$$

$$\rightarrow \text{cst. } \sigma \Phi_1 \left(\frac{\log(r^2 \bar{Q}_s^2)}{\sigma} \right)$$

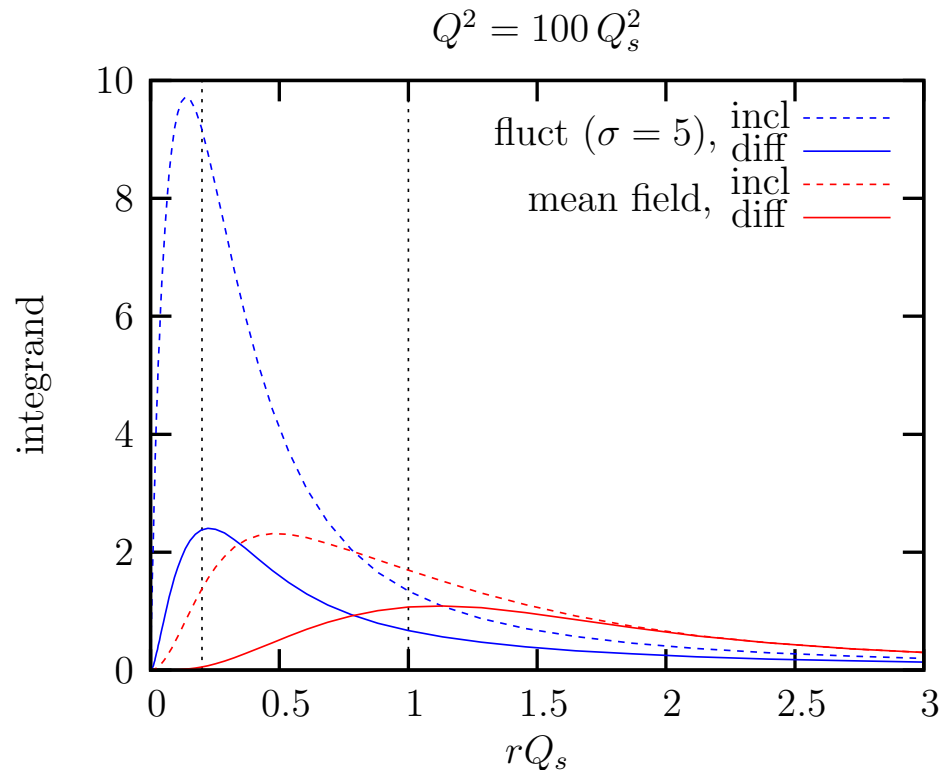
- Diffractive cross-section

$$\sigma_{\text{DDIS}} = \int dr |\Psi(r, Y; Q^2)|^2 \langle T(r, Y) \rangle^2 + (q\bar{q}g) + \dots$$

$$\rightarrow \text{cst. } \sigma \Phi_2 \left(\frac{\log(r^2 \bar{Q}_s^2)}{\sigma} \right)$$

\Rightarrow diffusive scaling for $\frac{1}{\sqrt{Y}} \sigma_{\text{DIS}}$ and $\frac{1}{\sqrt{Y}} \sigma_{\text{DDIS}}$

- Typical dipole scales in $|\Psi|^2 \otimes \langle T \rangle^{(1,2)}$:

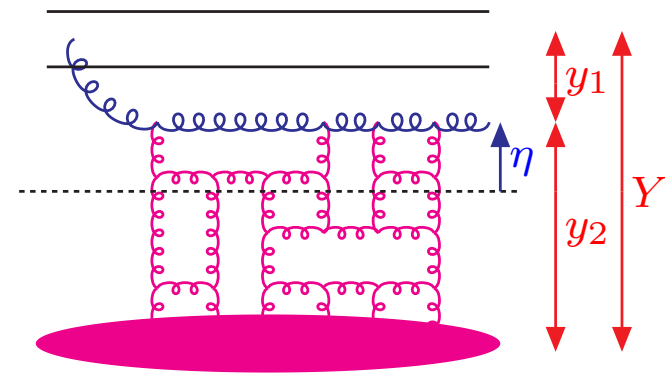


	DIS	DDIS
saturation	$r \sim 1/Q$	$r \sim 1/Q_s$
fluctuations	$r \sim 1/Q$	$r \sim 1/Q$

- Diffraction dominated by elastic amplitudes

dense-dilute scattering:

- dA or pp at forward rapidities
- dilute projectile \rightarrow dipoles
- gluon at rapidity η

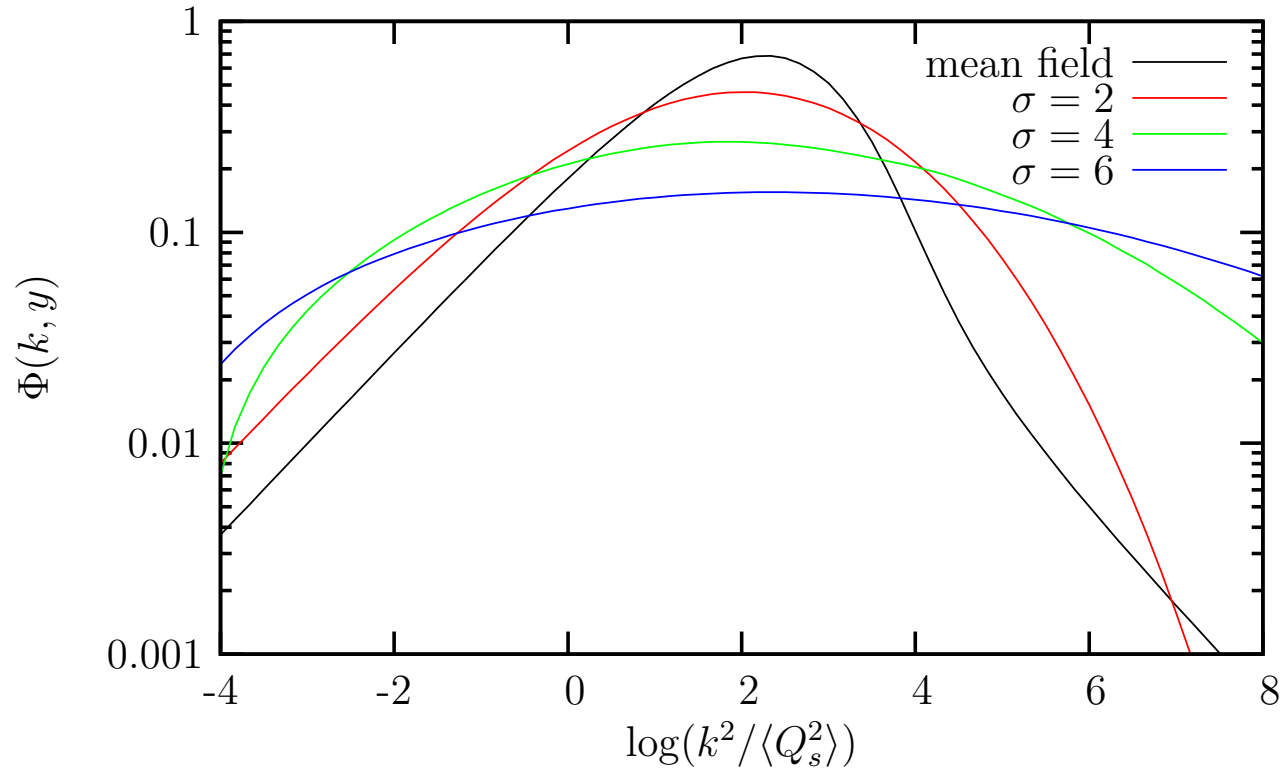


$$\frac{d\sigma}{d\eta d^2k d^2b} = \frac{\bar{\alpha}}{k^2} \int \frac{d^2p}{(2\pi)^2} \phi(\mathbf{p}, y_1) \Phi(\mathbf{k} - \mathbf{p}, y_2)$$

Projectile unintegrated gluon density $\phi(\mathbf{p}, y_1) = \int \frac{d^2r}{2\pi} e^{i\mathbf{p}\cdot\mathbf{r}} n(\mathbf{r}, y_1)$

Target contribution $\Phi(\mathbf{k}, y_2) = \int d^2r e^{i\mathbf{k}\cdot\mathbf{r}} \nabla_{\mathbf{r}}^2 \langle 2T(r, y_2) - T^2(r, y_2) \rangle$

[E. Iancu, C. Marquet, G.S., 06]



$$\Phi(k, Y) \rightarrow \frac{1}{\sigma} \exp \left[\frac{\log^2(k^2 / \bar{Q}_s^2)}{\sigma^2} \right] \Rightarrow \text{diffusive scaling for } \sqrt{Y} \Phi(k, Y)$$

Part 1: Evolution equations towards high-energy

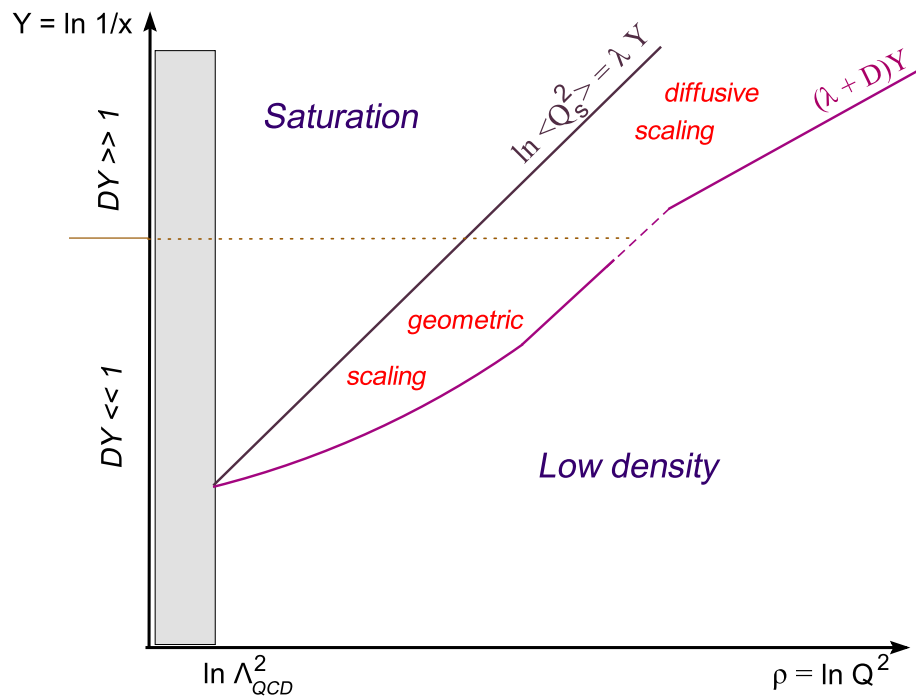
Infinite hierarchy:

contribution	$\partial_Y \langle T^k \rangle$	importance	diagrams
BFKL	$\langle T^k \rangle$	resums $\alpha_s^n \log^n(s)$	ladders
saturation	$\langle T^{k+1} \rangle$	near unitarity: $T \lesssim 1$	fan
fluctuations	$\langle T^{k-1} \rangle$	dilute tail: $T \gtrsim \alpha_s^2$	splittings & loops

Perspectives:

- beyond 2 gluon-exchange approximation
([J.T.Amaral, E.Iancu, G.S., D.Triantfyllopoulos, to appear])
- beyond large- N_c approximation

Part 2: Solutions for scattering amplitudes



Geometric scaling

$$T = T(rQ_s)$$

$$Q_s = \exp(\lambda Y)$$

Diffusive scaling

$$T = T[\log(rQ_s)/\sigma]$$

$$Q_s = \exp(\lambda Y), \sigma^2 = DY$$

General predictions of saturation
even when $T \ll 1$

Note: Knowledge of preasympt.

Perspectives:

- Better analytic control of the fluctuation effects
- include impact-parameter dependence

Part 3: Phenomenological consequences

● HERA:

- geometric scaling for F_2 , DVCS and VM-production
⇒ indications for saturation
- diffusive scaling for F_2 and F_2^D at higher energy

● LHC:

- diffusive scaling predicted for dense-dilute collisions (dA or forward pp)

Perspectives:

- Control of the interplay between geometric and diffusive scaling (HERA ?)
- More predictions for LHC
- Applications to dense-dense collisions

- generic scaling laws from high-energy QCD
- interesting links with statistical physics
- hints from HERA and TEVATRON

