

# *Towards a standardised approach to jet reconstruction*

Grégory Soyez

IPhT, CEA Saclay

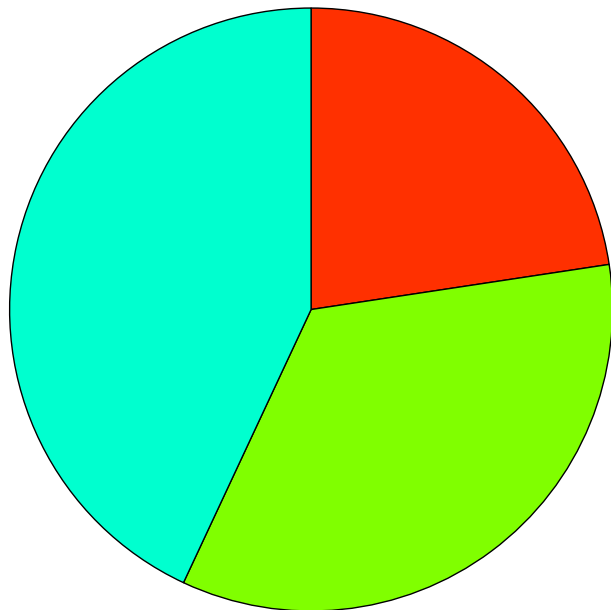
with Matteo Cacciari, Gavin Salam

MIT — May 20th 2013

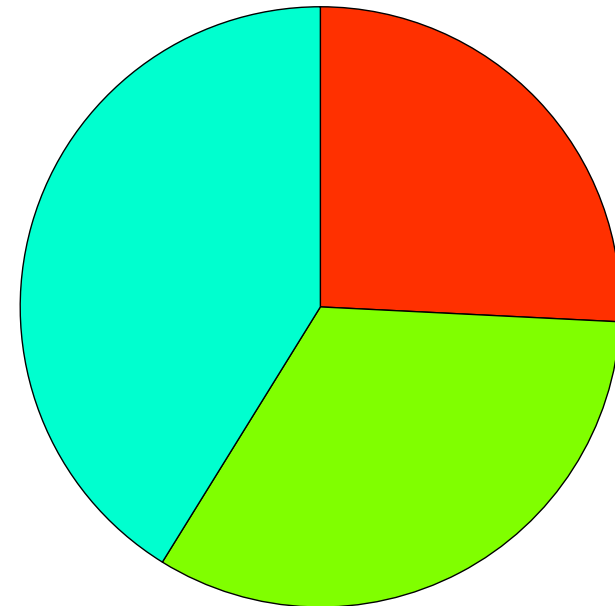
# *Jets are widely used objects at the LHC*

~ 60% of the LHC paper use jets!

CMS



ATLAS



■ "jet" in title  
■ "jets" used  
■ others

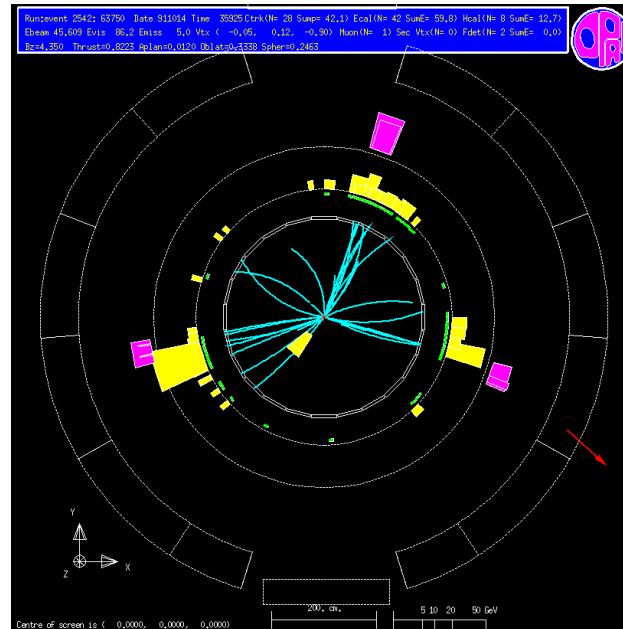
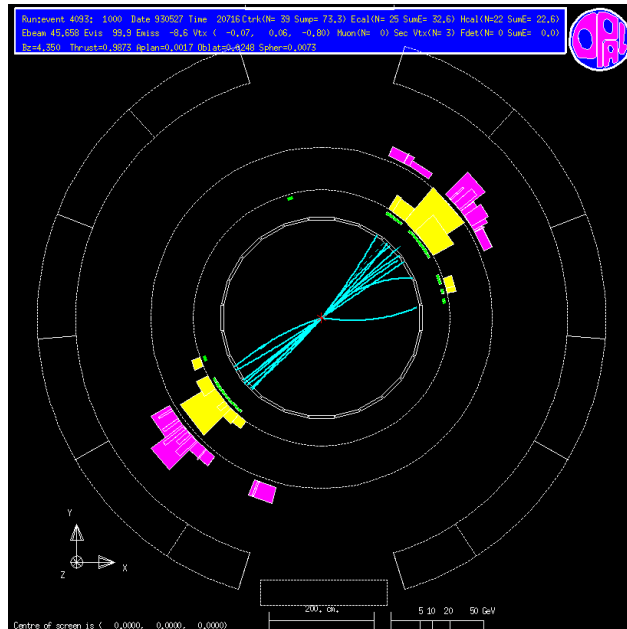
# Brief plan

- Jets (pre-LHC history)  
Concept and importance, *jet definitions*, illustrations
- Goal #1: robust/standard jet definitions  
anti- $k_t$  algorithm
- Goal #2: public and standard interface  
FastJet and its contrib
- Goal #3: jets at high luminosity *i.e.* handling pileup  
context, area-median pileup subtraction
- Goal #4: Jet substructure  
tagging boosted objects

***What is a “jet”?***

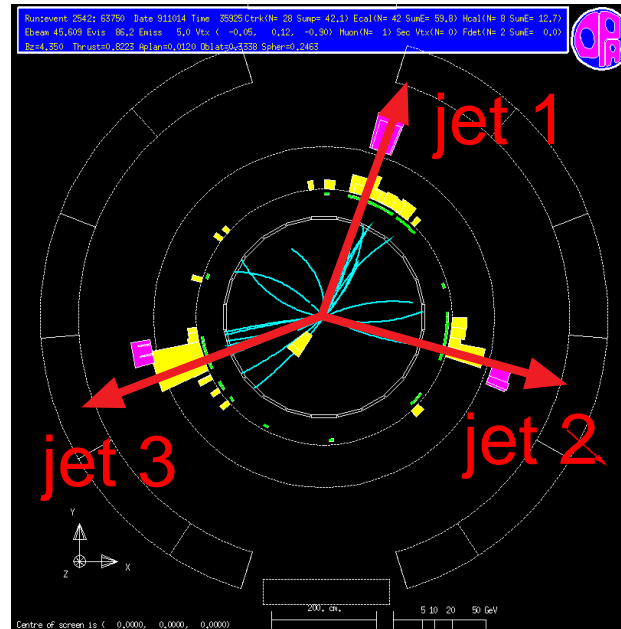
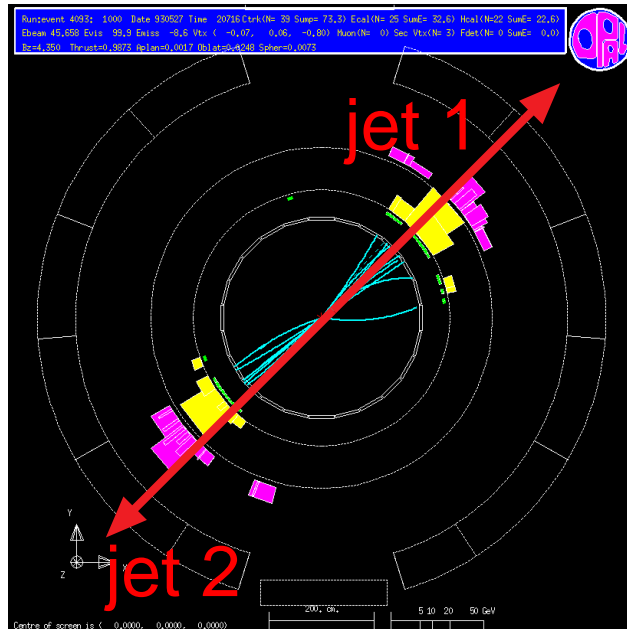
***concept/idea***

- Final-state events are pencil-like already observed in  $e^+e^-$  collisions:



- Consequence of the collinear divergence  
QCD (quark & gluon) branching proba:  $\frac{dP}{d\theta} \propto \frac{\alpha_s}{\theta}$

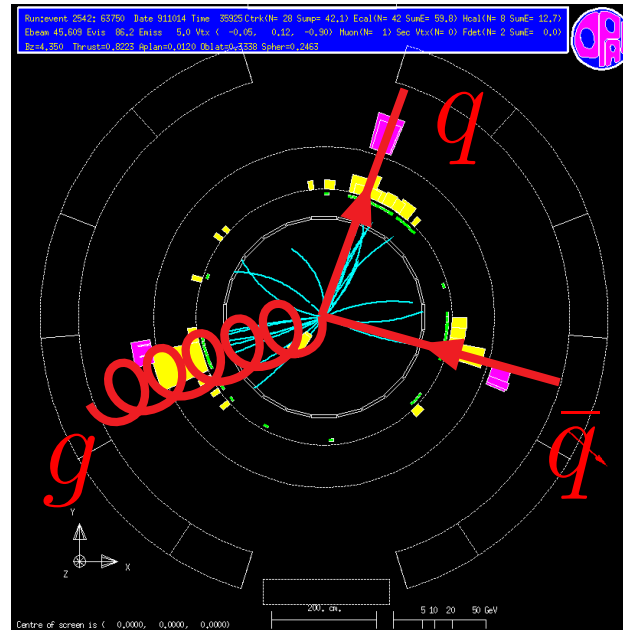
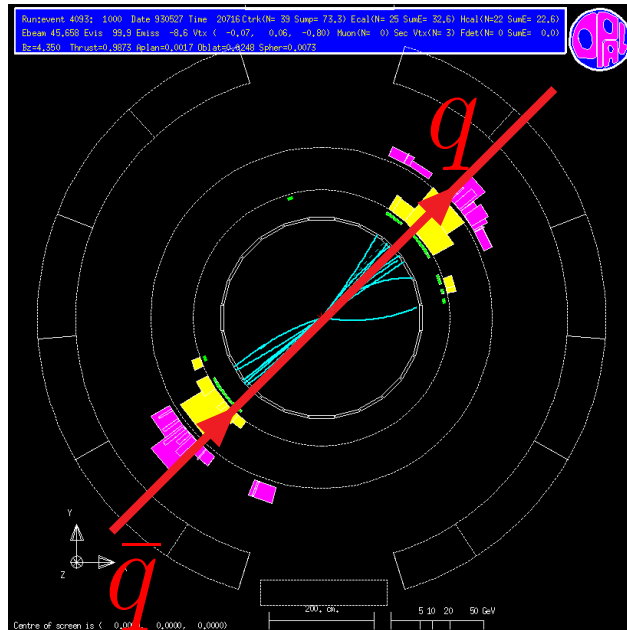
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“Jets”  $\equiv$  bunch of collimated particles

- Final-state events are pencil-like already observed in  $e^+e^-$  collisions:



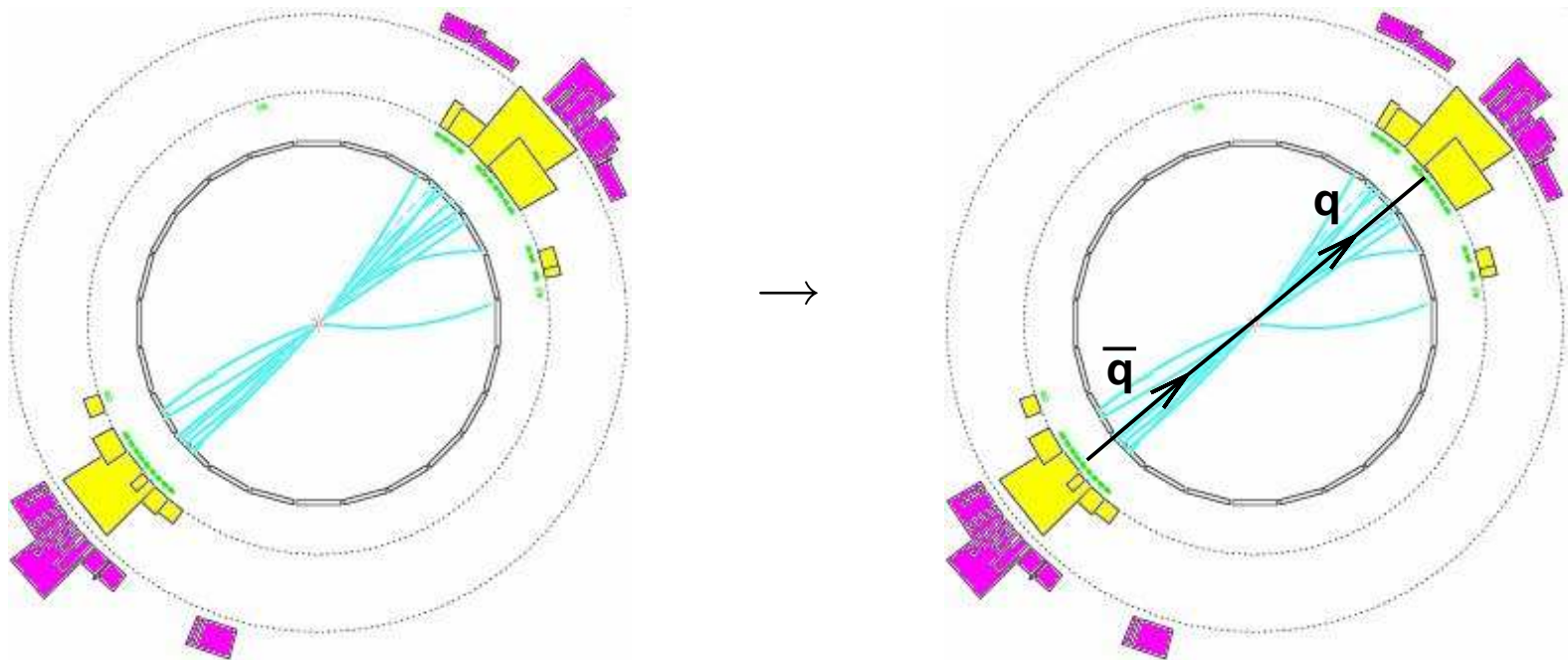
- Consequence of the collinear divergence  
 QCD (quark & gluon) branching proba:  $\frac{dP}{d\theta} \propto \frac{\alpha_s}{\theta}$

“Jets”  $\equiv$  bunch of collimated particles  $\cong$  hard partons

# Jets and partons

“Jets”  $\equiv$  bunch of collimated particles  $\cong$  hard partons

obviously 2 jets

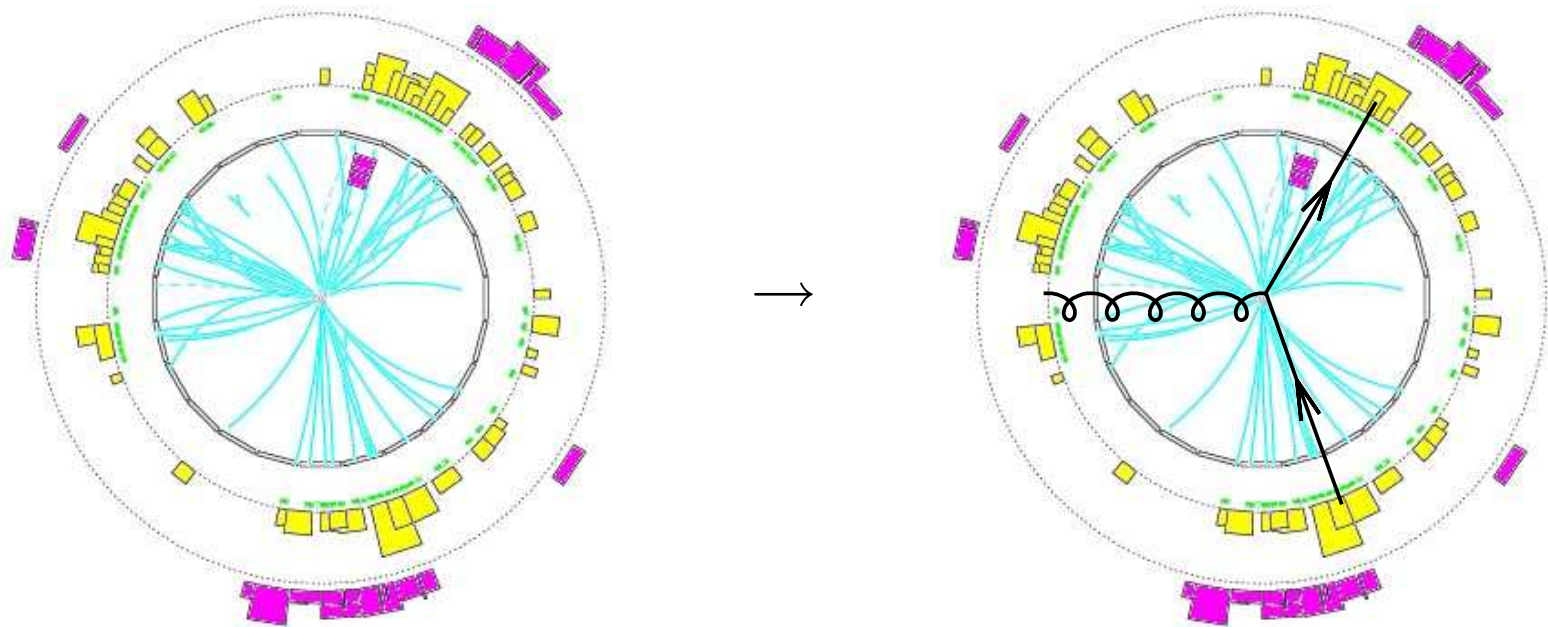




# Jets and partons

“Jets”  $\equiv$  bunch of collimated particles  $\cong$  hard partons

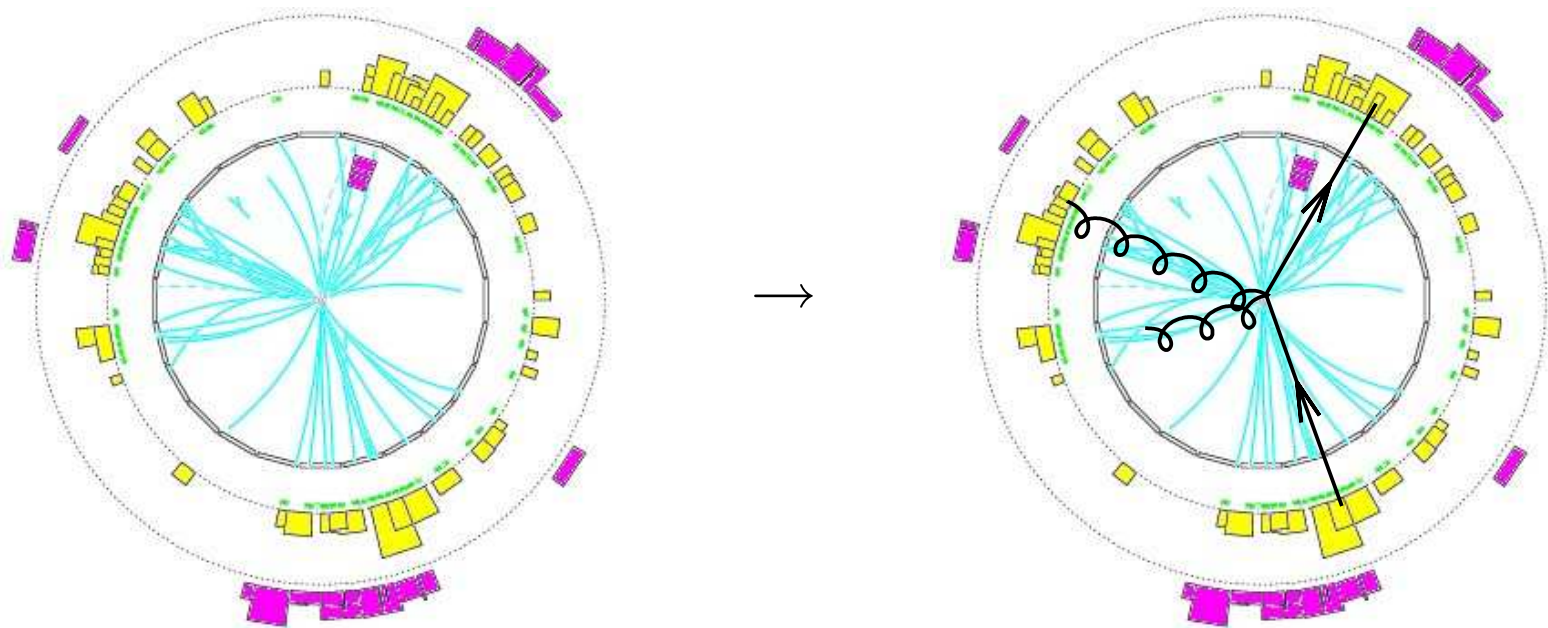
3 jets



# Jets and partons

“Jets”  $\equiv$  bunch of collimated particles  $\cong$  hard partons

3 jets... or 4?

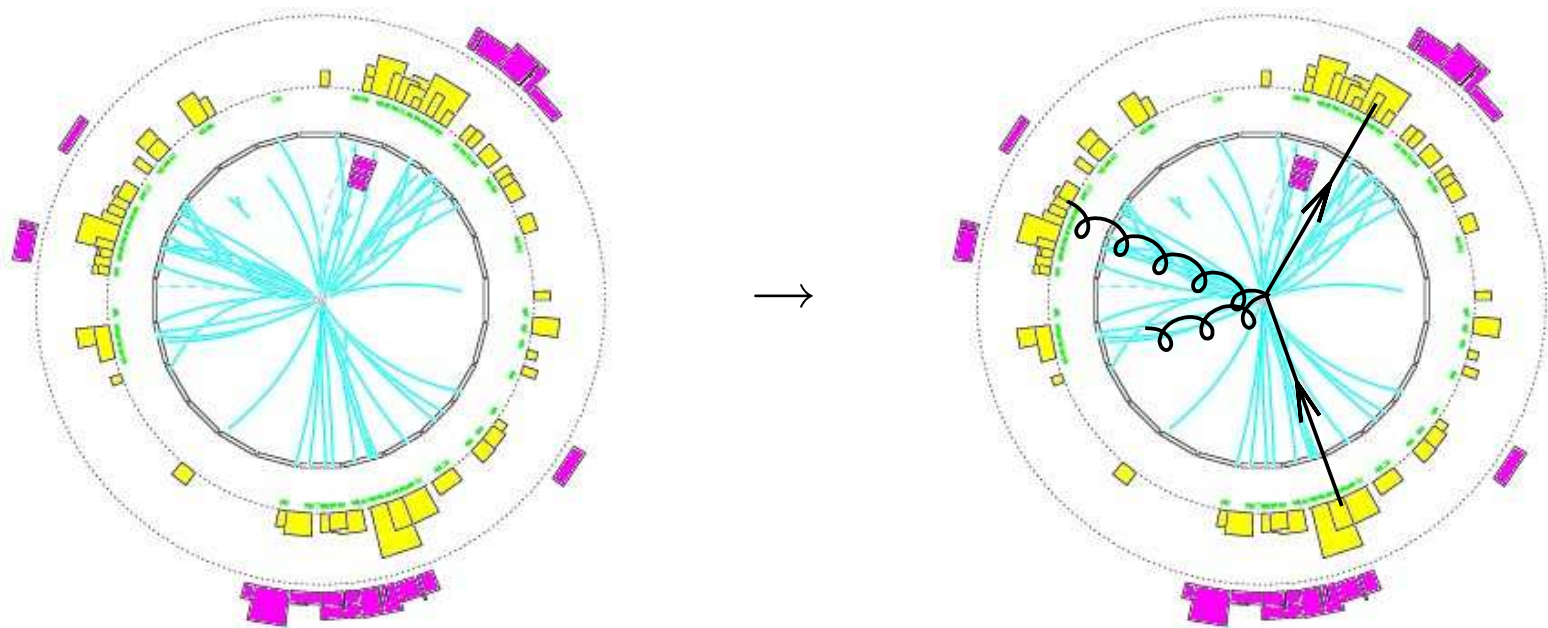


- “collinear” is arbitrary

# Jets and partons

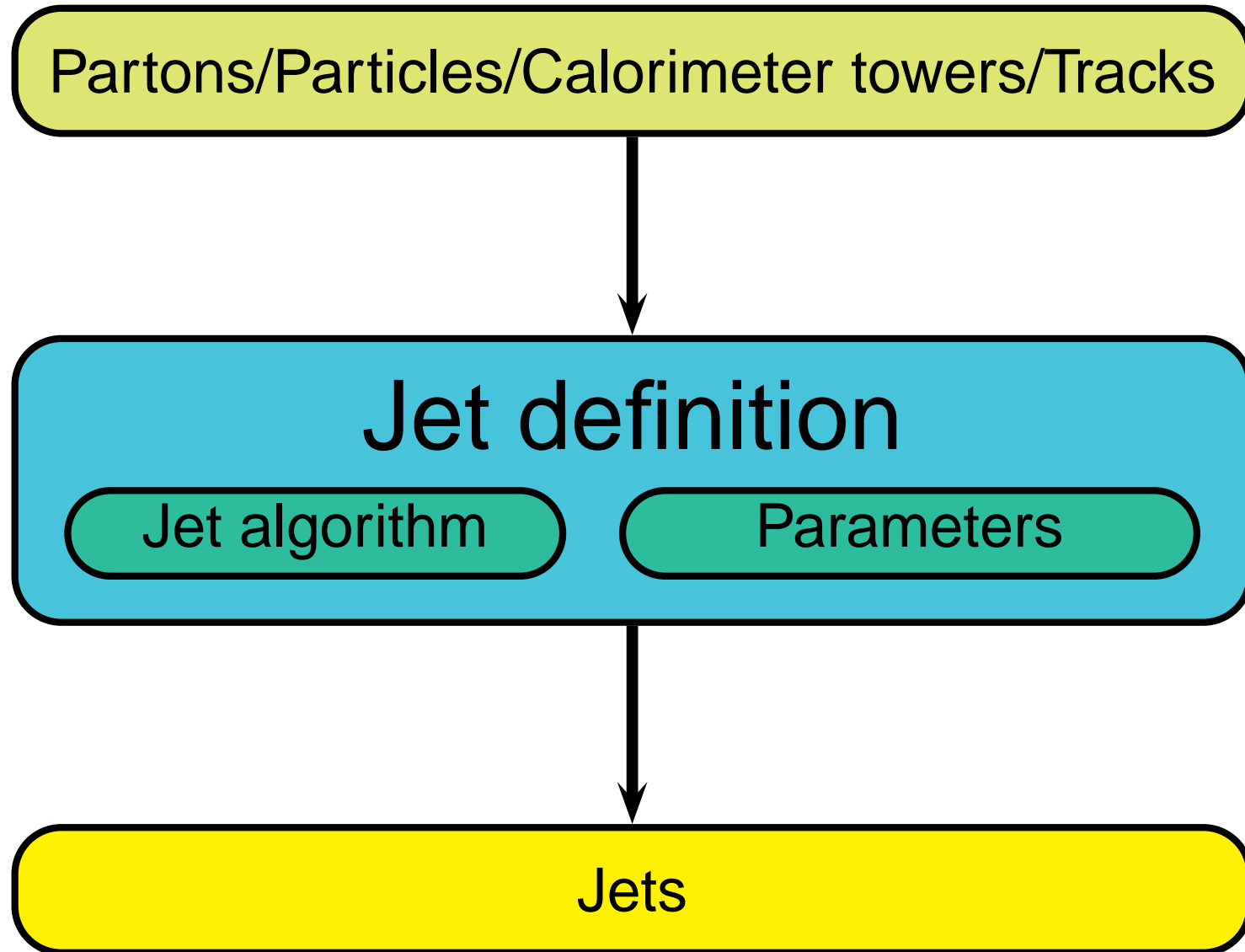
“Jets”  $\equiv$  bunch of collimated particles  $\cong$  hard partons

3 jets... or 4?



- “collinear” is arbitrary
- “parton” concept strictly valid only at LO

# Jet definition



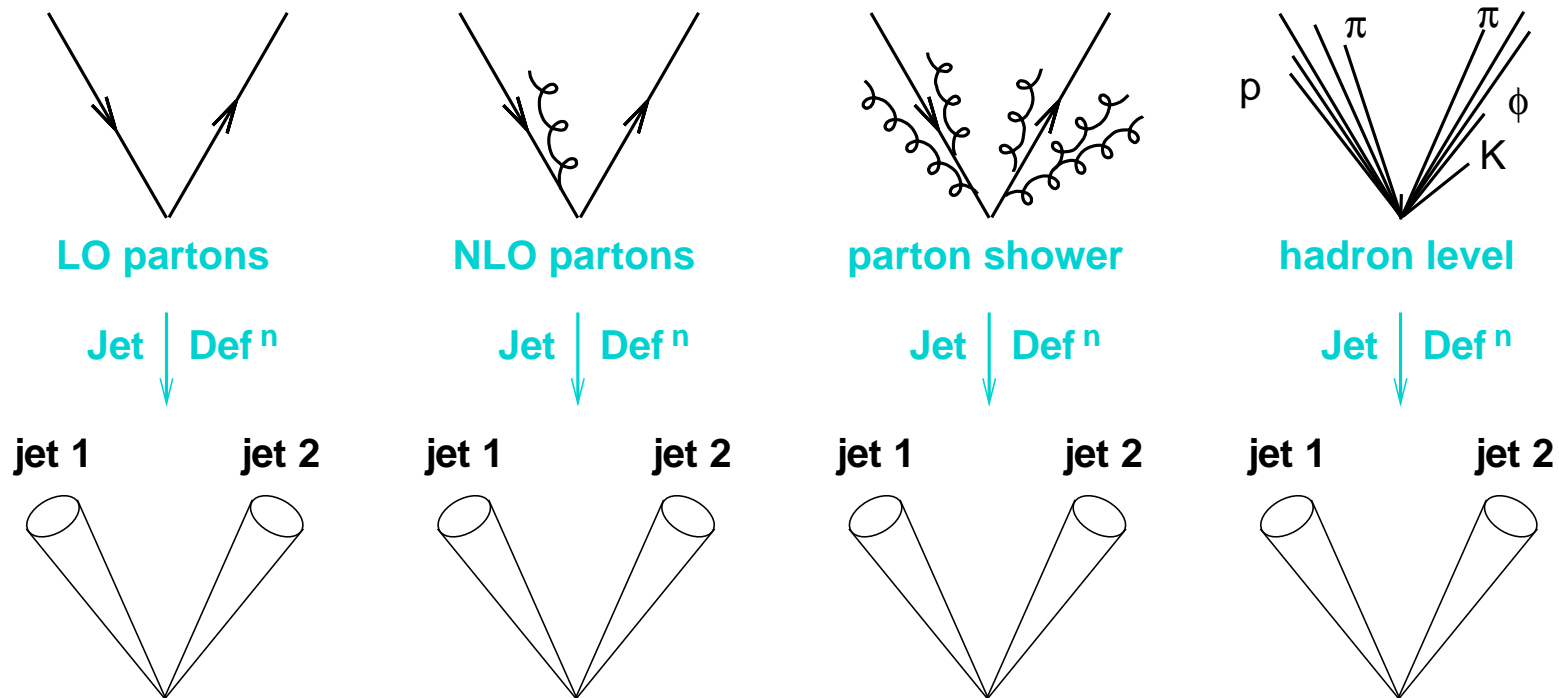
***What is a “jet”?***

***jet definition(s)***

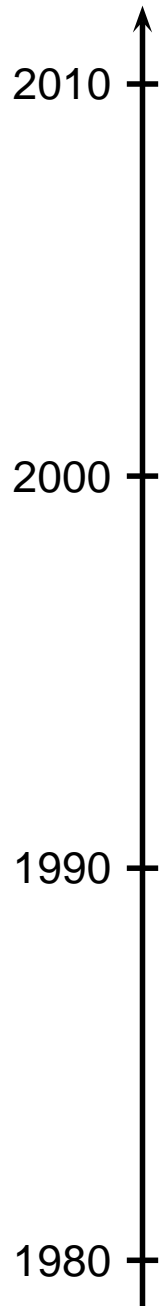
# Jet definition

A jet definition is supposed to

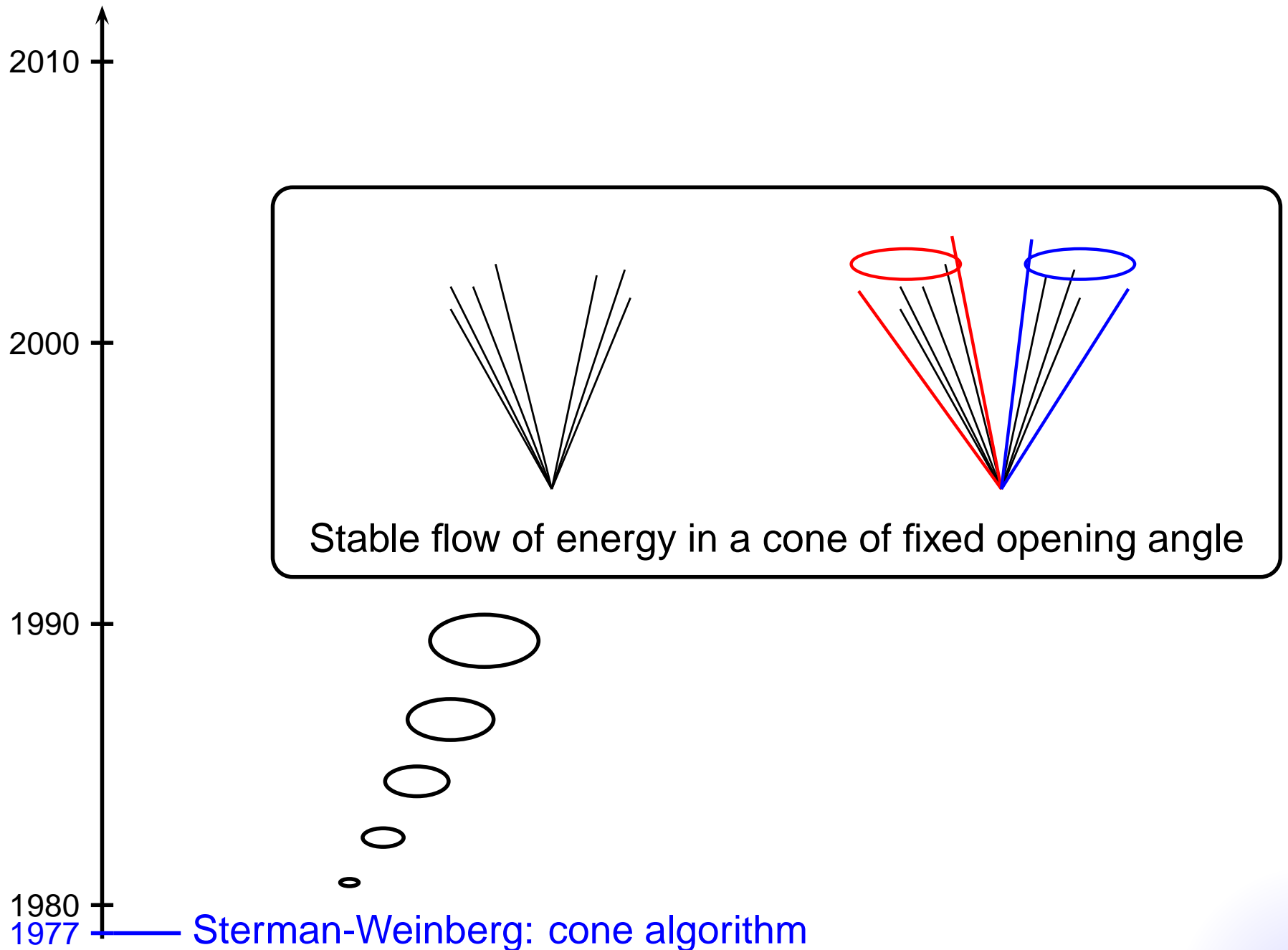
- give finite jet cross sections (th)
- be fast enough (exp)
- be (as) consistent (as possible) across different view of an event (th&exp)



# *A brief/rough flight over the history of jets*

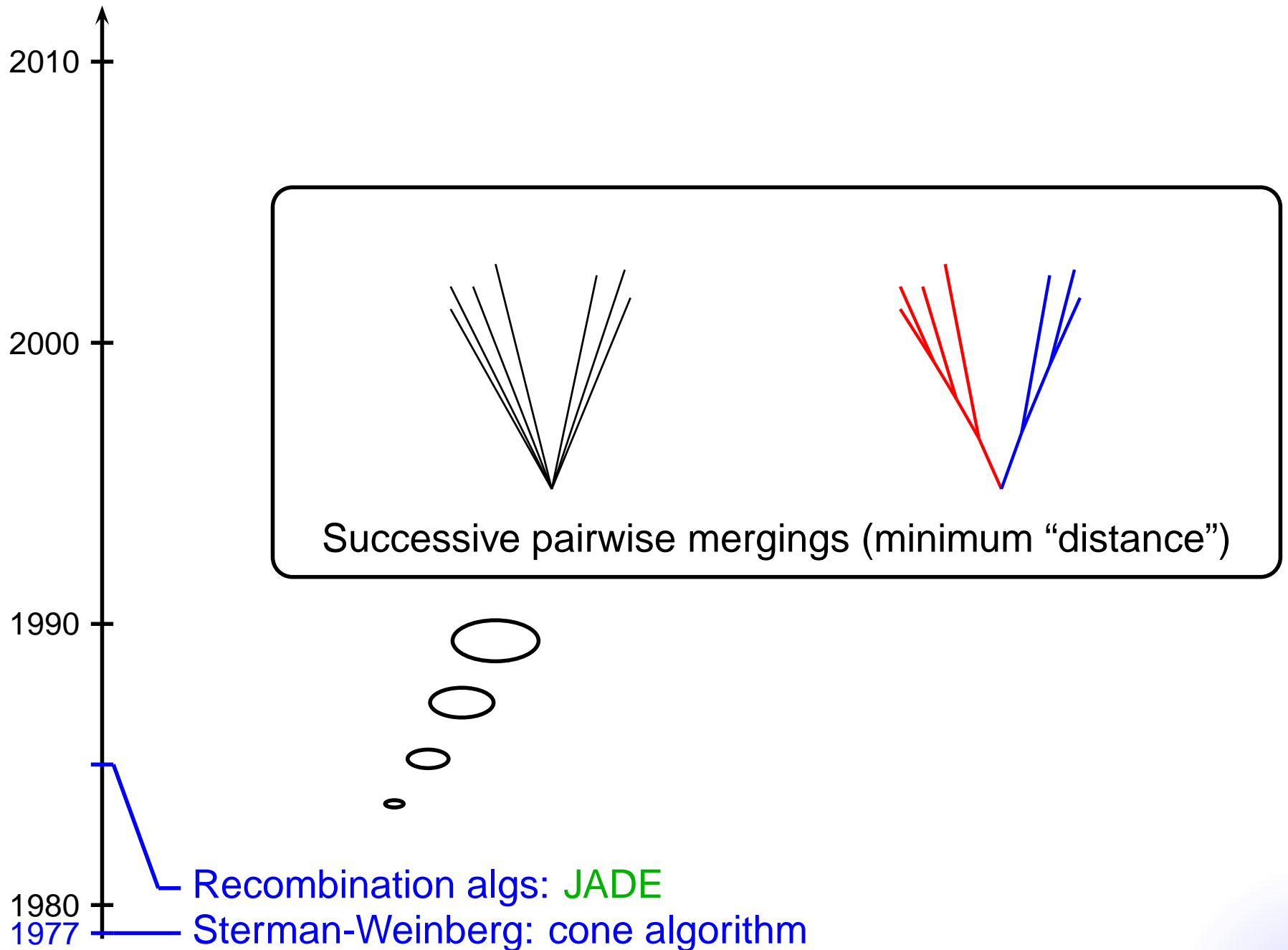


# A brief/rough flight over the history of jets

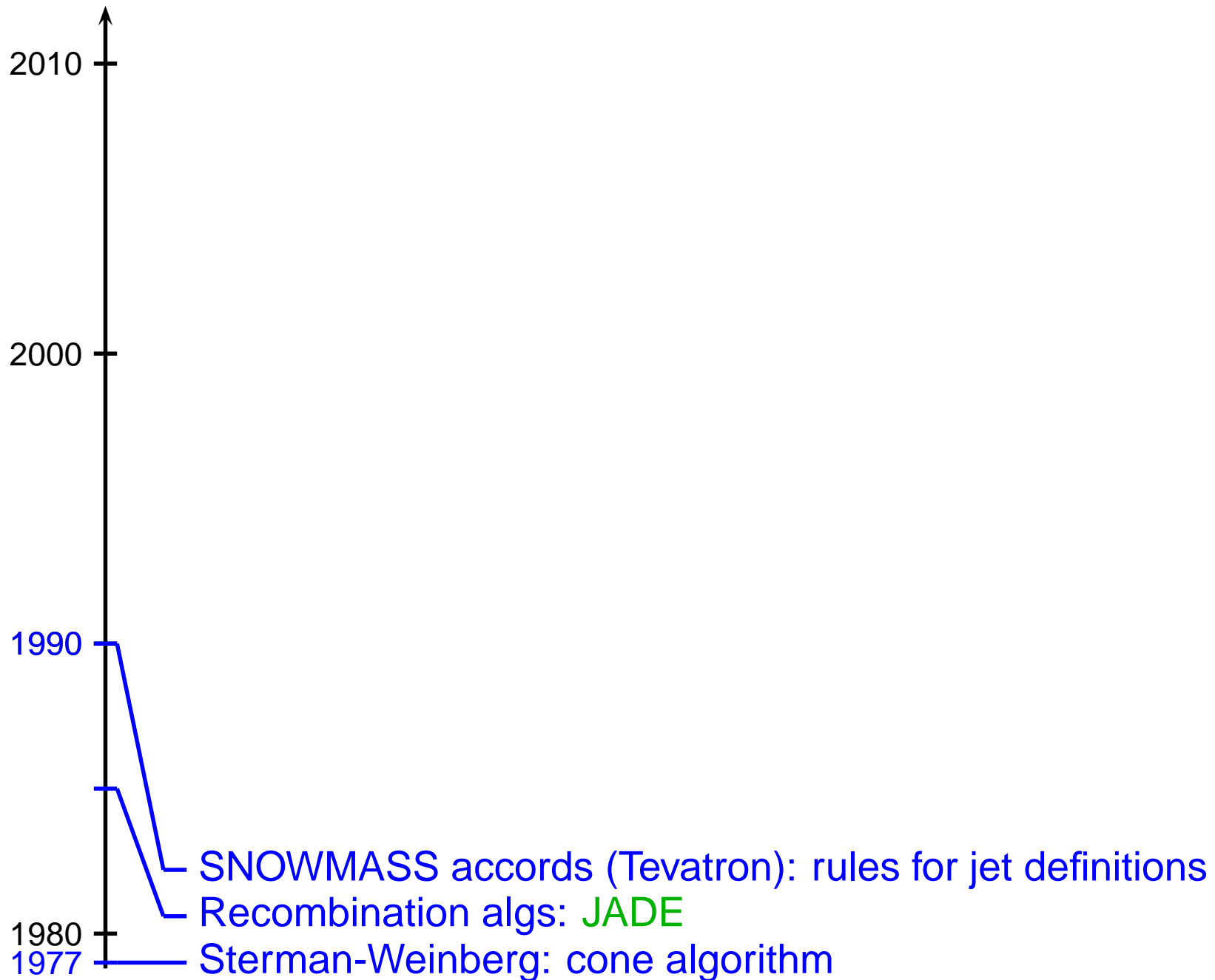




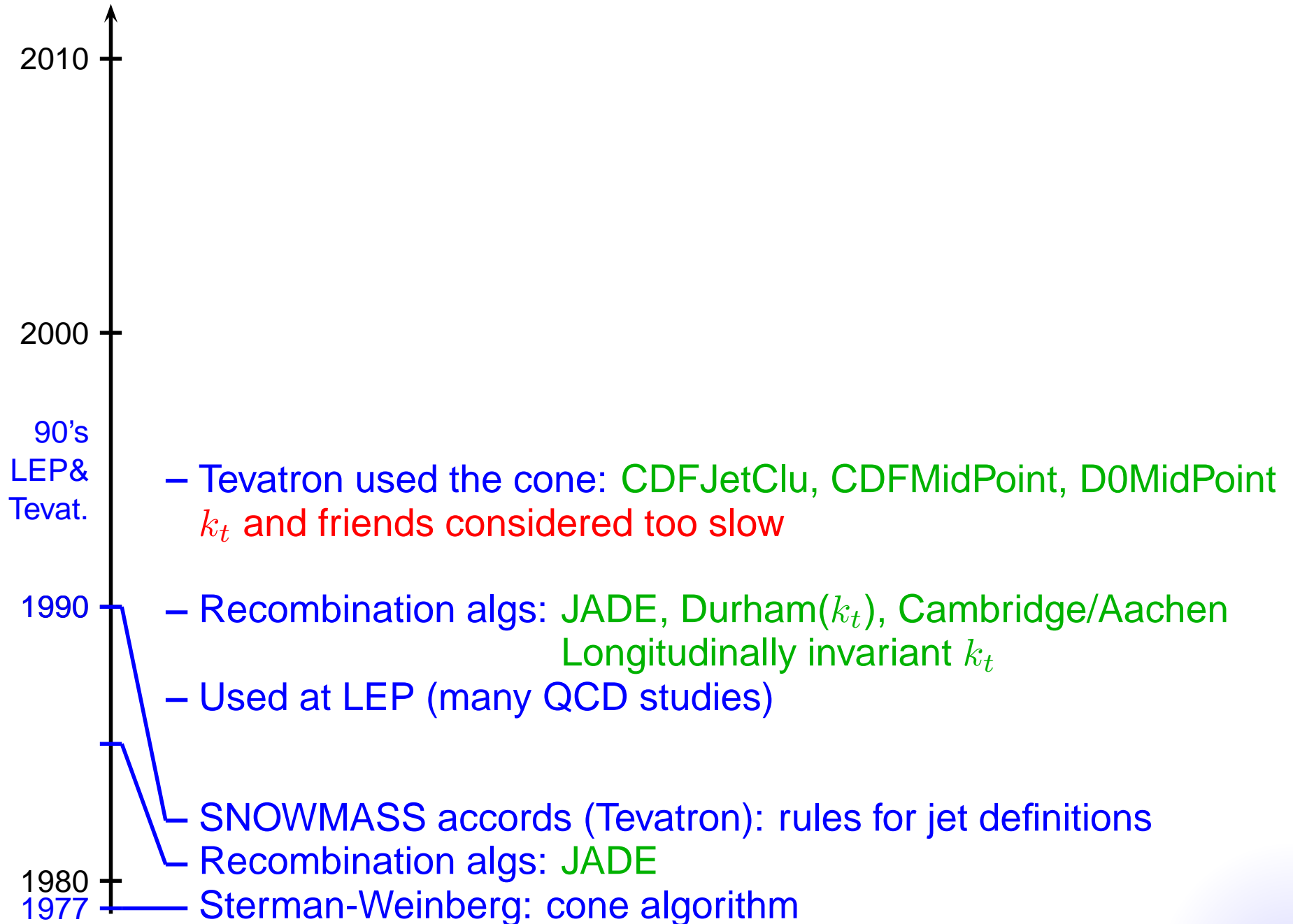
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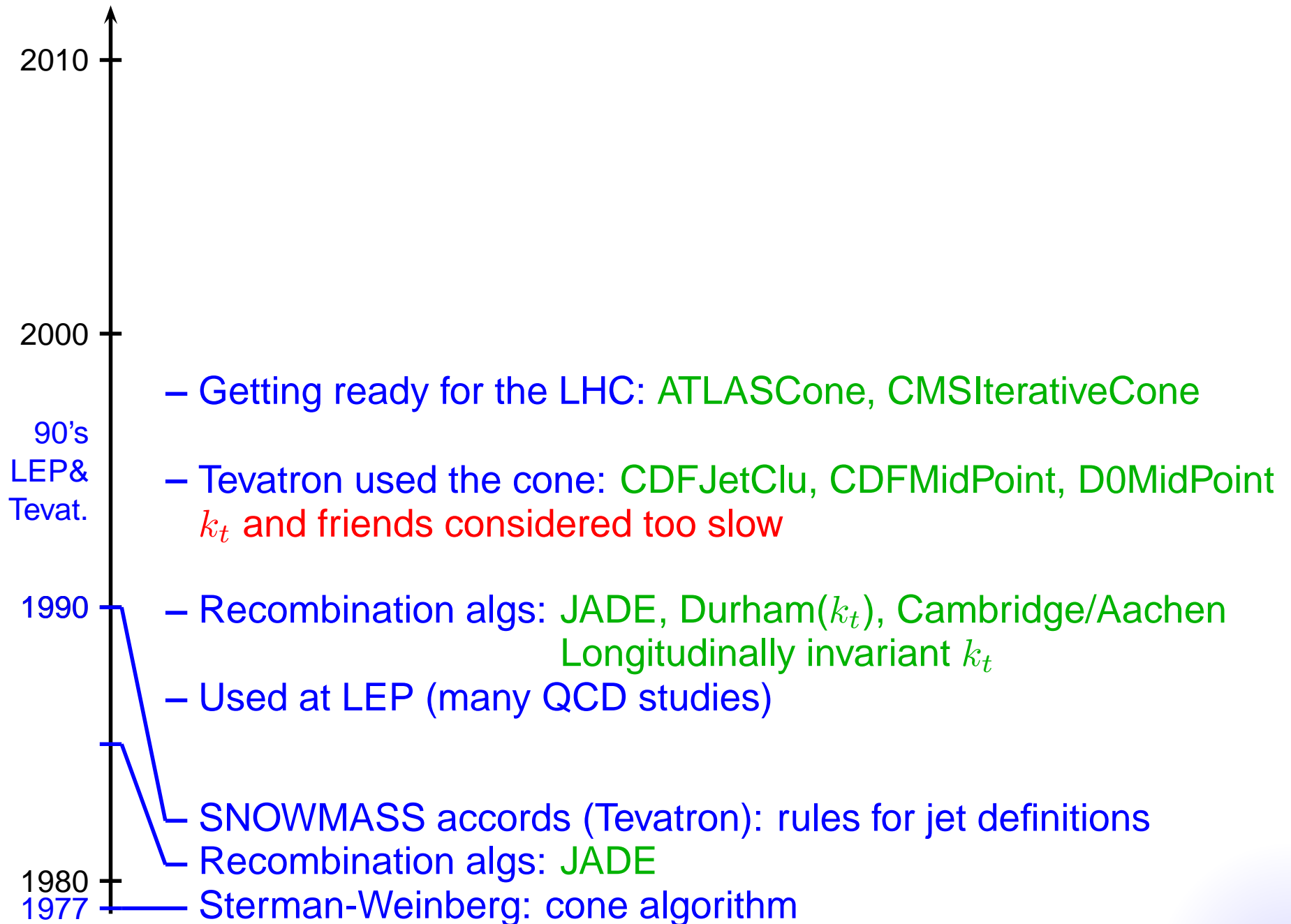
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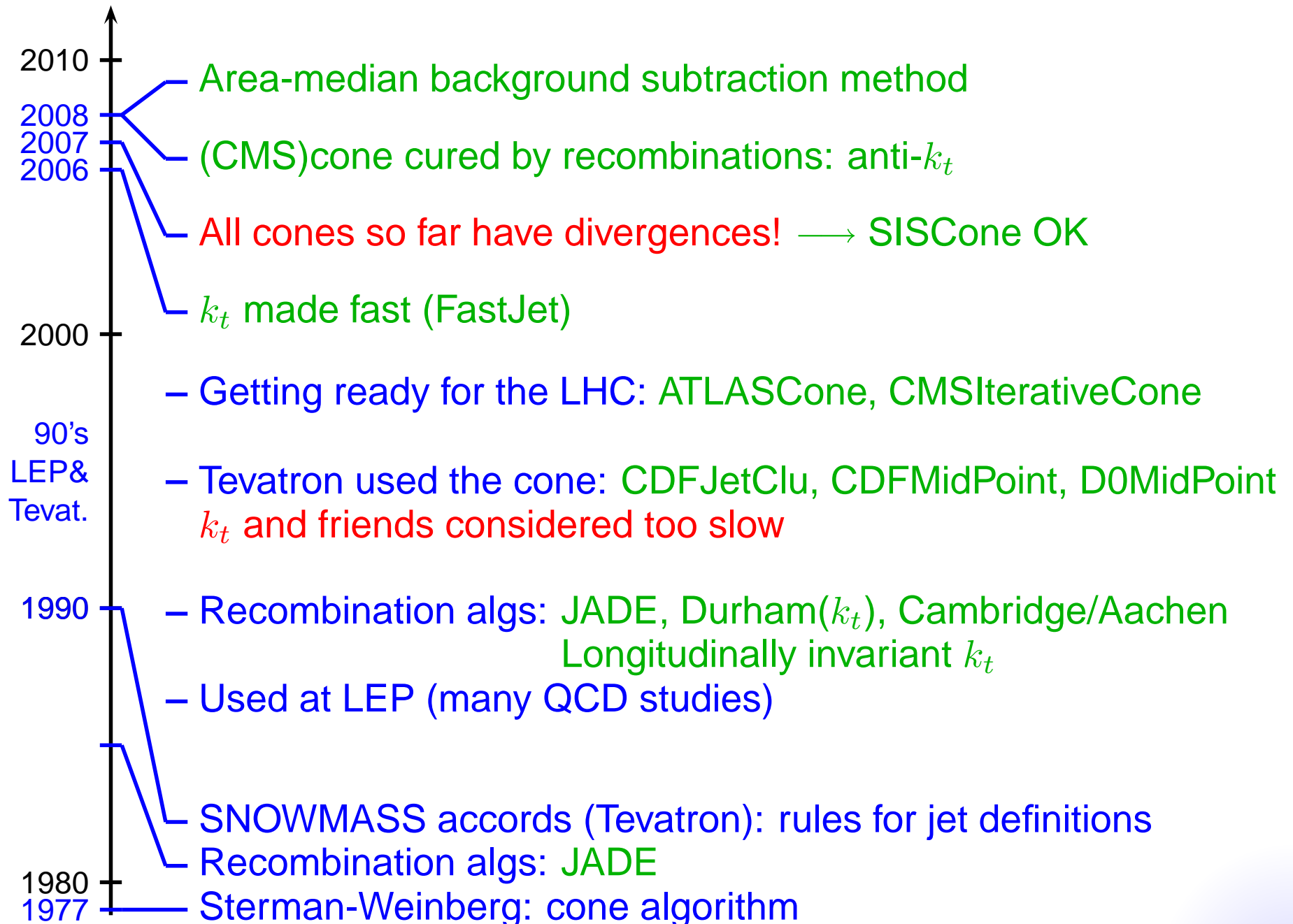
# A brief/rough flight over the history of jets



# A brief/rough flight over the history of jets



# A brief/rough flight over the history of jets



# A brief/rough flight over the history of jets



*What is a “jet”?*

*jets at the LHC*

# The anti- $k_t$ jets

- All experiments use the anti- $k_t$  algorithm:

[M. Cacciari, G. Salam, GS, 2008]

- From all the objects, define the distances

$$d_{ij} = \min(k_{t,i}^{-2}, k_{t,j}^{-2})(\Delta y_{ij}^2 + \Delta\phi_{ij}^2), \quad d_{iB} = k_{t,i}^{-2} R^2$$

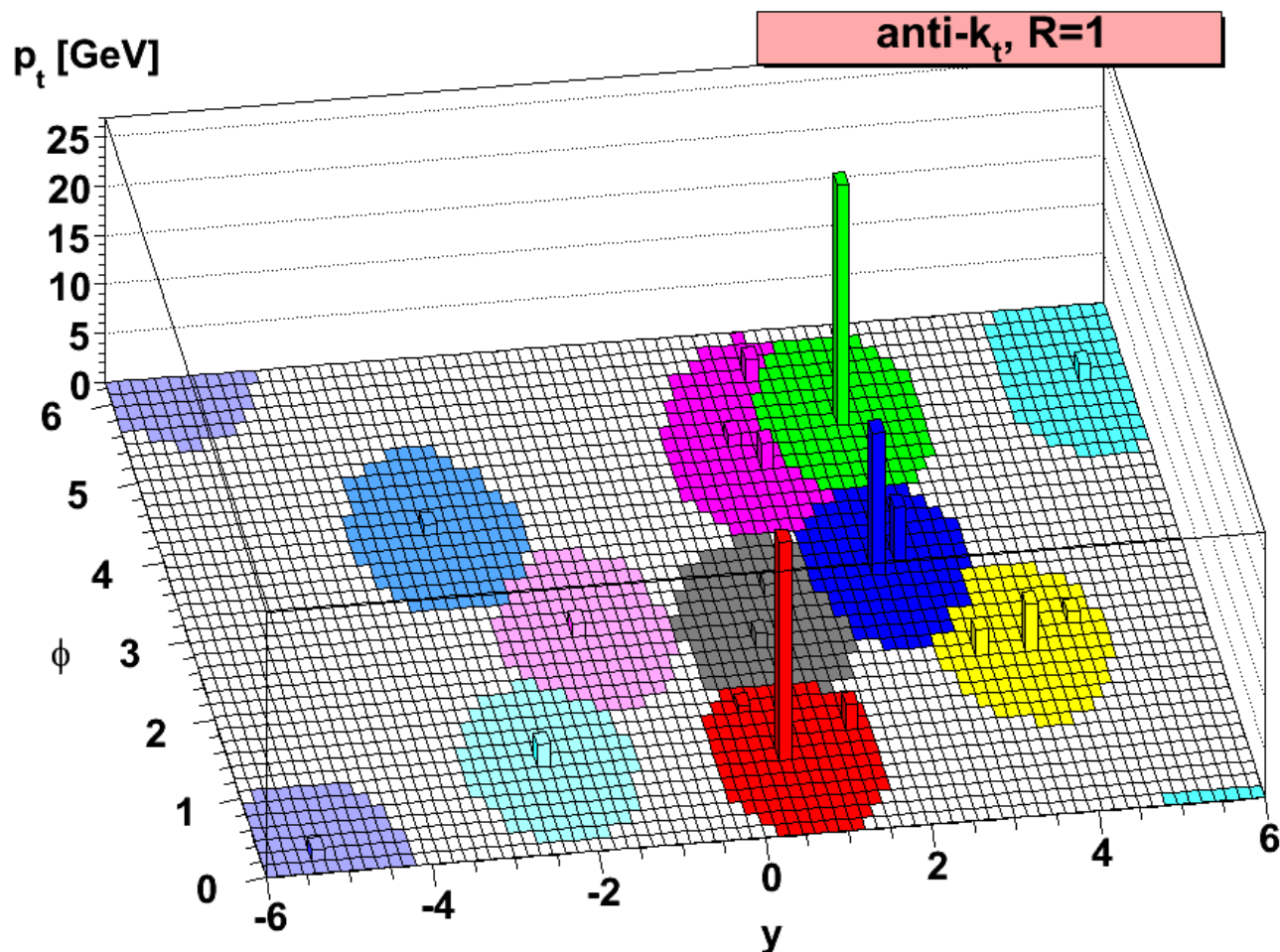
- repeatedly find the minimal distance
  - if  $d_{ij}$ : recombine  $i$  and  $j$  into  $k = i + j$
  - if  $d_{iB}$ : call  $i$  a jet

- $R$  is a size parameter (e.g. CMS: 0.5,0.7, ATLAS: 0.4,0.6)



# The anti- $k_t$ jets

Main property: hard jets are circular

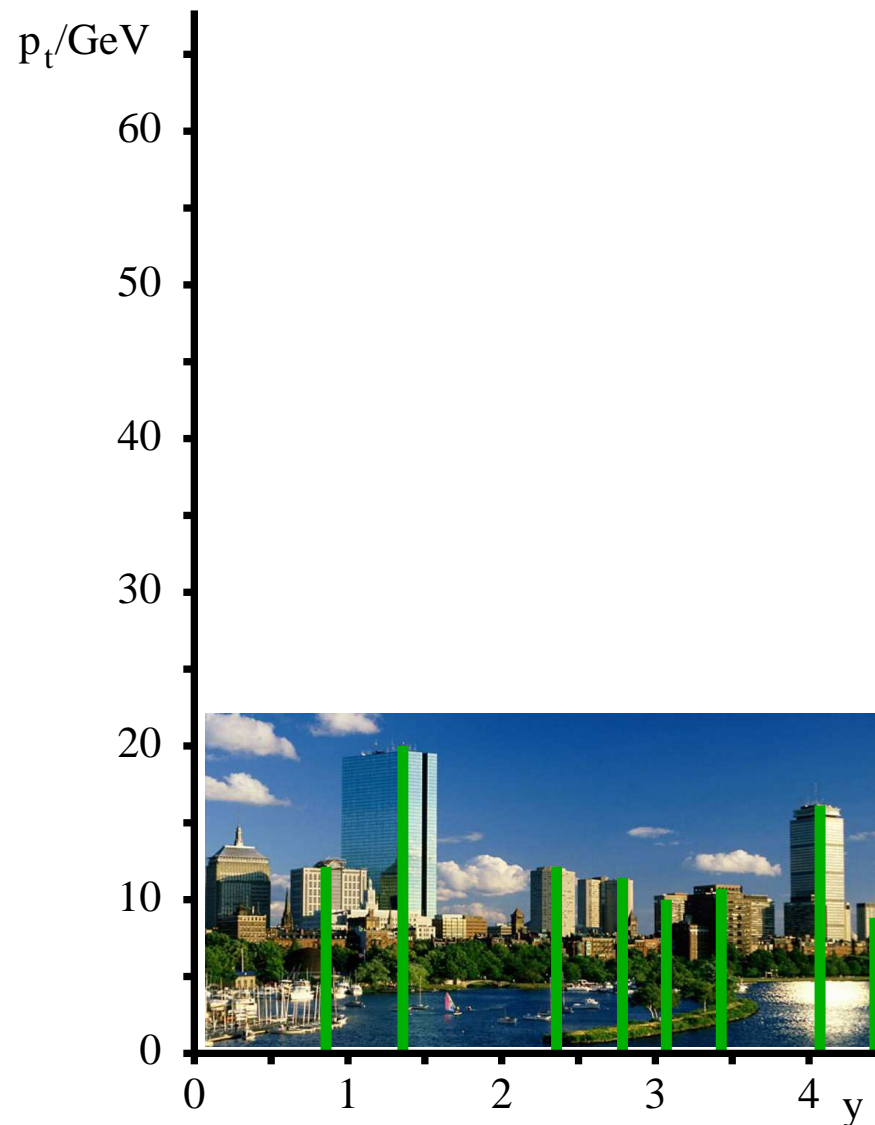


# Clustering in action: anti- $k_t$ ( $R = 0.7$ )

Start with your  
favourite picture

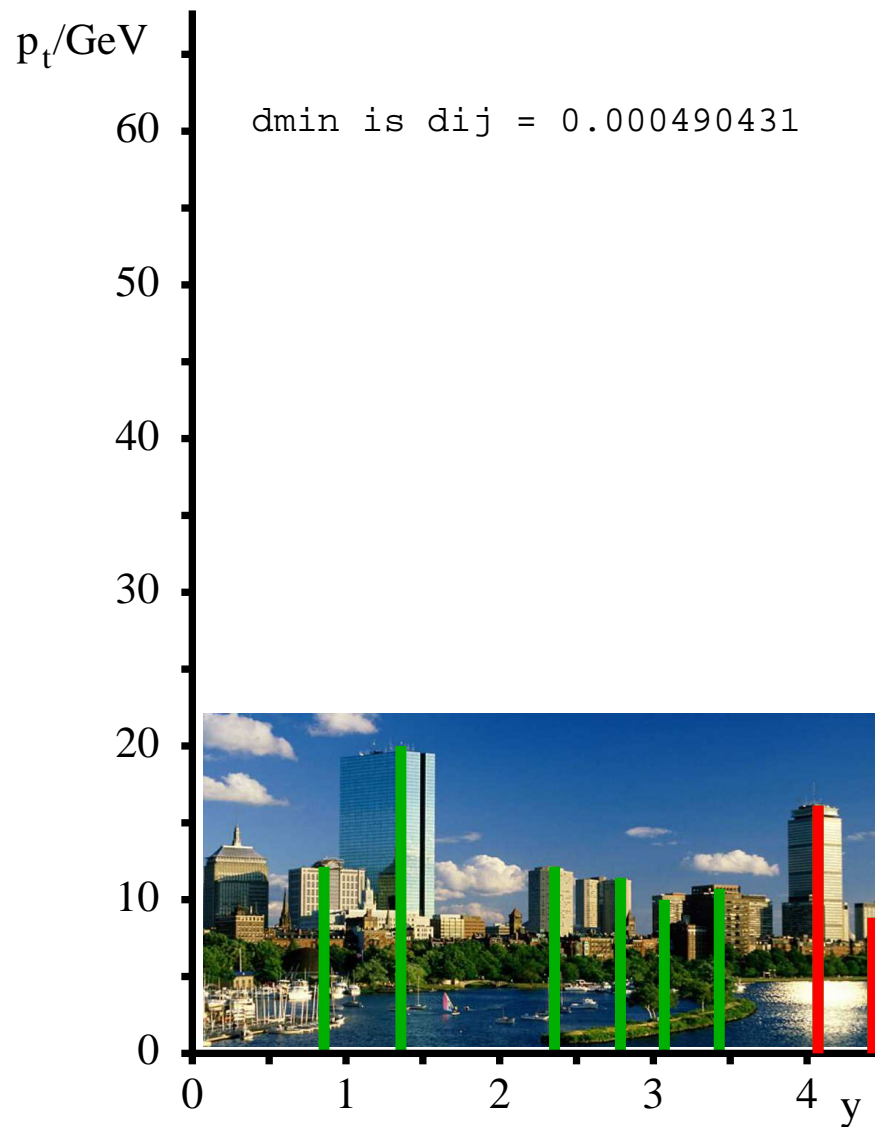


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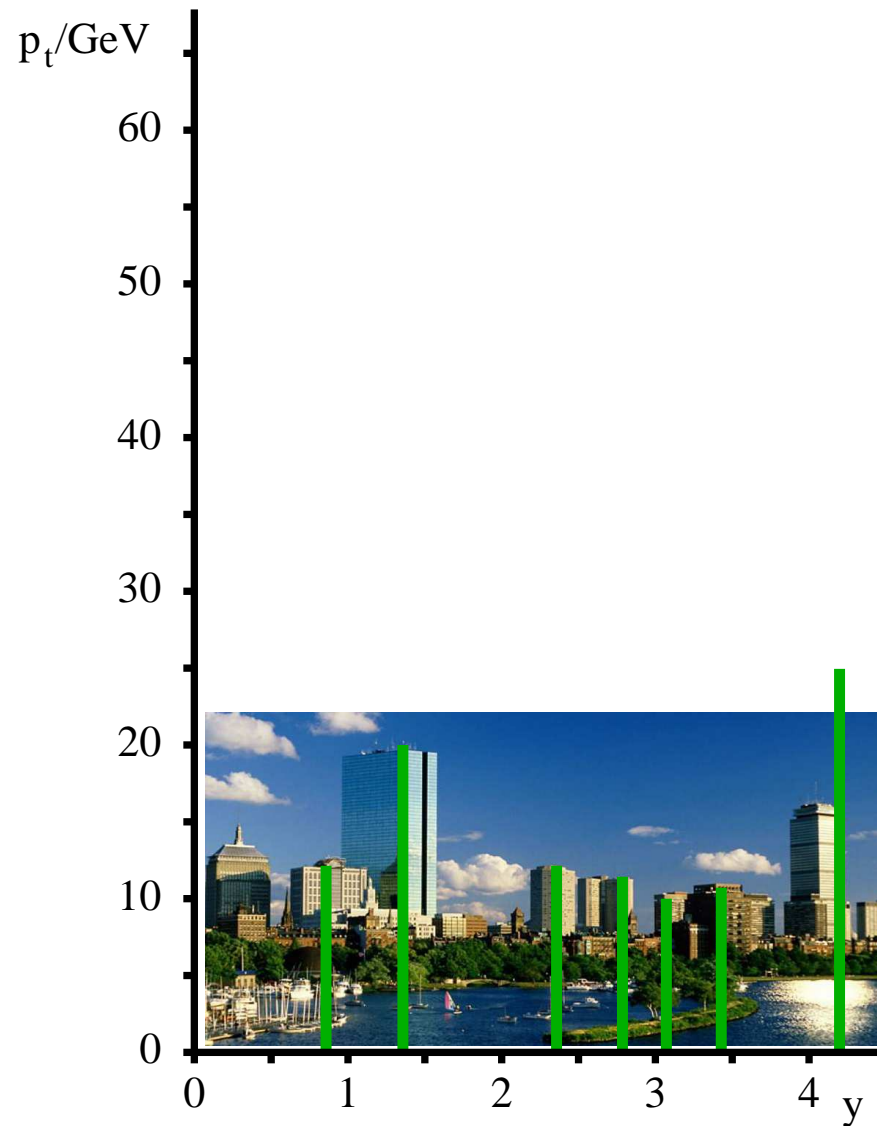
Start with your  
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# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



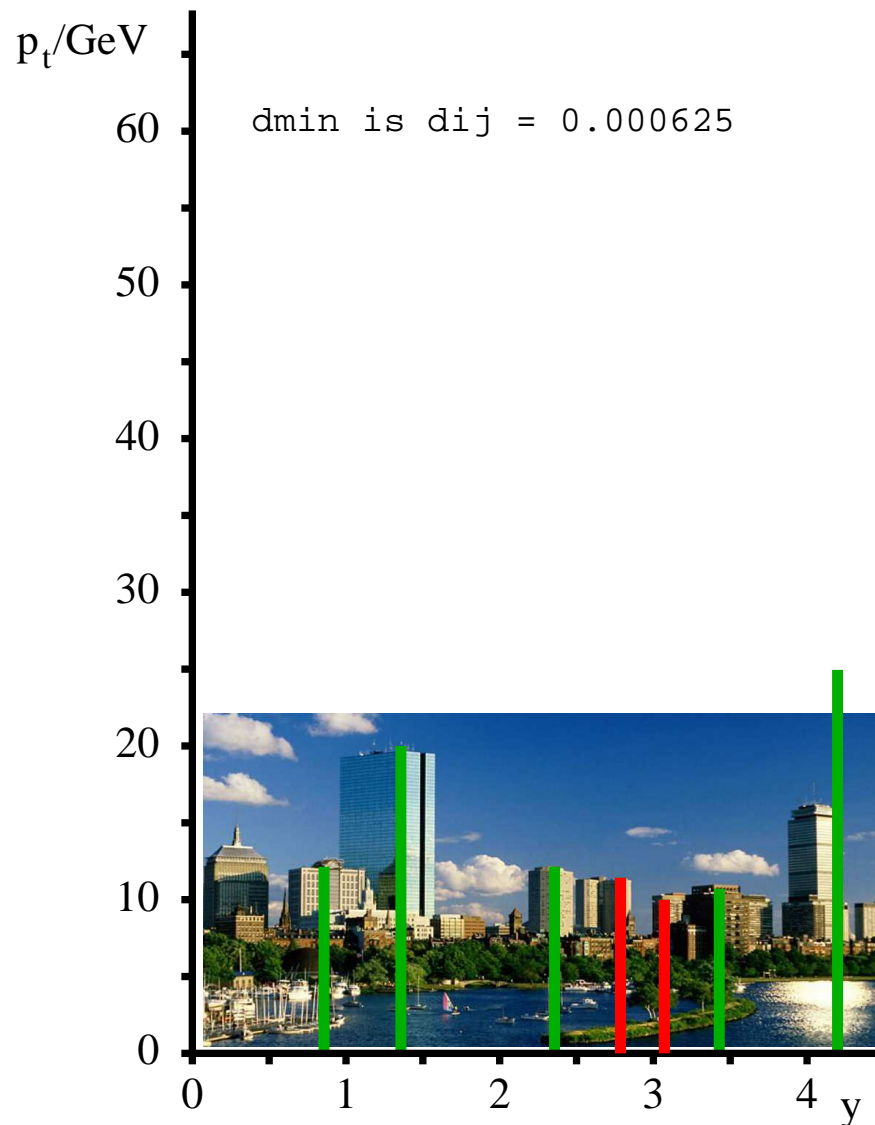
min is  $d_{ij} = 4.9 \cdot 10^{-4}$

# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



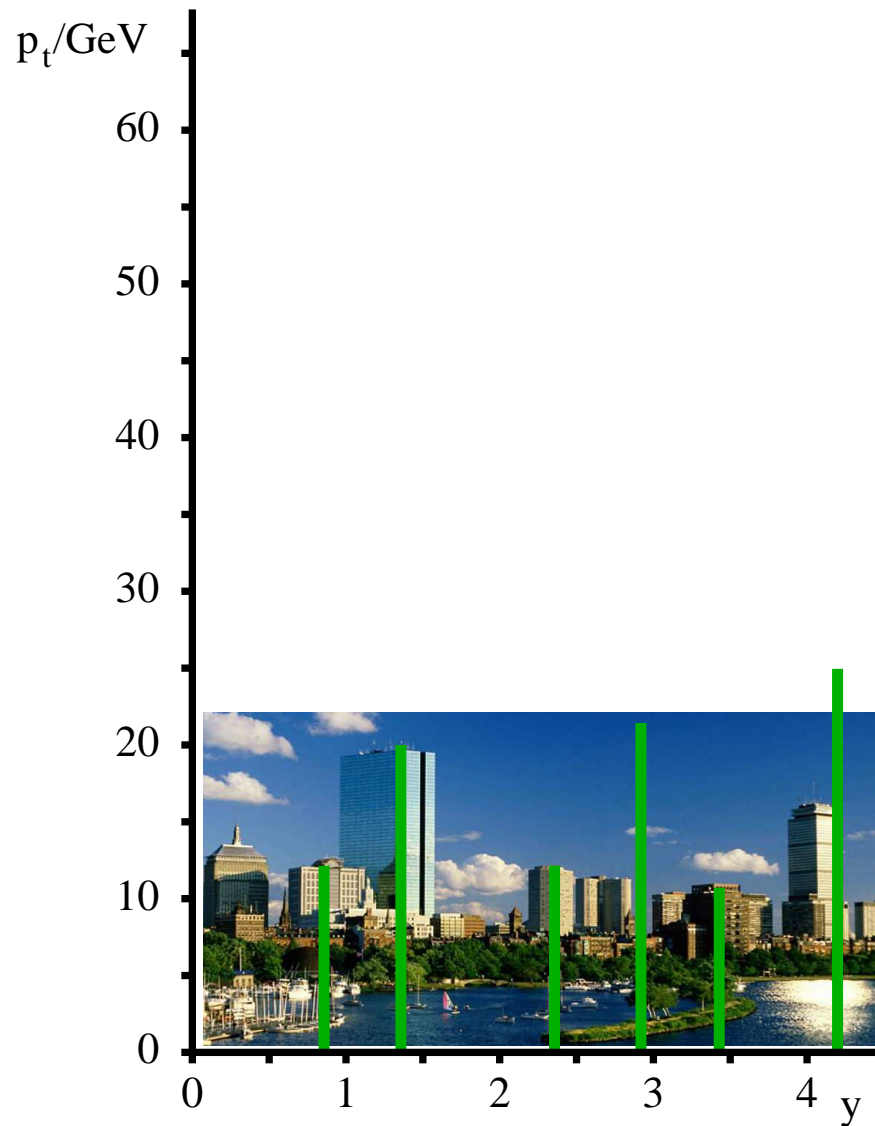
recombine them

# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



min is  $d_{ij} = 6.3 \cdot 10^{-4}$

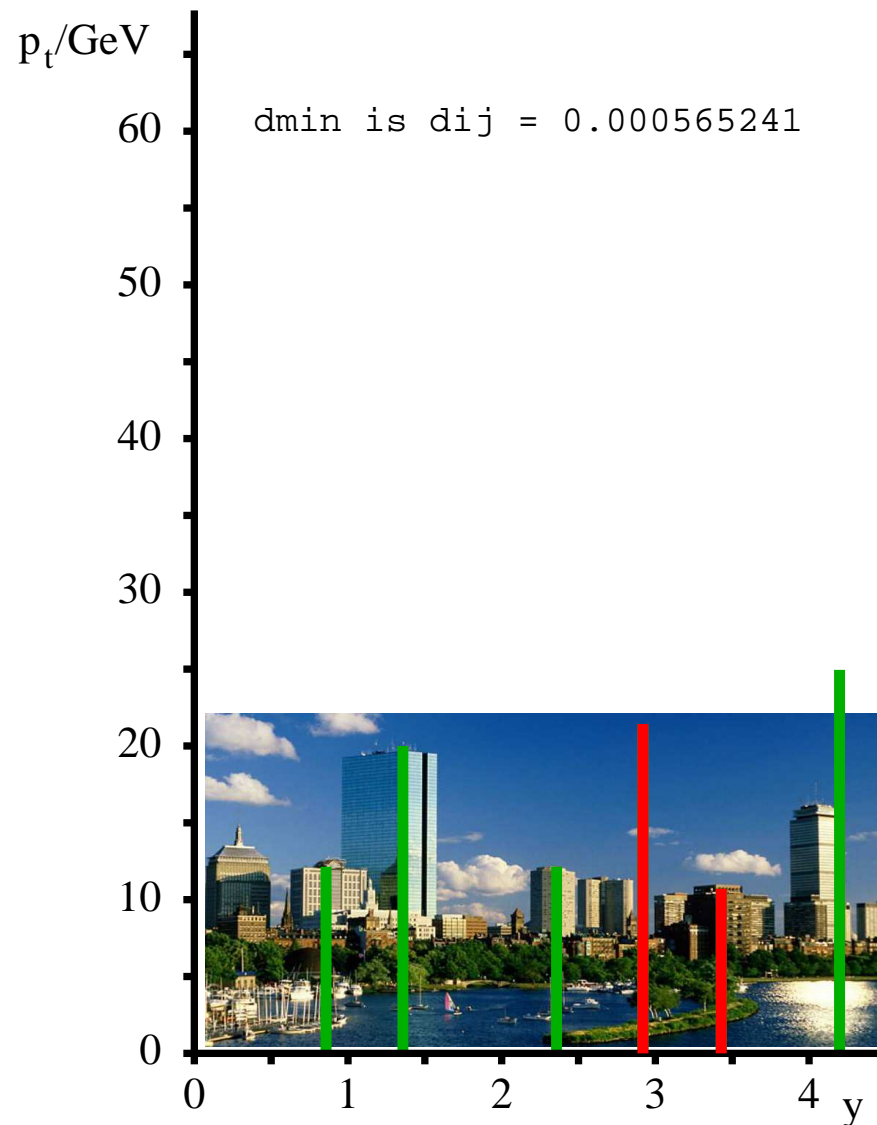
# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



recombine them



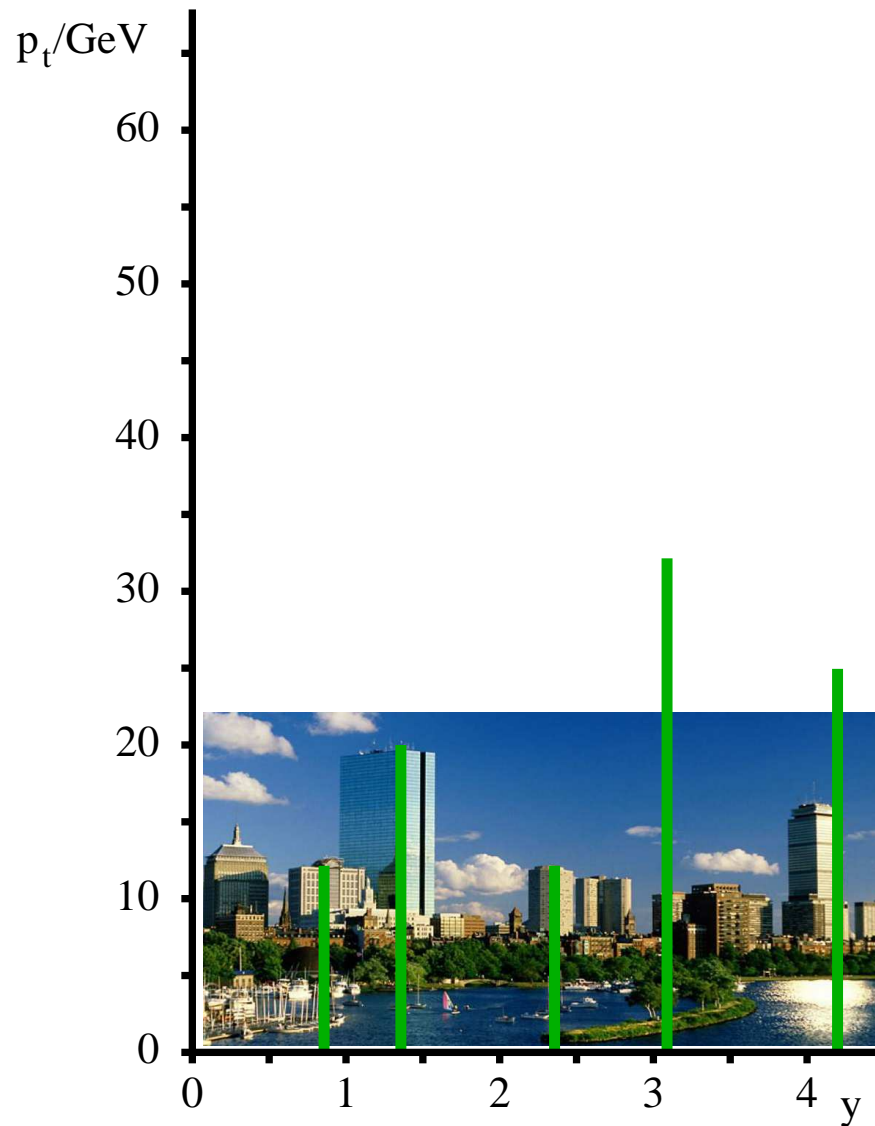
# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



min is  $d_{ij} = 5.7 \cdot 10^{-4}$

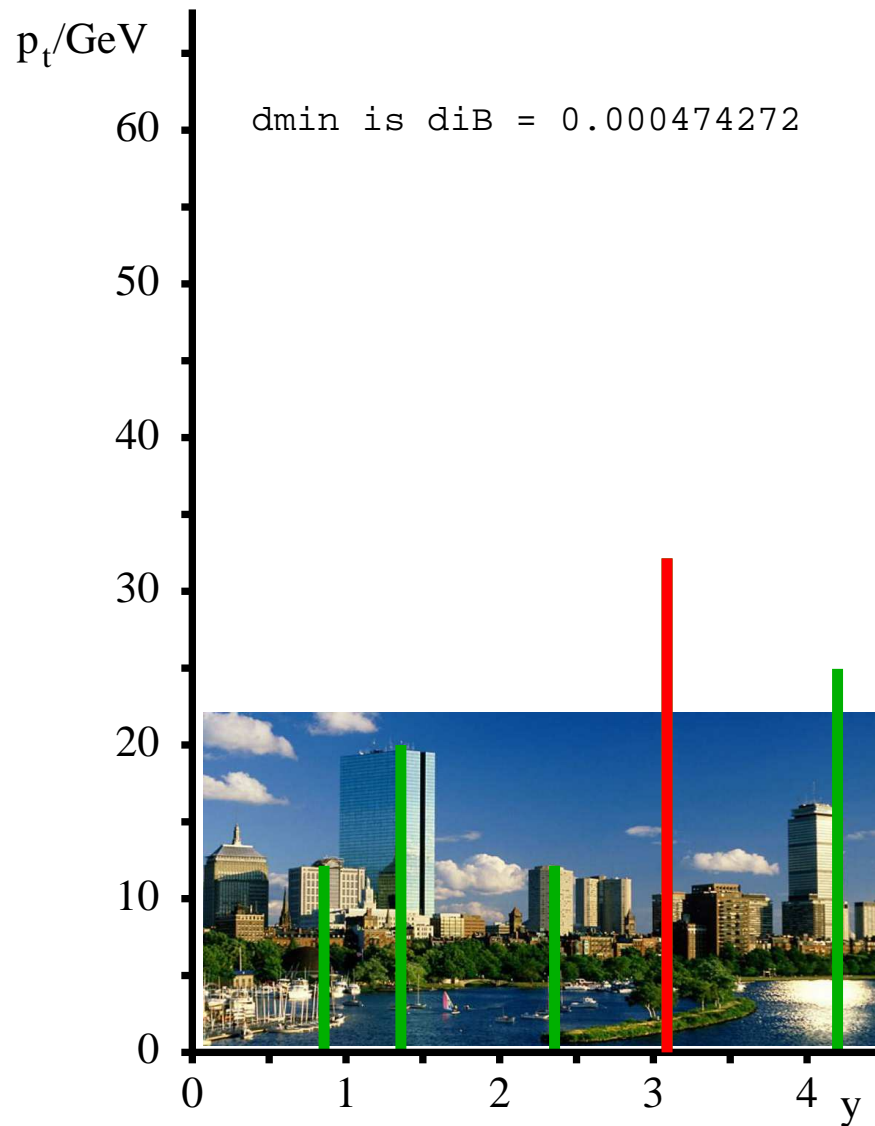


# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



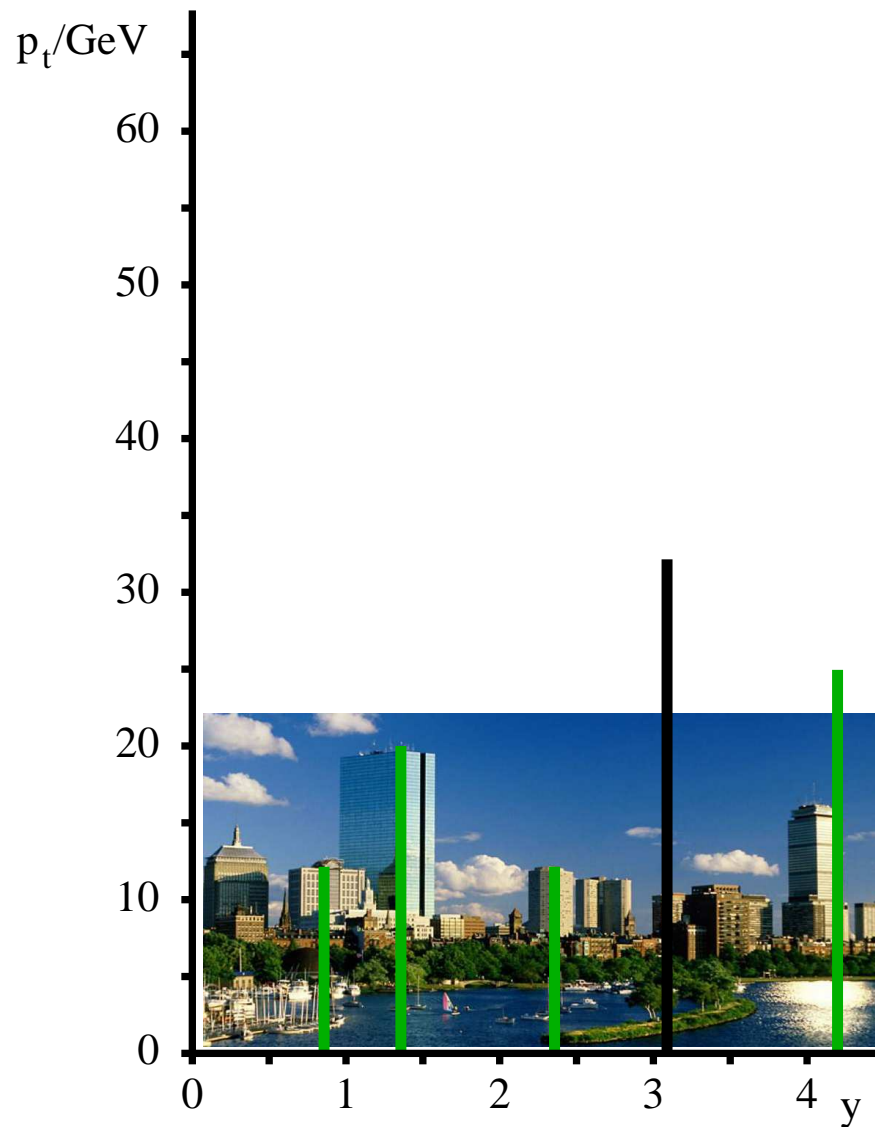
recombine them

# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



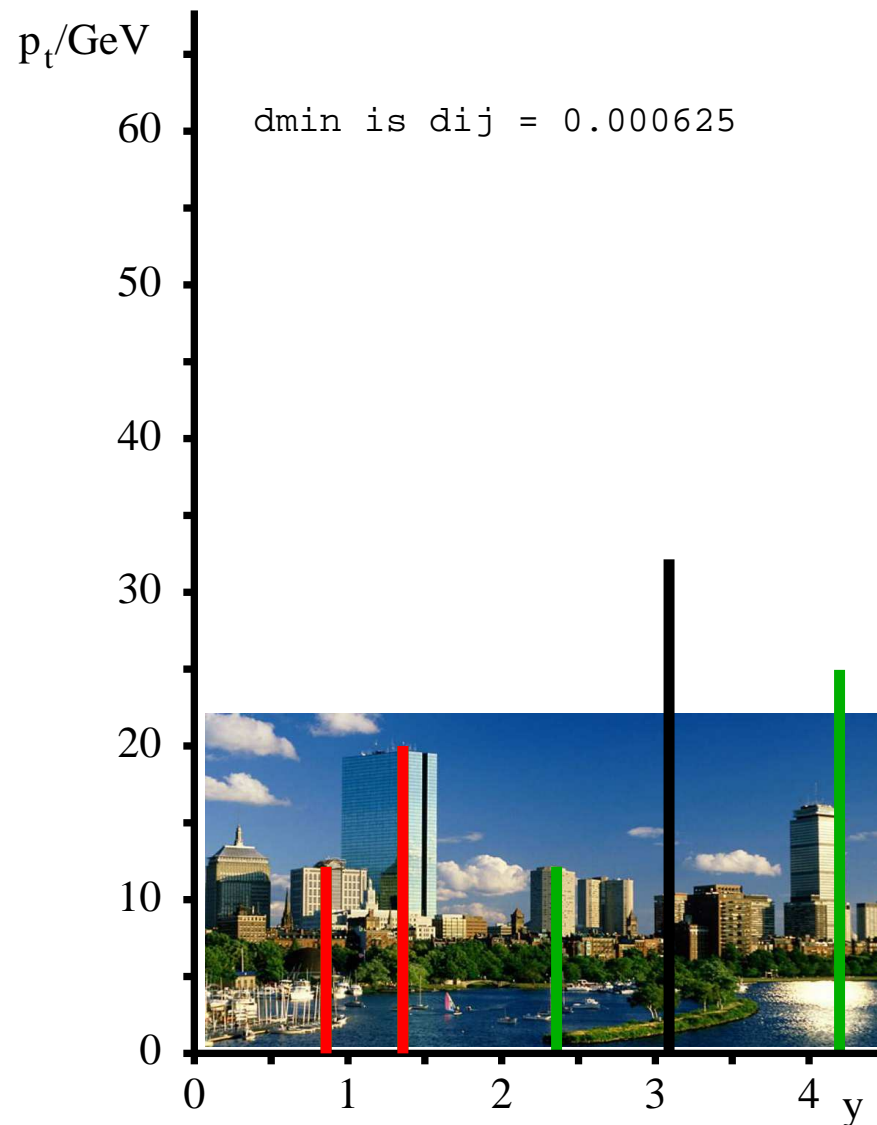
min is  $d_{iB} = 4.7 \cdot 10^{-4}$

# Clustering in action: anti- $k_t$ ( $R = 0.7$ )

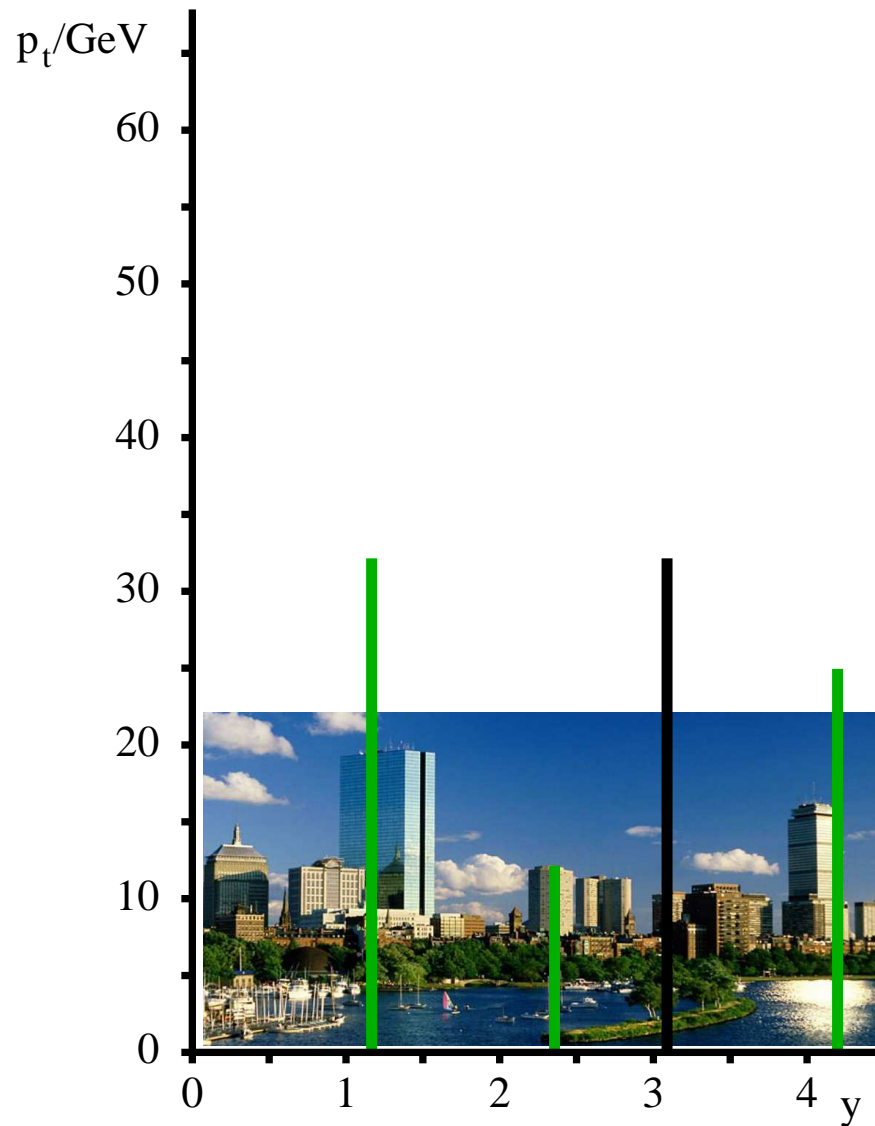


declare as a jet

# Clustering in action: anti- $k_t$ ( $R = 0.7$ )

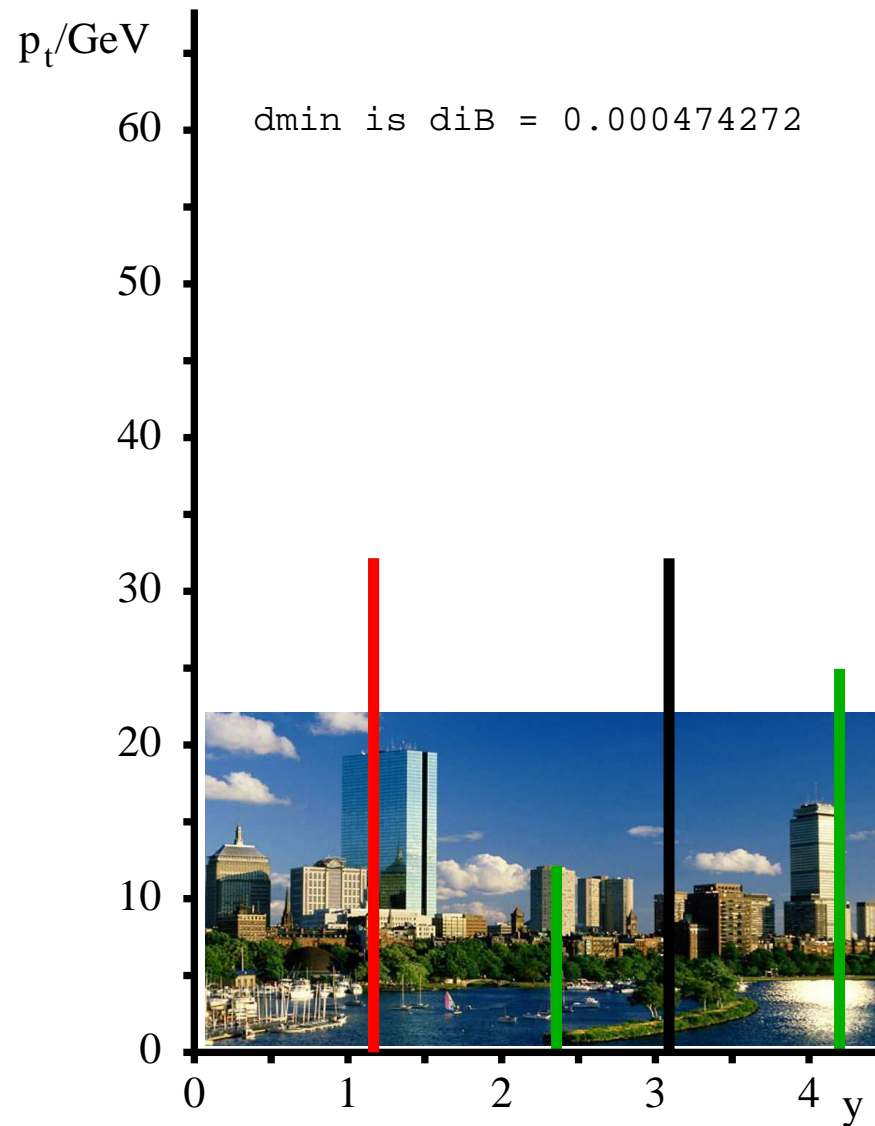


# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



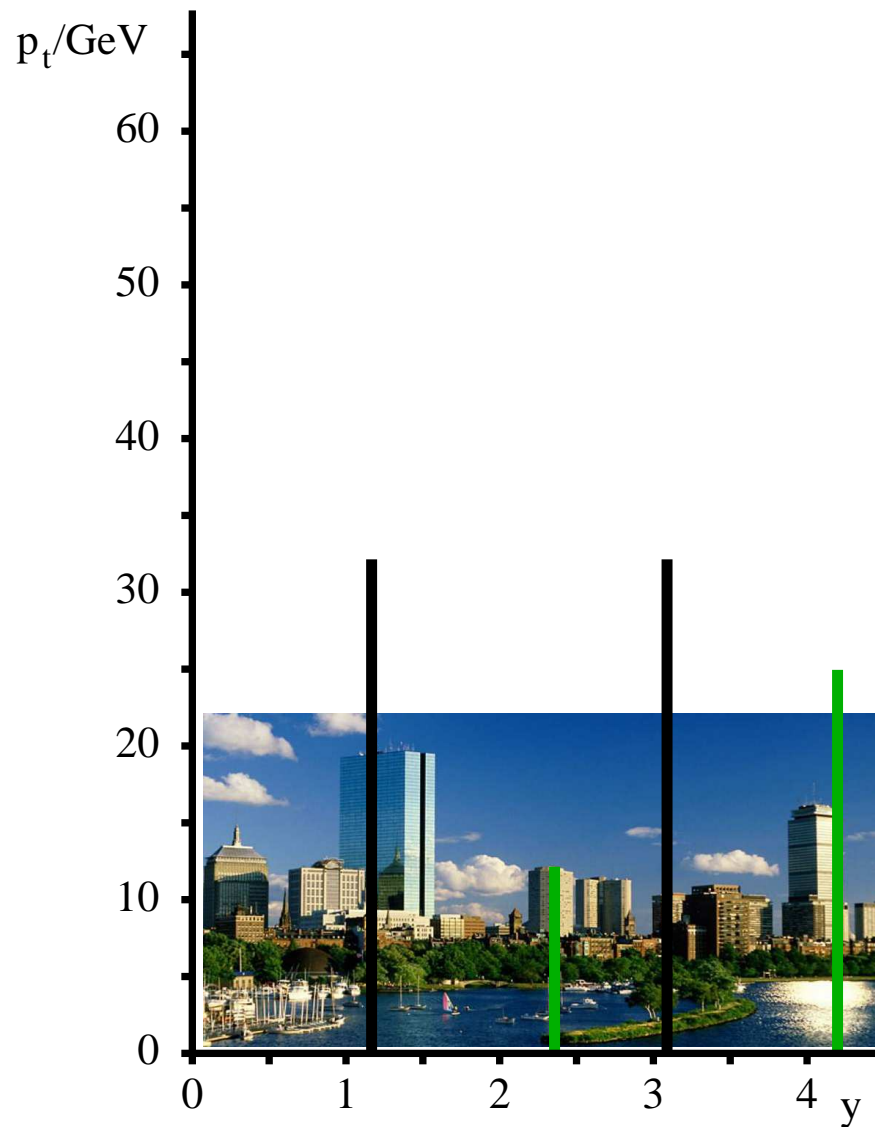
recombine them

# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



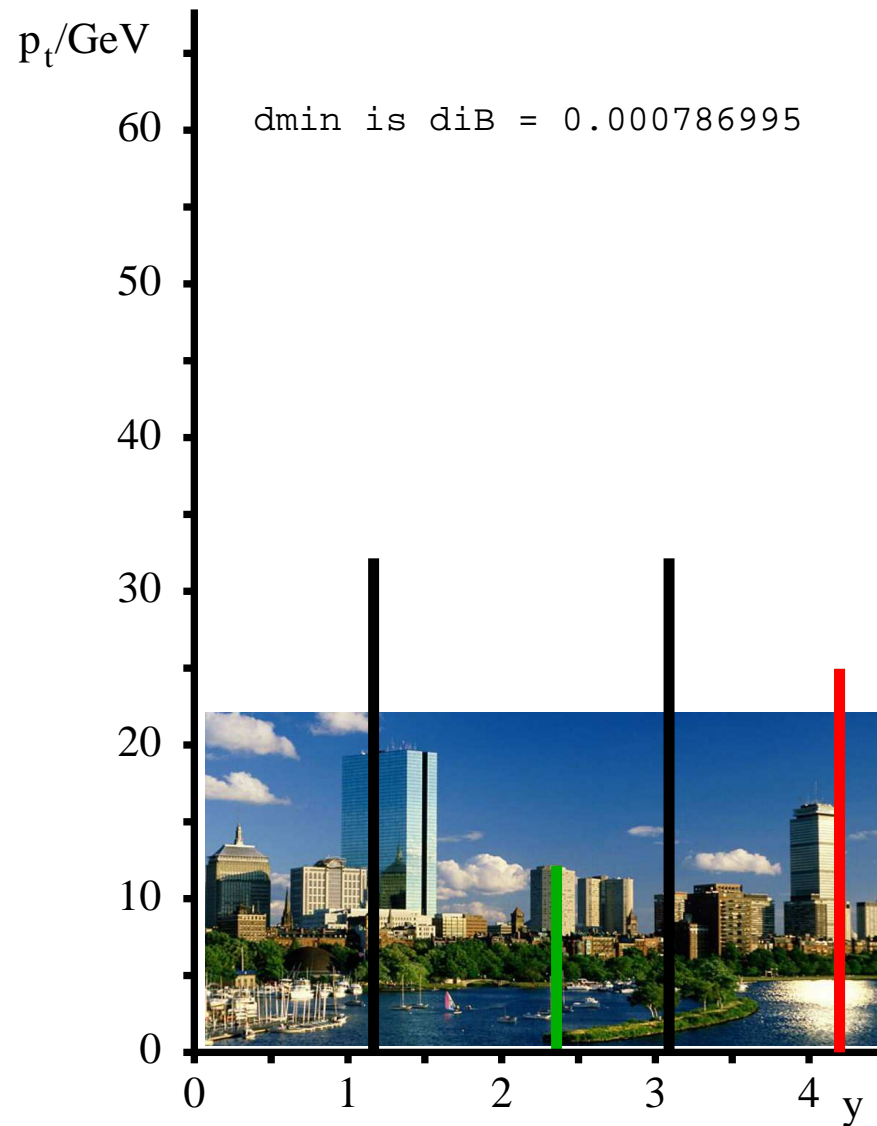
min is  $d_{iB} = 4.7 \cdot 10^{-4}$

# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



declare as a jet

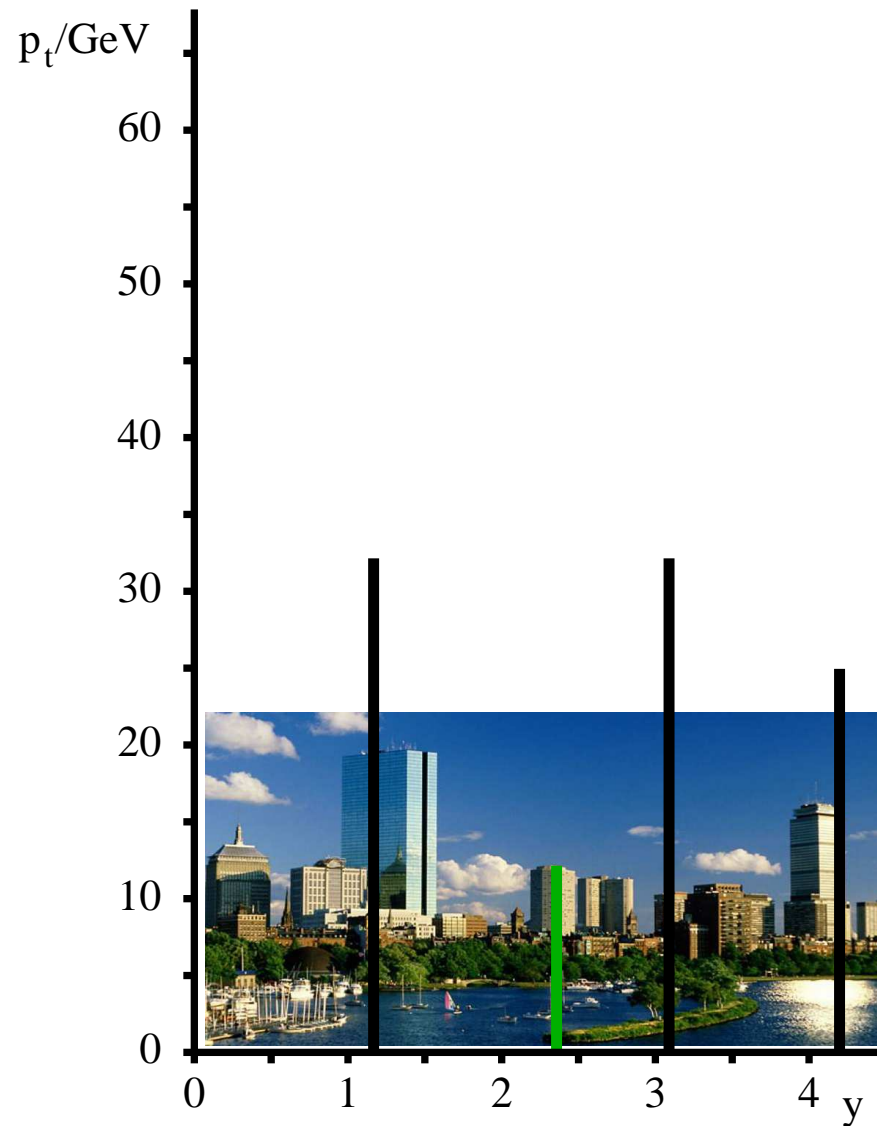
# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



min is  $d_{iB} = 7.9 \cdot 10^{-4}$

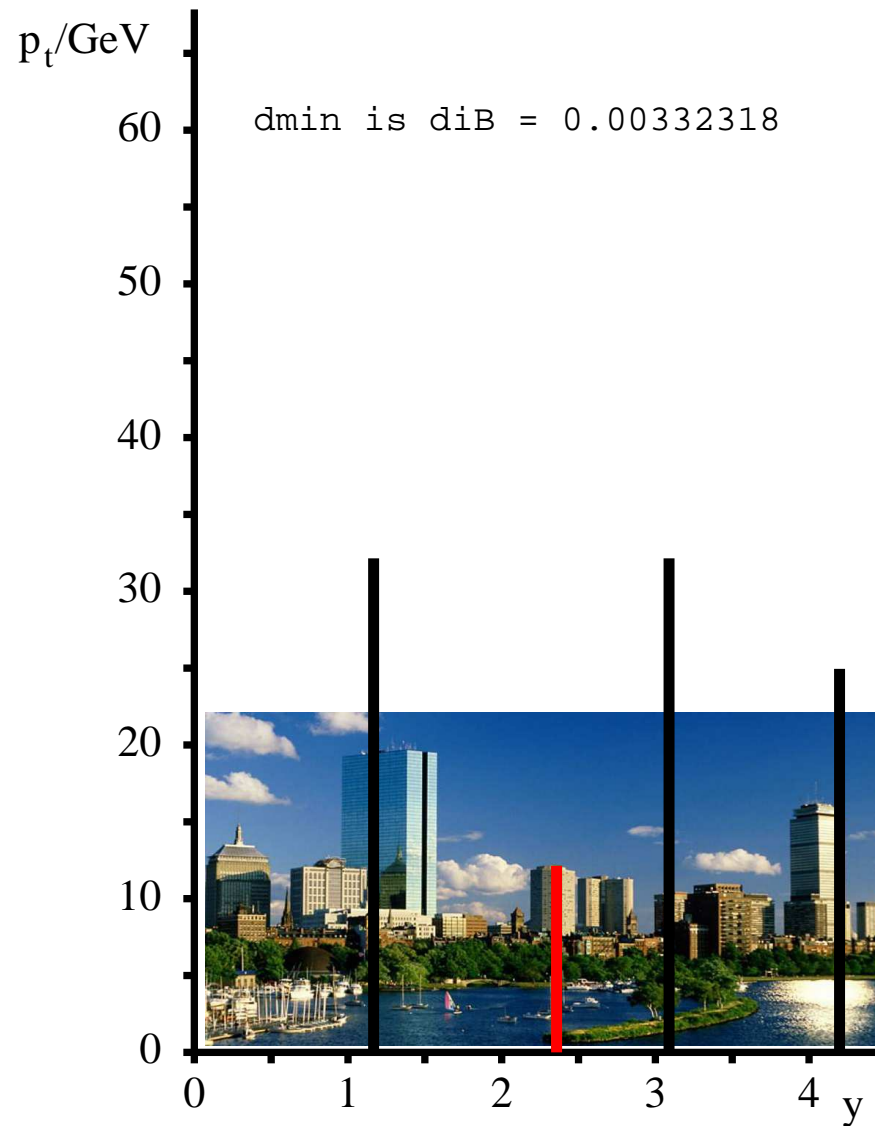


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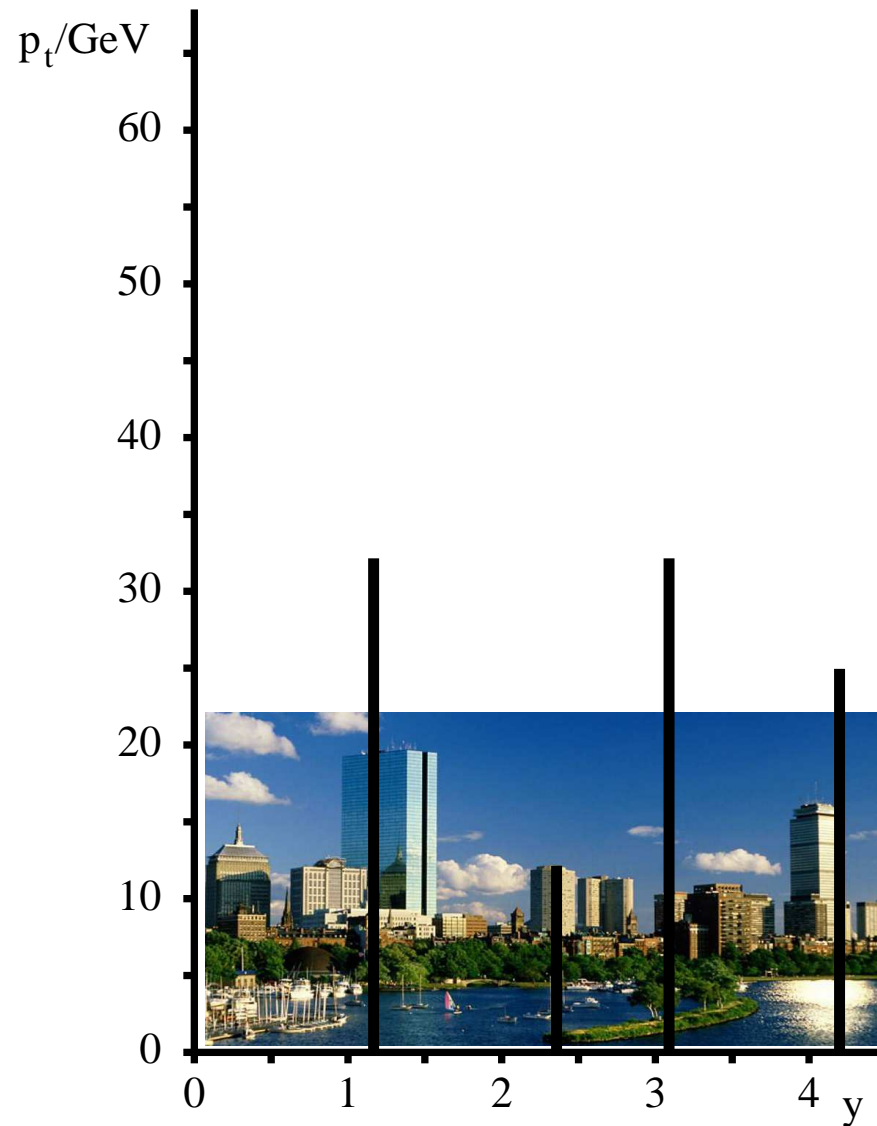
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# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



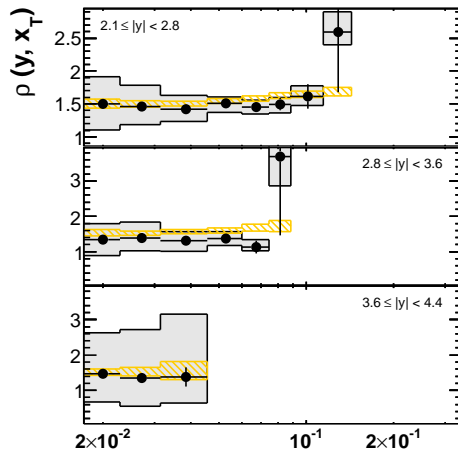
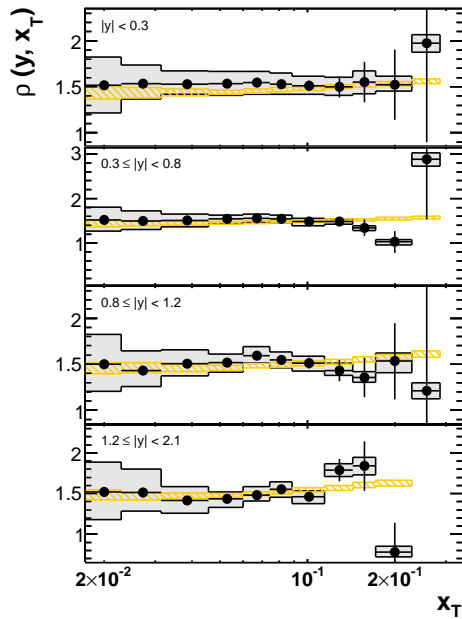
min is  $d_{iB} = 3.3 \cdot 10^{-3}$

# Clustering in action: anti- $k_t$ ( $R = 0.7$ )



declare as a jet

# Examples



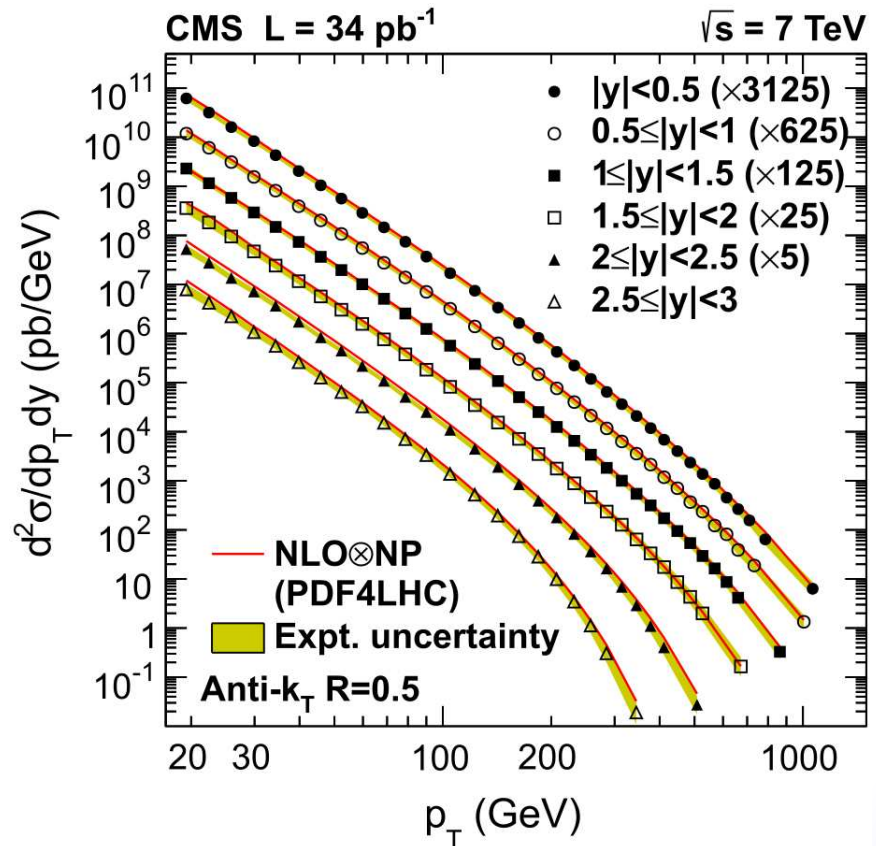
## ATLAS

$$\int L dt = 0.20 \text{ pb}^{-1}$$

$$\rho = \left[ \frac{2.76 \text{ TeV}}{7 \text{ TeV}} \right]^3 \frac{\sigma_{\text{jet}}^{2.76 \text{ TeV}}}{\sigma_{\text{jet}}^{7 \text{ TeV}}}$$

anti- $k_r$ ,  $R = 0.6$

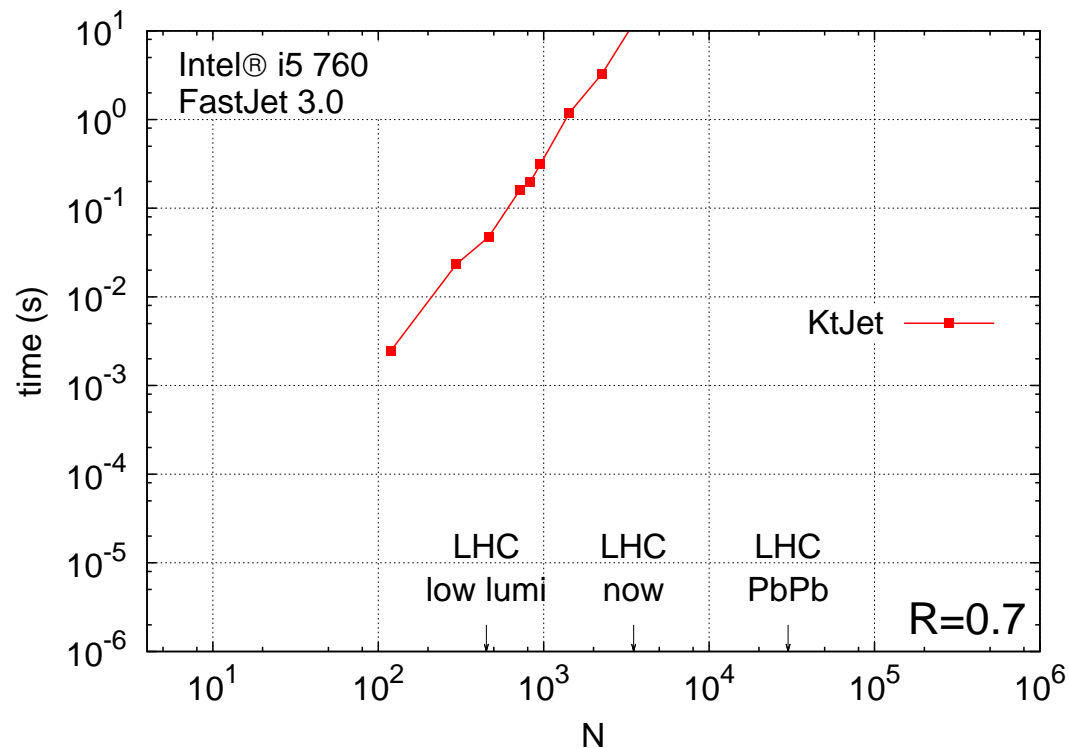
- Data with statistical uncertainty
- Systematic uncertainties
- ⊗ NLO pQCD
- ▨ non-pert. corr. (CT10,  $\mu = p_T^{\text{max}}$ )



# *Implementation*

[M.Cacciari, G.Salam, 2005]

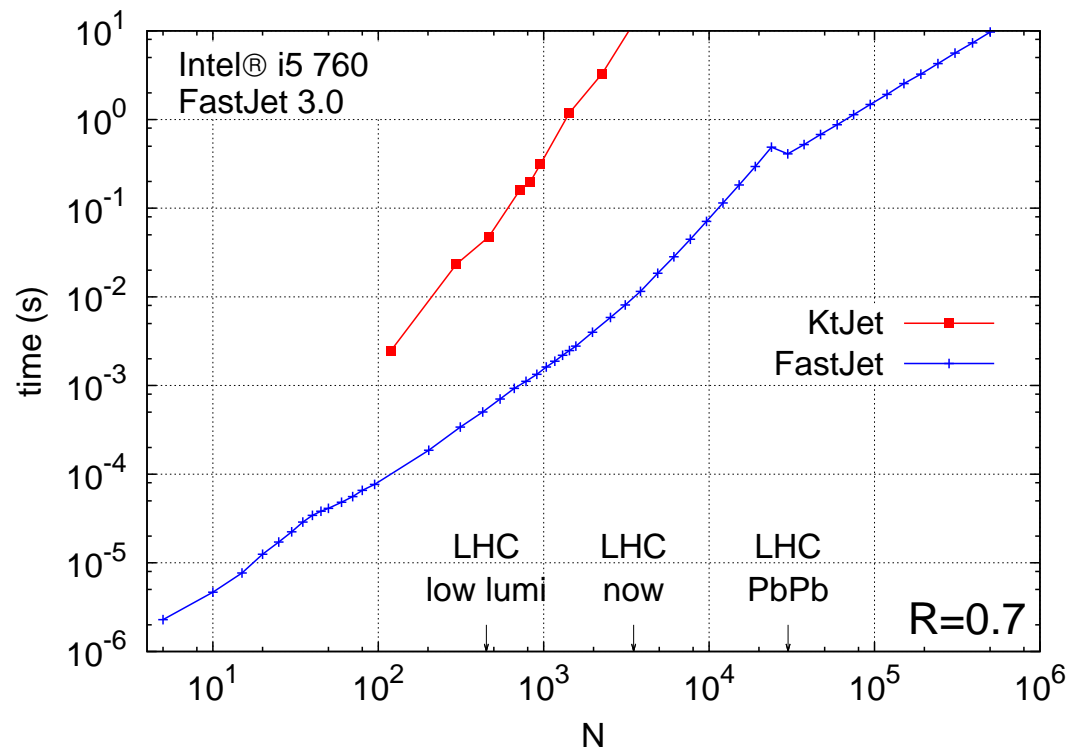
- Tevatron era:  $k_t$  too slow:  $\mathcal{O}(N^3)$  for  $N$  particles



# FastJet (1/2)

[M.Cacciari, G.Salam, 2005]

- Tevatron era:  $k_t$  too slow:  $\mathcal{O}(N^3)$  for  $N$  particles
- Now: (anti-) $k_t$  very fast:  $\mathcal{O}(N^2)$  or even  $\mathcal{O}(N \log(N))$ 
  - the “FastJet lemma”: min distance is a Nearest Neighbour
  - use of computational geometry *e.g.* Voronoi diagram



[M.Cacciari, G.Salam, GS, 2007-2013]

- Grown way beyond just fast recombinations:
  - plugins for used jet definitions
  - jet areas and background subtraction (see below)
  - tools for manipulating jets
  - more to come...
- FastJet 3.0.3 released in June 2012  
see [www.fastjet.fr](http://www.fastjet.fr)
- Standard interface for jet physics  
for both theorists and experimentalists



# FastJet contrib (New: Feb 2013)

- [fastjet.fr](#)
- [fastjet-contrib](#)
- [contrib svn](#)

## FastJet Contrib

The fastjet-contrib space is intended to provide a common location for access to 3rd party extensions of FastJet.

**Download** the current version: [fjcontrib-1.003](#) (released 1 May 2013), which contains [these contributions](#). Changes relative to earlier versions are briefly described in the [NEWS](#) file.

Package	Version	Information
<a href="#">GenericSubtractor</a>	1.1.0	<a href="#">README</a> <a href="#">NEWS</a>
<a href="#">JetFFMoments</a>	1.0.0	<a href="#">README</a> <a href="#">NEWS</a>
<a href="#">VariableR</a>	1.0.1	<a href="#">README</a> <a href="#">NEWS</a>
<a href="#">Nsubjettiness</a>	1.0.2	<a href="#">README</a> <a href="#">NEWS</a>
<a href="#">EnergyCorrelator</a>	1.0.1	<a href="#">README</a> <a href="#">NEWS</a>

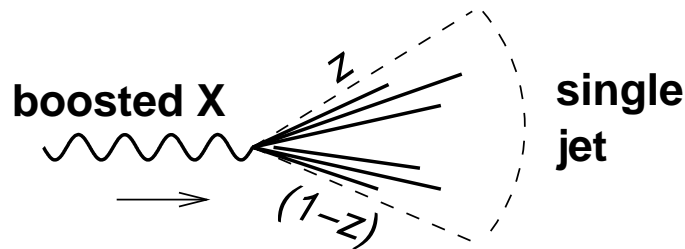
- a quick and uniform access to 3rd-party code
- contributors are welcome (please contact us)

***Jet substructure***  
***Boosted object taggers***

# Fat jets

## Problem:

boosted heavy object  $\Rightarrow$  decays in a **single jet**



$$R \gtrsim \frac{m}{p_t} \frac{1}{\sqrt{z(1-z)}}$$

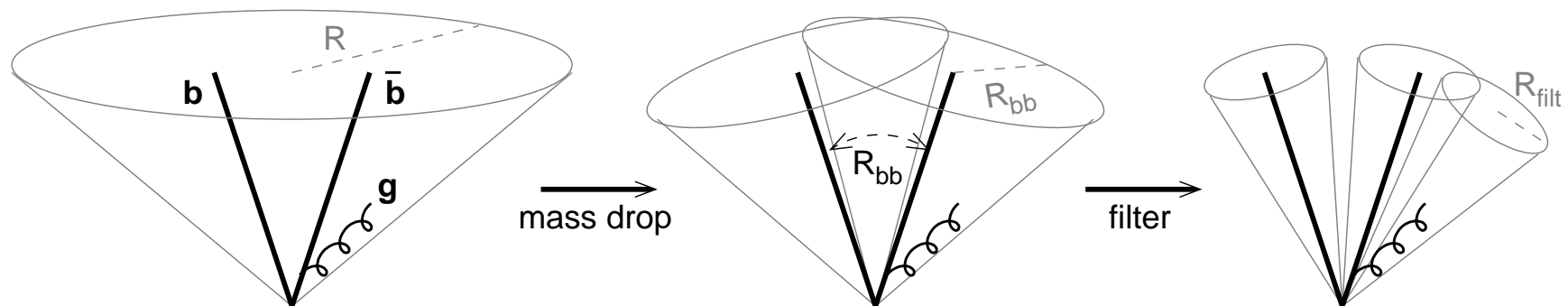
**How to disentangle that from a QCD jet?**

## Many applications: (examples)

- 2-pronged decay:  $W \rightarrow q\bar{q}$ ,  $H \rightarrow b\bar{b}$
- 3-pronged decay:  $t \rightarrow qqb$ ,  $\tilde{\chi} \rightarrow qqq$
- busier combinations:  $t\bar{t}H$
- new physics: e.g. heavy SUSY  $\rightarrow$  boosted top



# Many tools

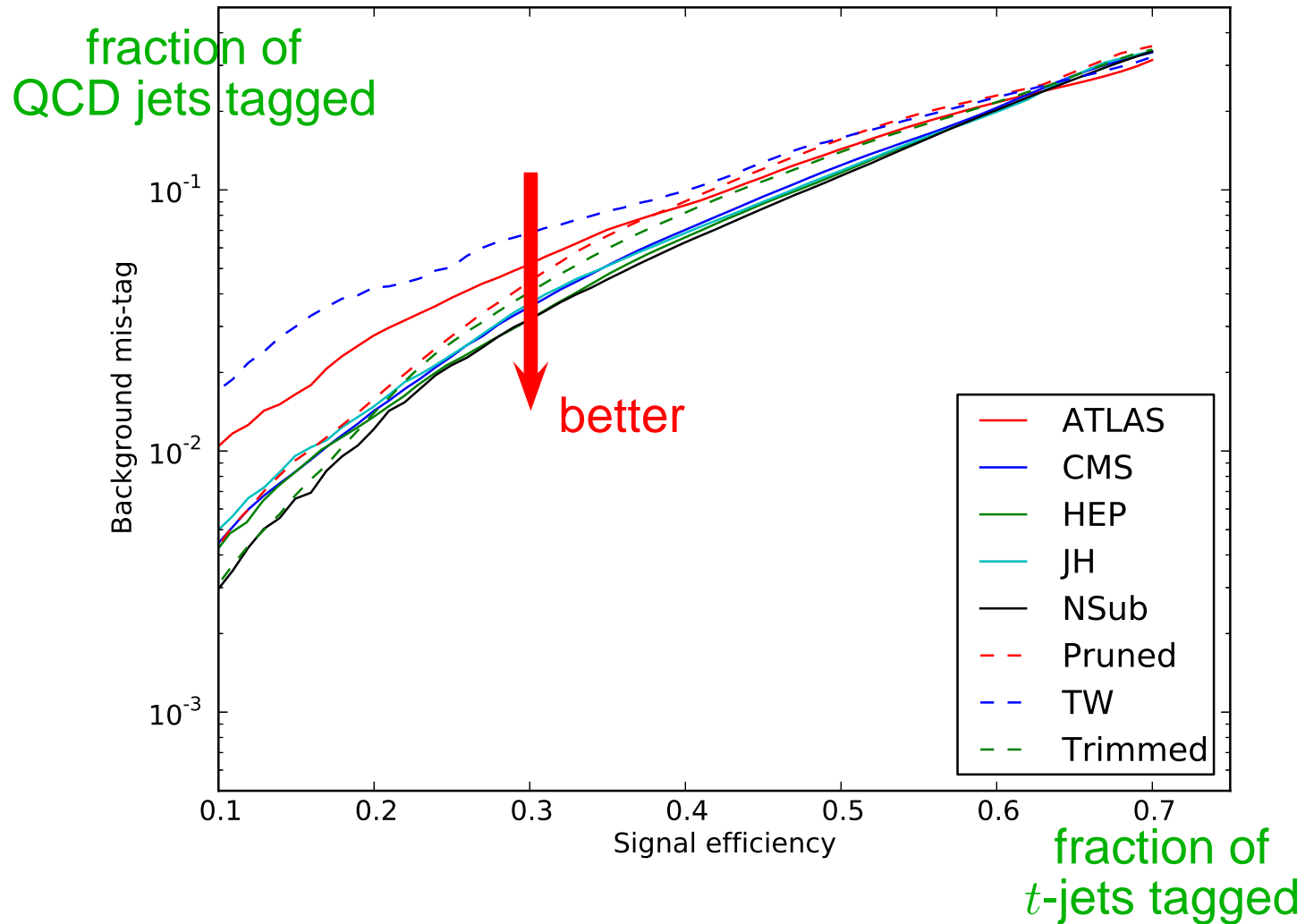


- Main idea: get rid of soft/collinear branchings (typical of QCD)
- Many approaches:
  - uncluster the jet into subjets/investigate the clustering history
  - constrain radiation pattern imposing kinematic cuts...
  - ... or using jet shapes (functions of jet constituents)
- Many tools: mass drop; filtering, trimming, pruning;  $N$ -subjettiness, planar flow, energy correlations, pull; template methods; Johns Hopkins top tagger, HEPTopTagger; ...

# Example: top tagging

[Boost 2011 proceedings]

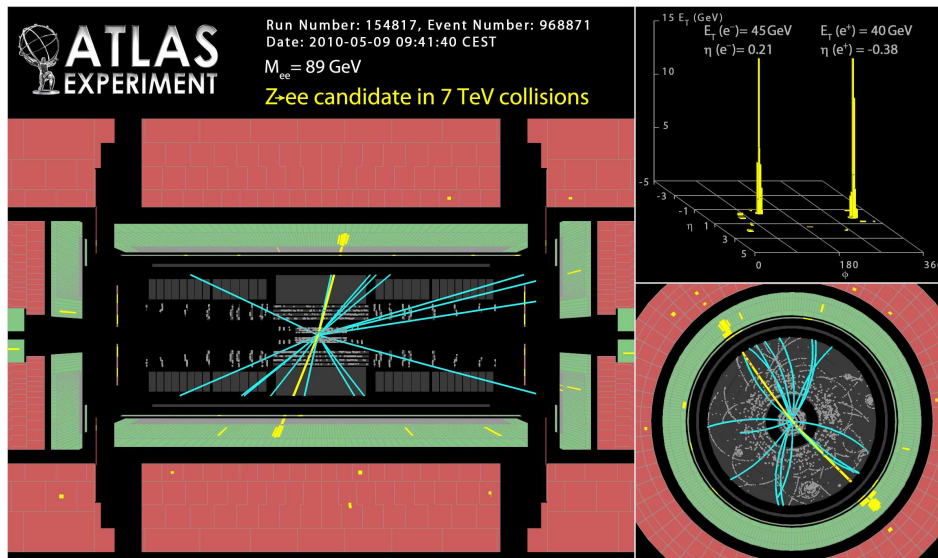
Sherpa 1.3.1 — anti- $k_t$  (R=0.1) jets,  $p_t > 200$  GeV



# ***Jets in soft background***

## $Z \rightarrow l^+ l^-$ candidate at ATLAS

Low luminosity  
(bunch population)

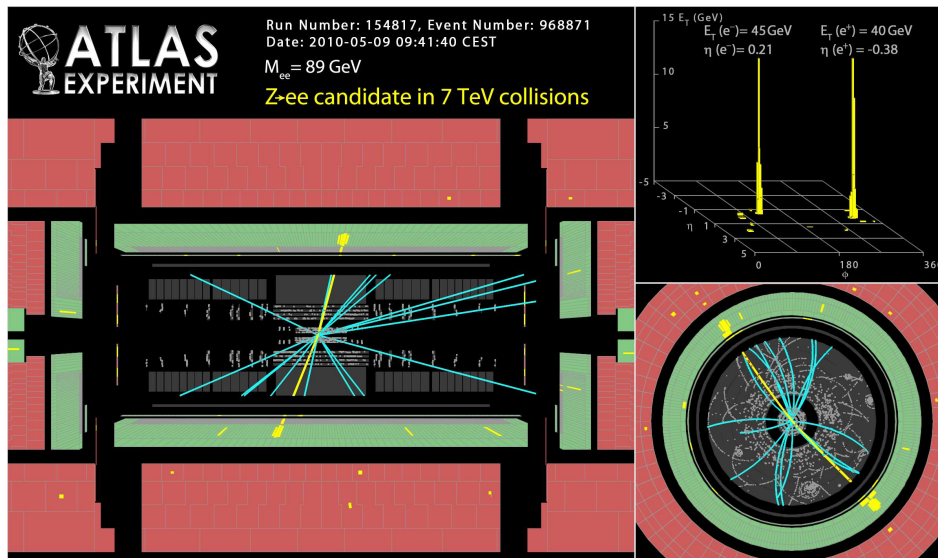




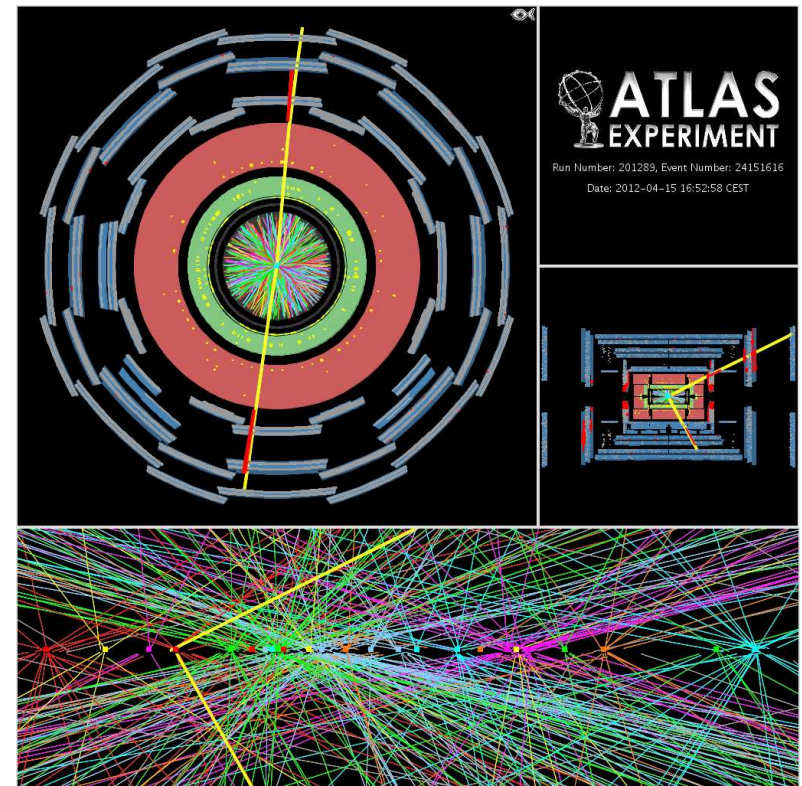
# Pileup

## $Z \rightarrow \ell^+ \ell^-$ candidate at ATLAS

Low luminosity  
(bunch population)

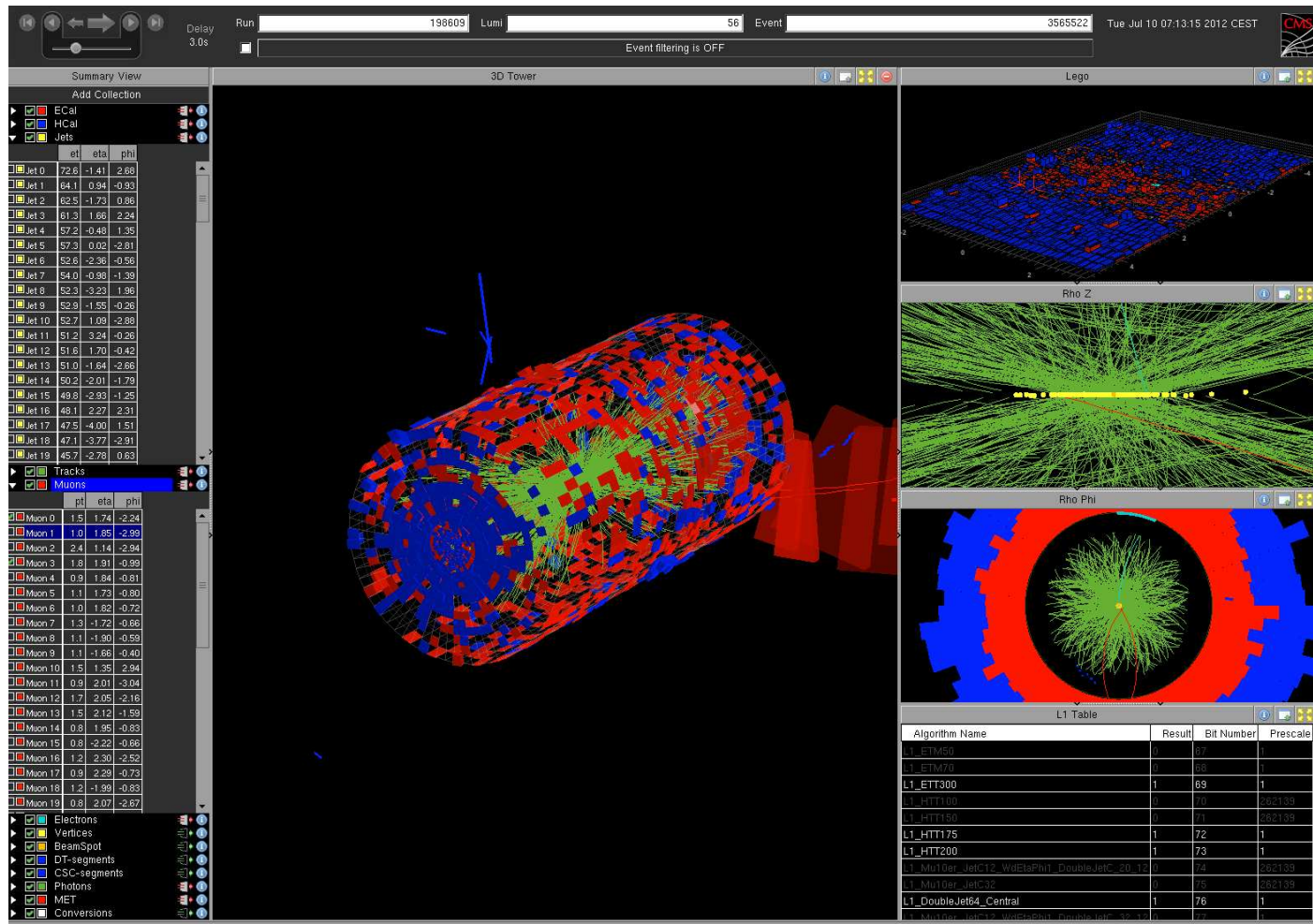


High luminosity  
(bunch population)



- many (soft)  $pp$  interactions with the hard one (here 25)
- soft background in the whole detector

## A CMS event with 78 pile-up vertices!

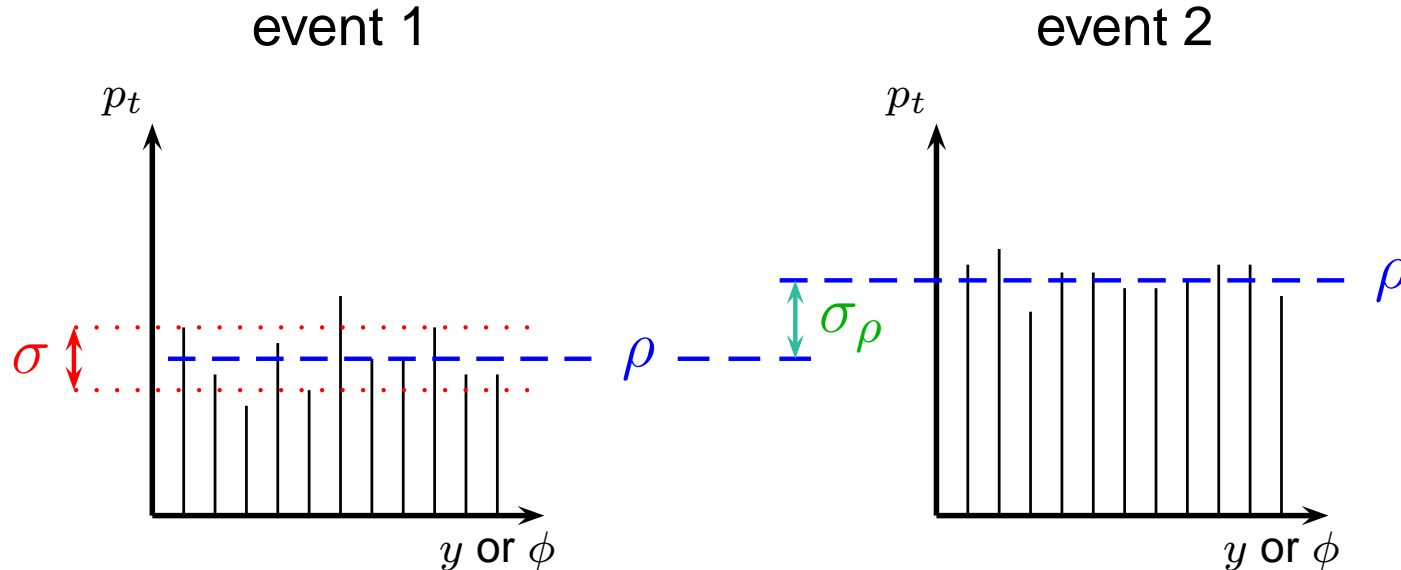


End of Run I: 30 PU vertices on average

# Basic characterisation

Pileup mostly characterised by 3 numbers (\*):

- $\rho$ : the average activity in an event (per unit area)
- $\sigma$ : the intra-event fluctuations (per unit area)
- $\sigma_\rho$ : the event-to-event fluctuations of  $\rho$



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Jet of momentum  $p_t$  and area  $A$ :

$$\text{one event: } p_t \rightarrow p_t + \rho A \pm \sigma \sqrt{A}$$

$$\text{event average: } p_t \rightarrow p_t + \langle \rho \rangle A \pm \sigma_\rho A \pm \sigma \sqrt{A}$$

# Basic characterisation

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Jet of momentum  $p_t$  and area  $A$ :

$$\begin{aligned} \text{one event: } p_t &\rightarrow p_t + \boxed{\rho A} \pm \boxed{\sigma \sqrt{A}} \\ \text{event average: } p_t &\rightarrow p_t + \boxed{\langle \rho \rangle A} \pm \boxed{\sigma_\rho A \pm \sigma \sqrt{A}} \end{aligned}$$

$p_t$  **shift**  $p_t$  **smearing**  
resolution degradation

(\*) valid also for the underlying event in heavy-ion collisions

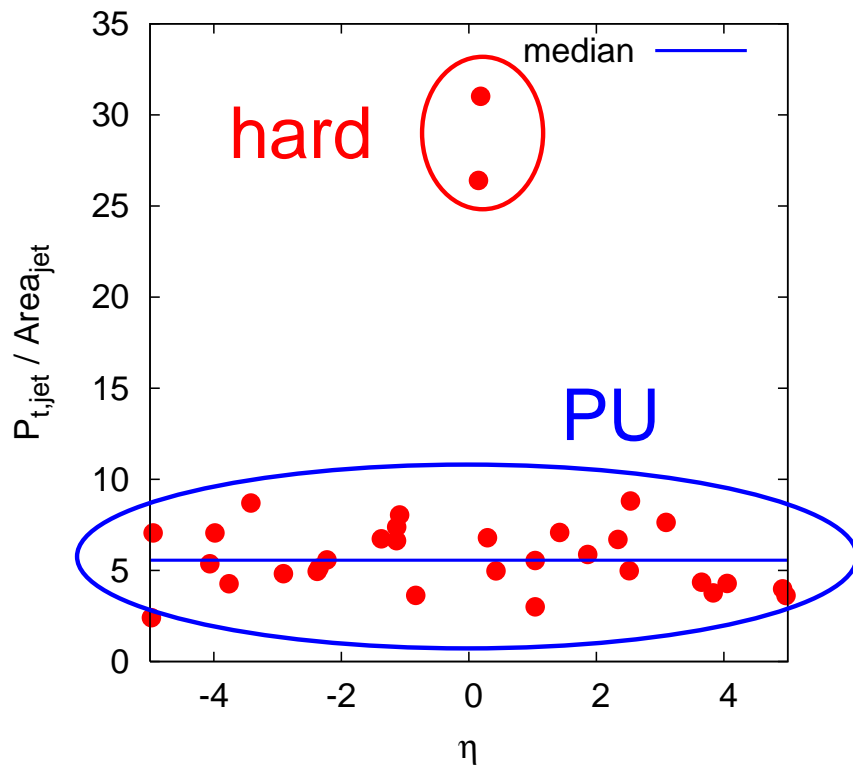
# Median-area-based subtraction

[M.Cacciari, G.P. Salam, 07; M.Cacciari, G.P. Salam, GS, 2008]

$$p_{t,\text{jet}}^{(\text{sub})} = p_{t,\text{jet}} - \rho_{\text{est}} A_{\text{jet}}$$

$$\rho_{\text{est}} = \text{median}_{j \in \text{patches}} \left\{ \frac{p_{t,j}}{A_j} \right\}$$

per event (typically)      per jet



break the event in  
patches of similar size  
e.g. cluster with  $k_t$




# Subtraction methods (correct for the shift)

one **subtracts** a contribution from individual jets

subtracted	PU kept
constant $p_t$ ( $\langle \rho A \rangle$ )	both flucts + area flucts
$\langle \rho \rangle \times A$	both flucts ( $\sigma \sqrt{A}$ & $\sigma_\rho A$ )
$\langle \rho \rangle_{\text{per PU vertex}} \times n_{PU} \times A$	$\sigma \sqrt{A}$ and part of $\sigma_\rho A$
$\rho_{\text{event}} \times A$	only $\sigma \sqrt{A}$

# Subtraction methods (correct for the shift)

one **subtracts** a contribution from individual jets

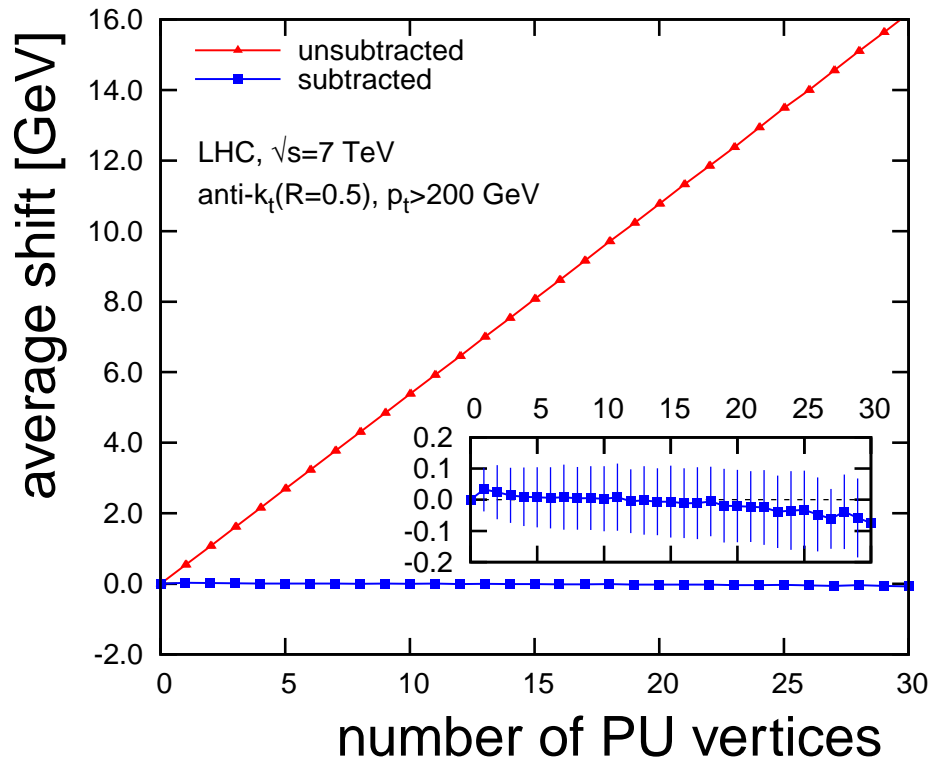
subtracted	PU kept	more averaged
constant $p_t$ ( $\langle \rho A \rangle$ )	<b>both flucts + area flucts</b>	 'event-by-event'
$\langle \rho \rangle \times A$	<b>both flucts (<math>\sigma\sqrt{A}</math> &amp; <math>\sigma_\rho A</math>)</b>	
$\langle \rho \rangle_{\text{per PU vertex}} \times n_{PU} \times A$	<b><math>\sigma\sqrt{A}</math> and part of <math>\sigma_\rho A</math></b>	
$\rho_{\text{event}} \times A$	<b>only <math>\sigma\sqrt{A}</math></b>	

**Event-by-event determinations of the shift (are expected to)  
reduce the smearing effects of PU**



# Subtraction benchmarks

average  $p_t$  shift



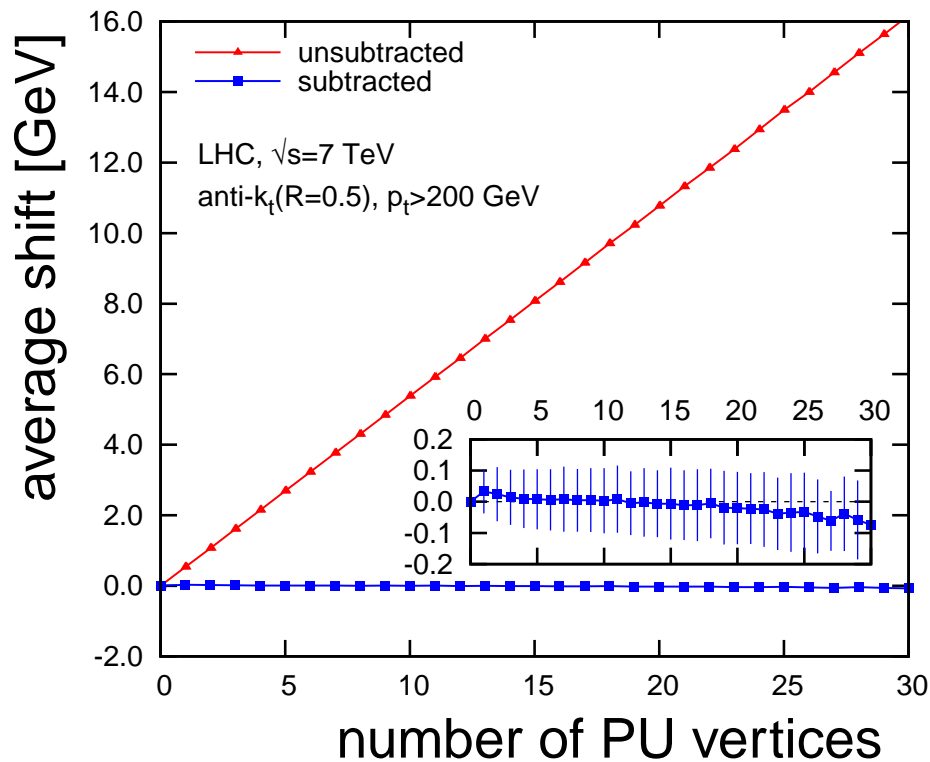
No subtraction

area-median subtraction

corrected for shift

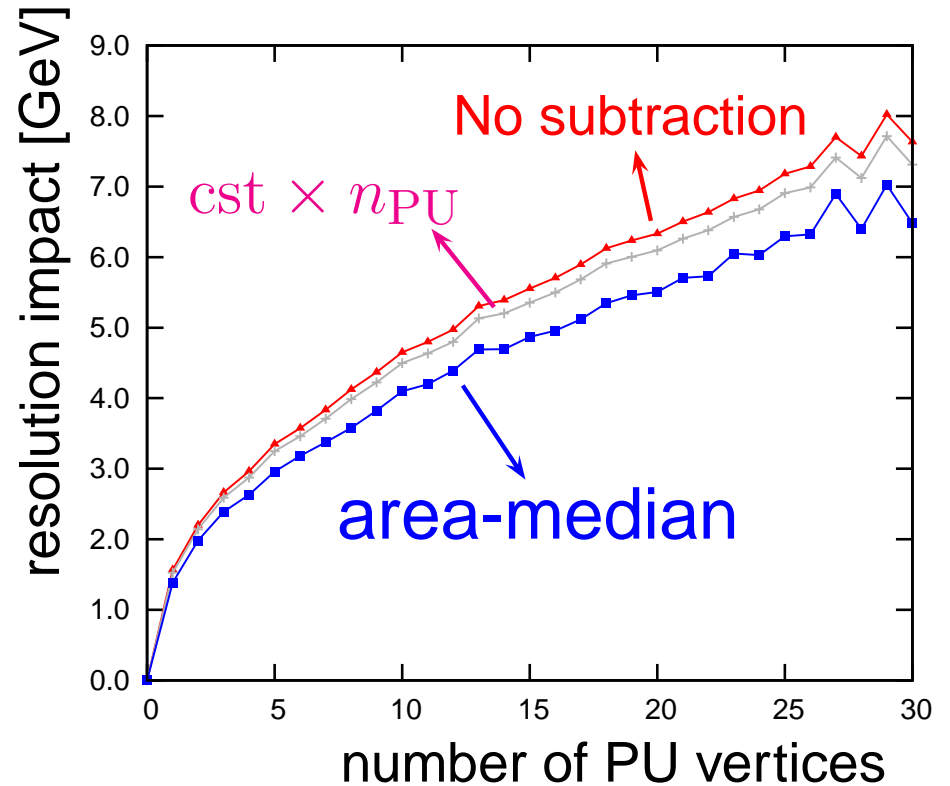
# Subtraction benchmarks

## average $p_t$ shift



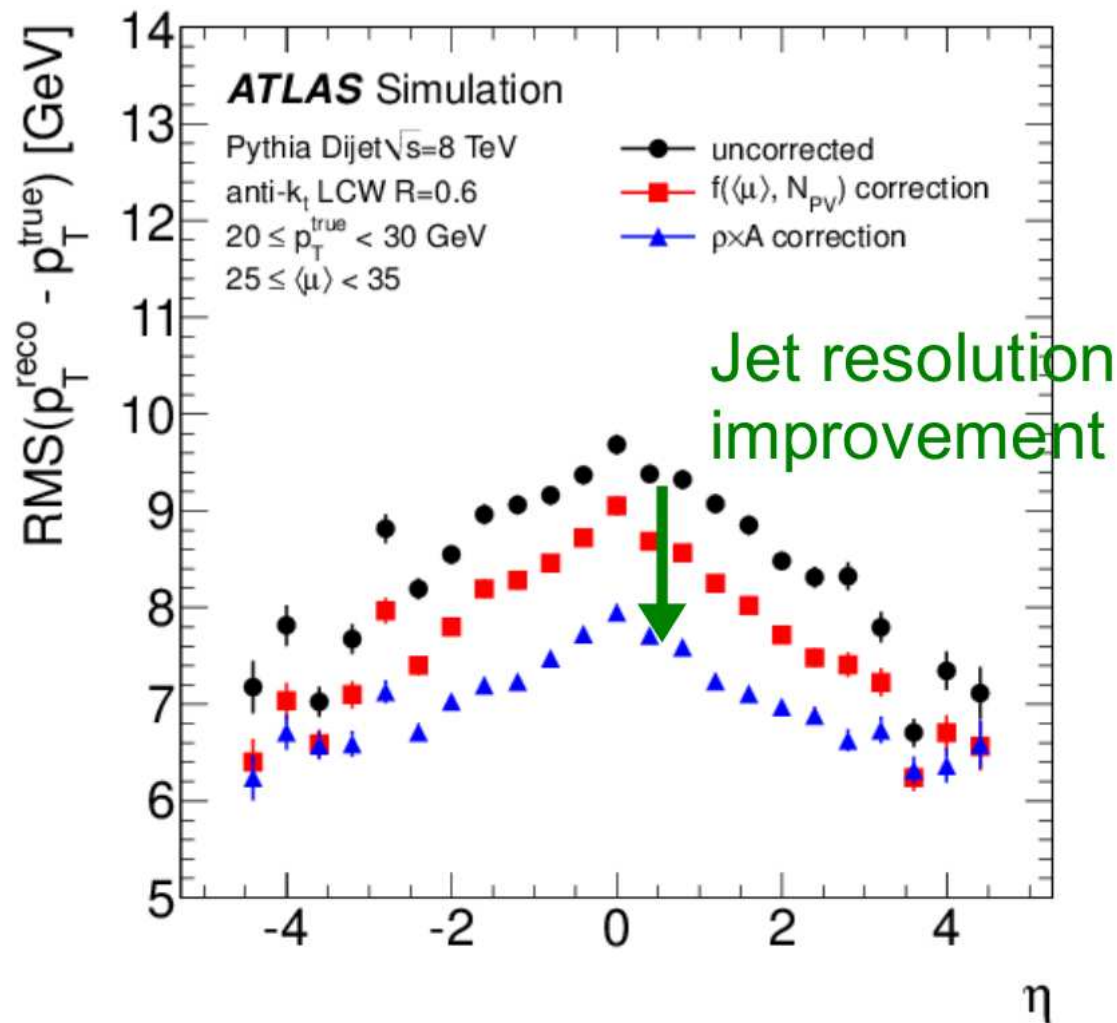
corrected for shift

## impact on resolution



resolution improved

# PU subtraction as seen in ATLAS



[B. Petersen, ATLAS Status report for the LHCC, 2013]

# Recent developments

## Improvements/extensions of the method

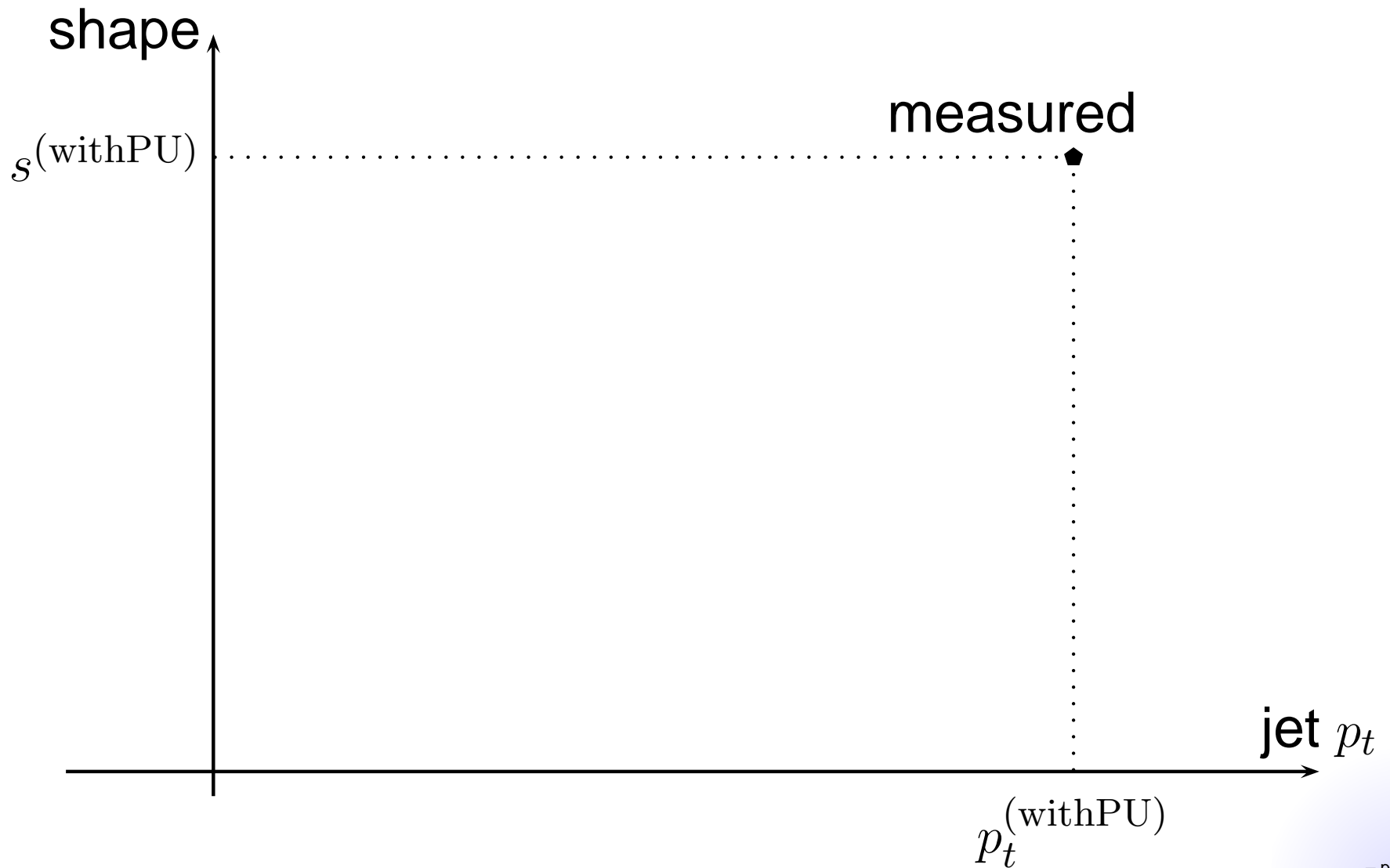
- Methods to handle **positional dependence of  $\rho$**   
Directly relevant for the LHC (e.g. rapidity dependence)  
[M.Cacciari,G.Salam,GS,2010-2011]
- Subtraction for **jet mass and jet shapes**  
Important for jet tagging (“ $q$  v.  $g$  jet”,  $b$  jet, top jet,  $H \rightarrow b\bar{b}$ )  
[GS,G.Salam,J.Kim,S.Dutta,M.Cacciari,2013]
- Subtraction of **fragmentation function (moments)**  
Useful for quenching in  $PbPb$  collisions  
[M.Cacciari,P.Quiroga,G.Salam,GS,2012]

## ***Example: jet shapes***

***(function of the jet constituents – e.g. the jet mass)***

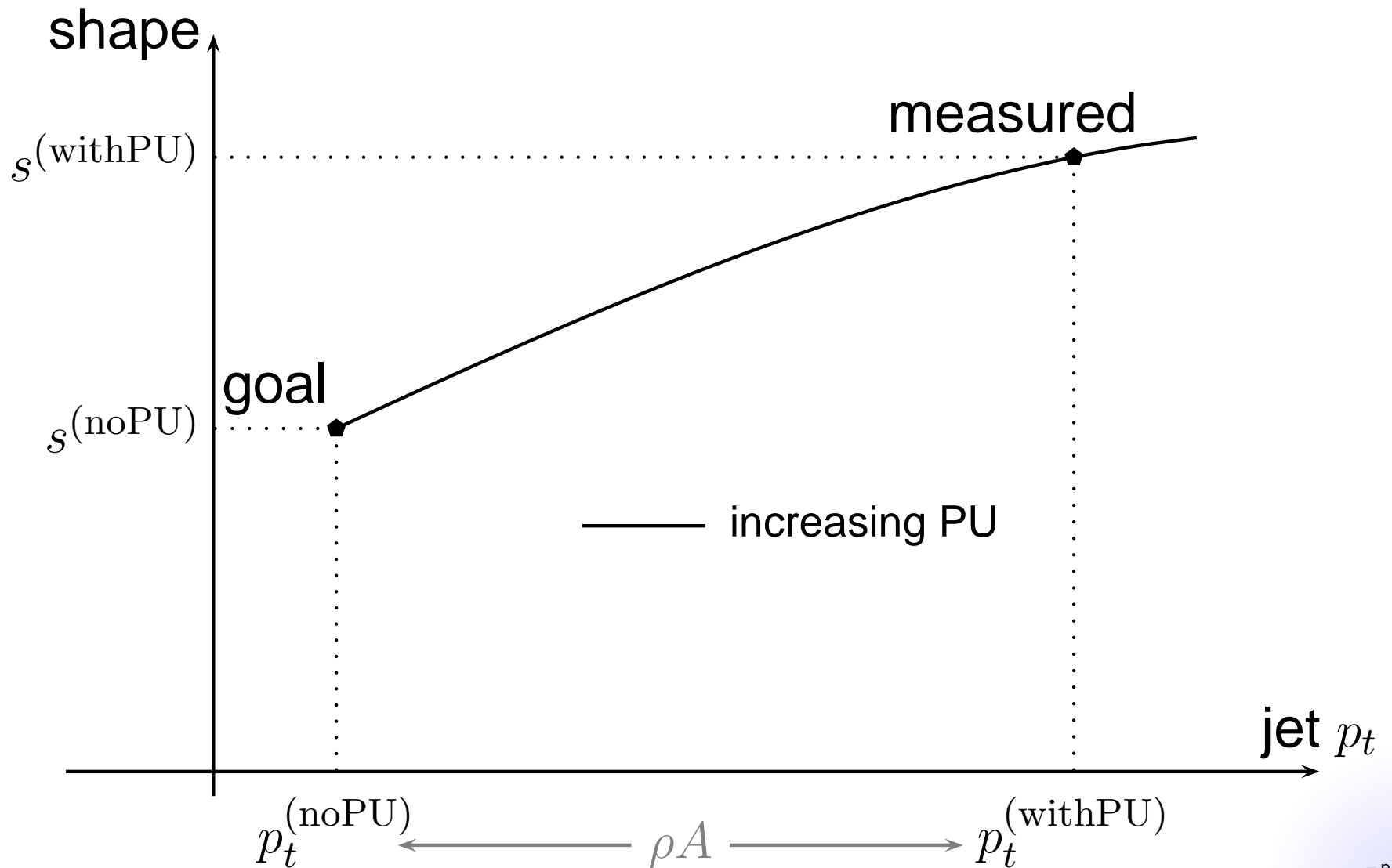
# Idea: area-median + extrapolation to 0 PU

[M.Cacciari, S.Dutta, J.Kim, G.Salam, GS, 2013]



# Idea: area-median + extrapolation to 0 PU

[M.Cacciari, S.Dutta, J.Kim, G.Salam, GS, 2013]

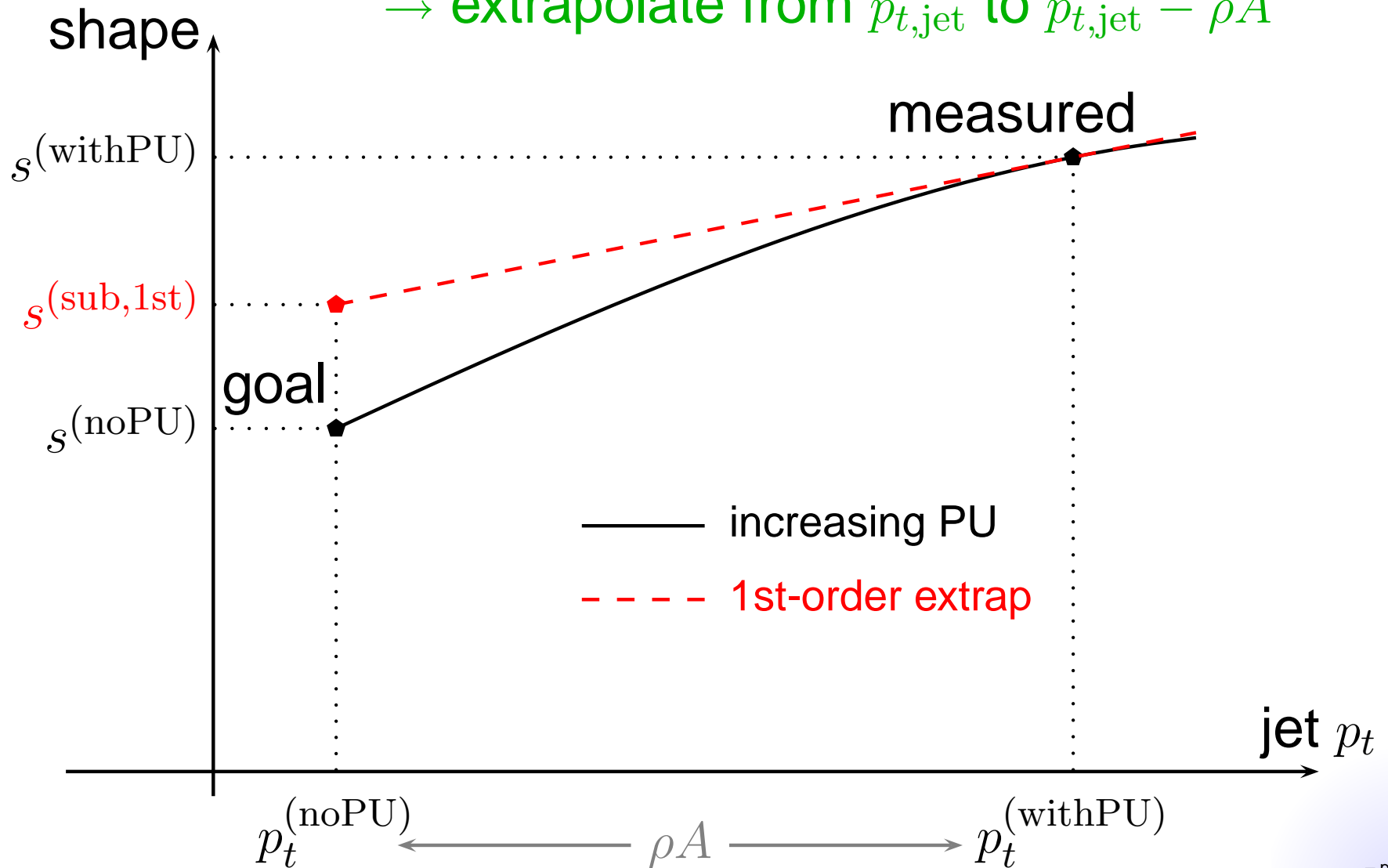


# Idea: area-median + extrapolation to 0 PU

[M.Cacciari, S.Dutta, J.Kim, G.Salam, GS, 2013]

knowledge of the derivatives wrt uniform shift of PU

→ extrapolate from  $p_{t,\text{jet}}$  to  $p_{t,\text{jet}} - \rho A$



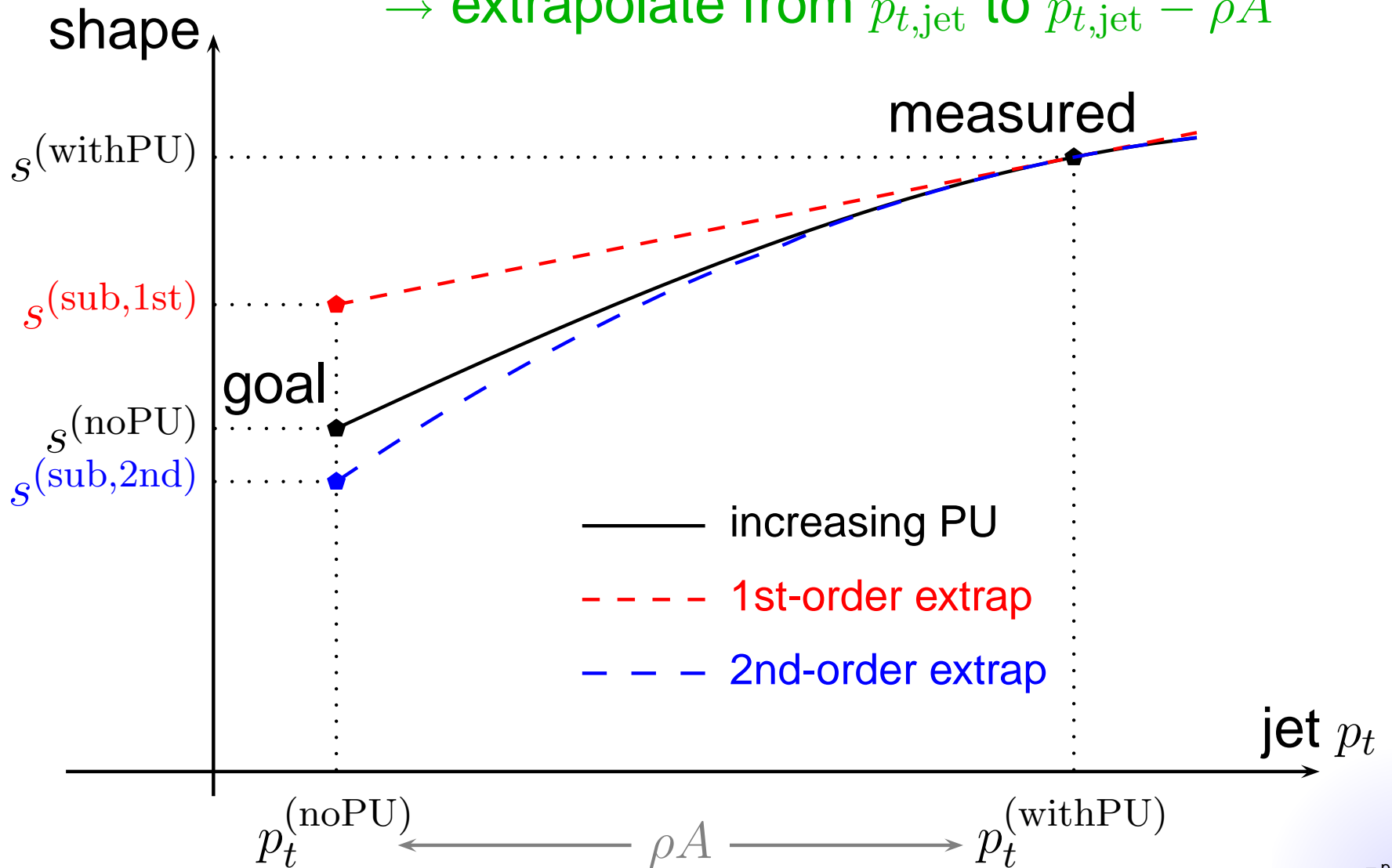


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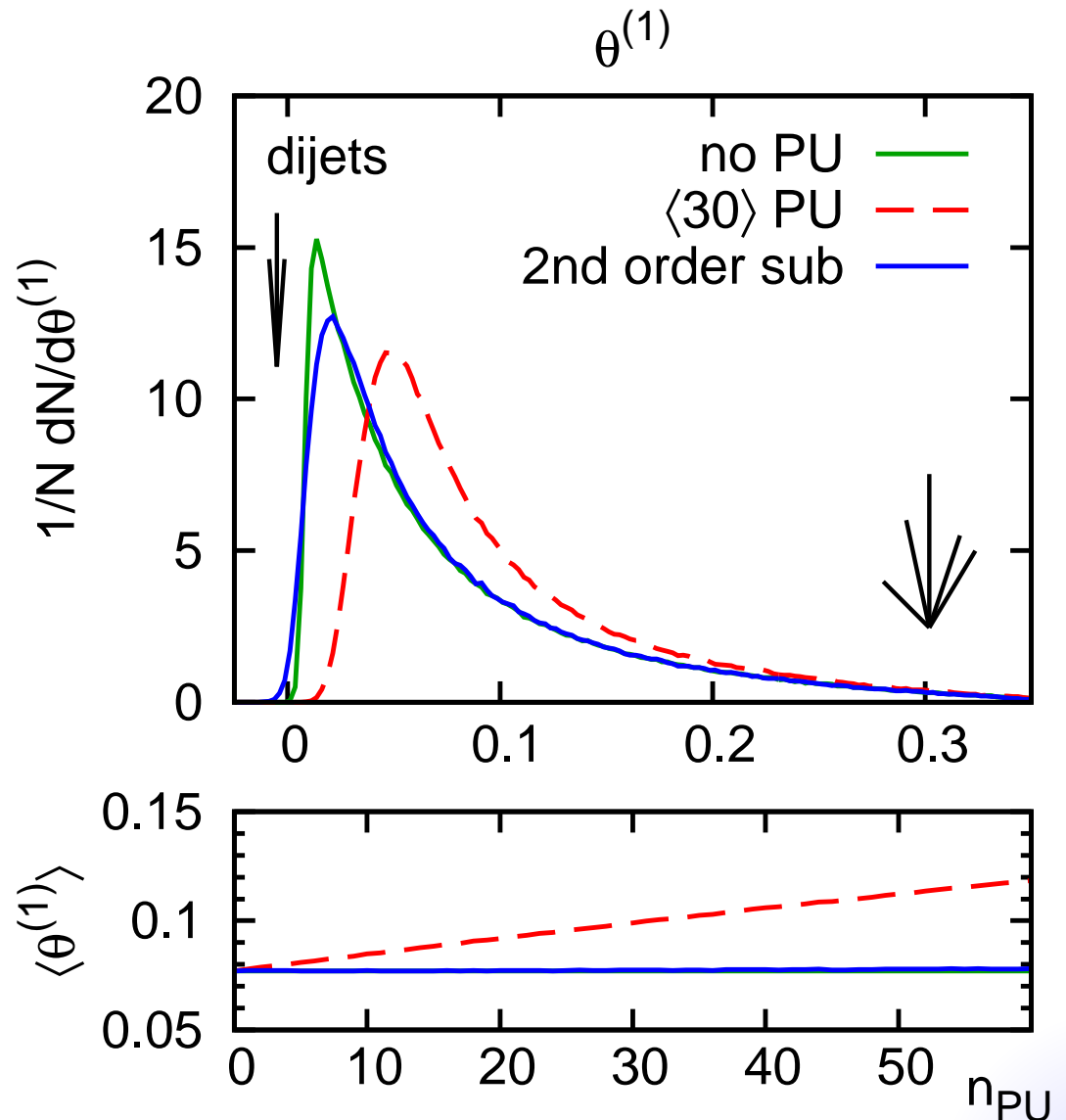
→ extrapolate from  $p_{t,\text{jet}}$  to  $p_{t,\text{jet}} - \rho A$



# Application 1: quark/gluon discrimination

Use a cut on girth/broadening/width  $\theta^{(1)} < 0.05$

$$\theta^{(1)}(\text{jet}) = \frac{1}{\tilde{p}_t} \sum_{i \in \text{jet}} p_{t,i} \Delta R_{i,\text{jet}}$$

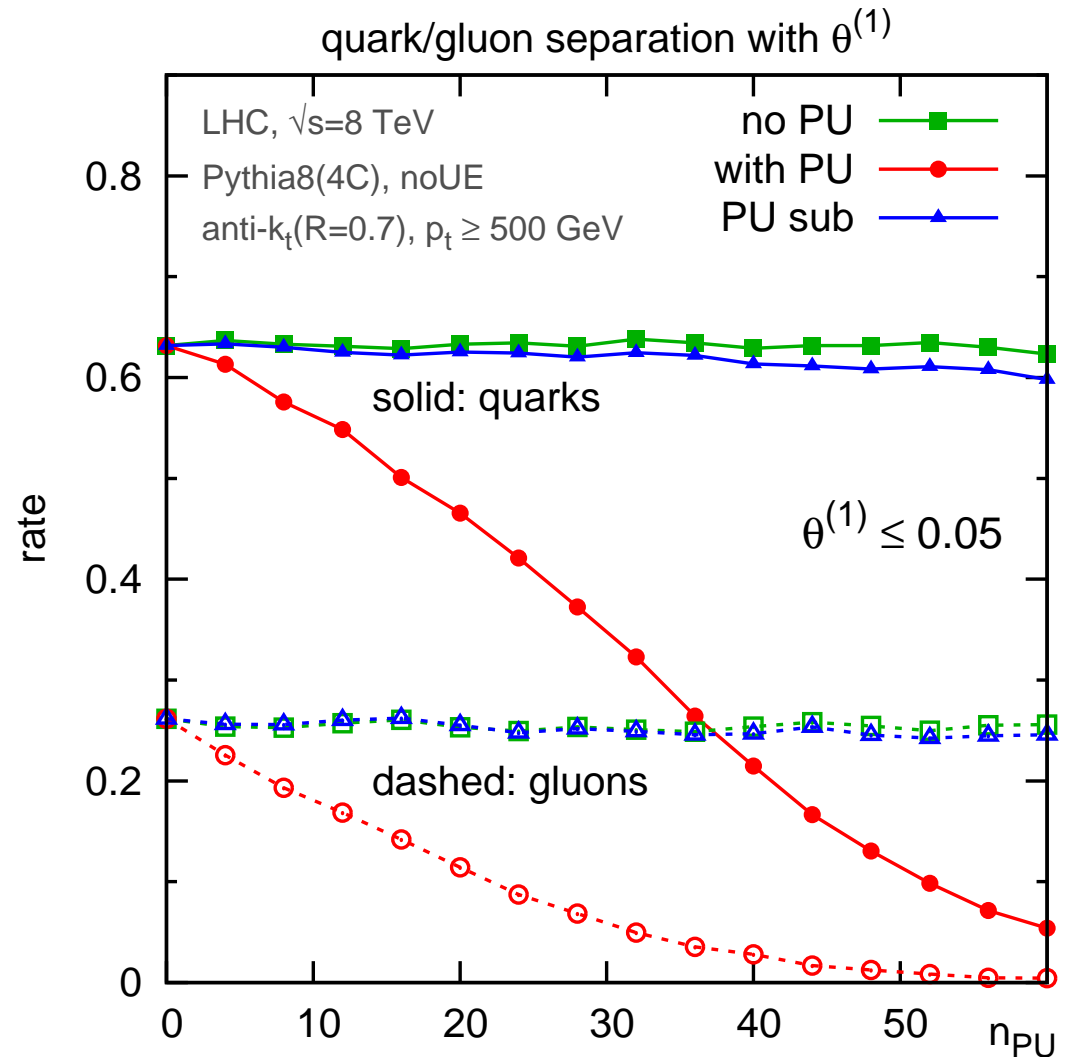


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efficiencies very well recovered

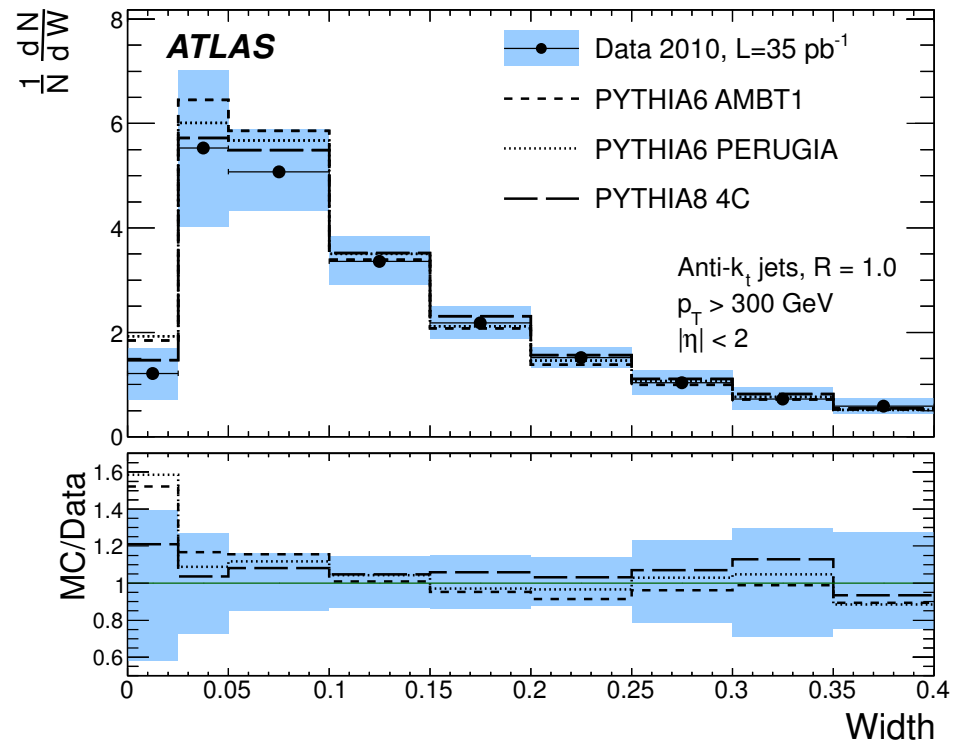
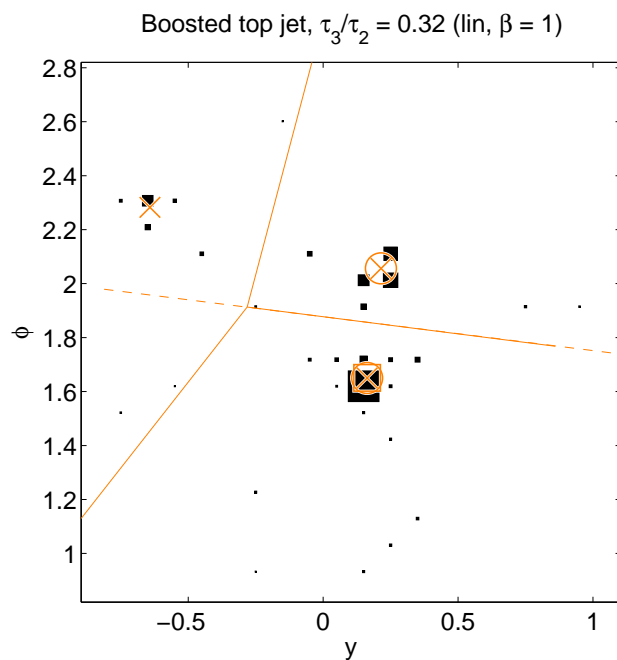


# Application 2: $N$ -subjettiness top tagging

Top tagging: cut on  $\tau_3/\tau_2$  and on the filtered mass

$N$  axes (e.g. excl.  $k_t$  subjets)

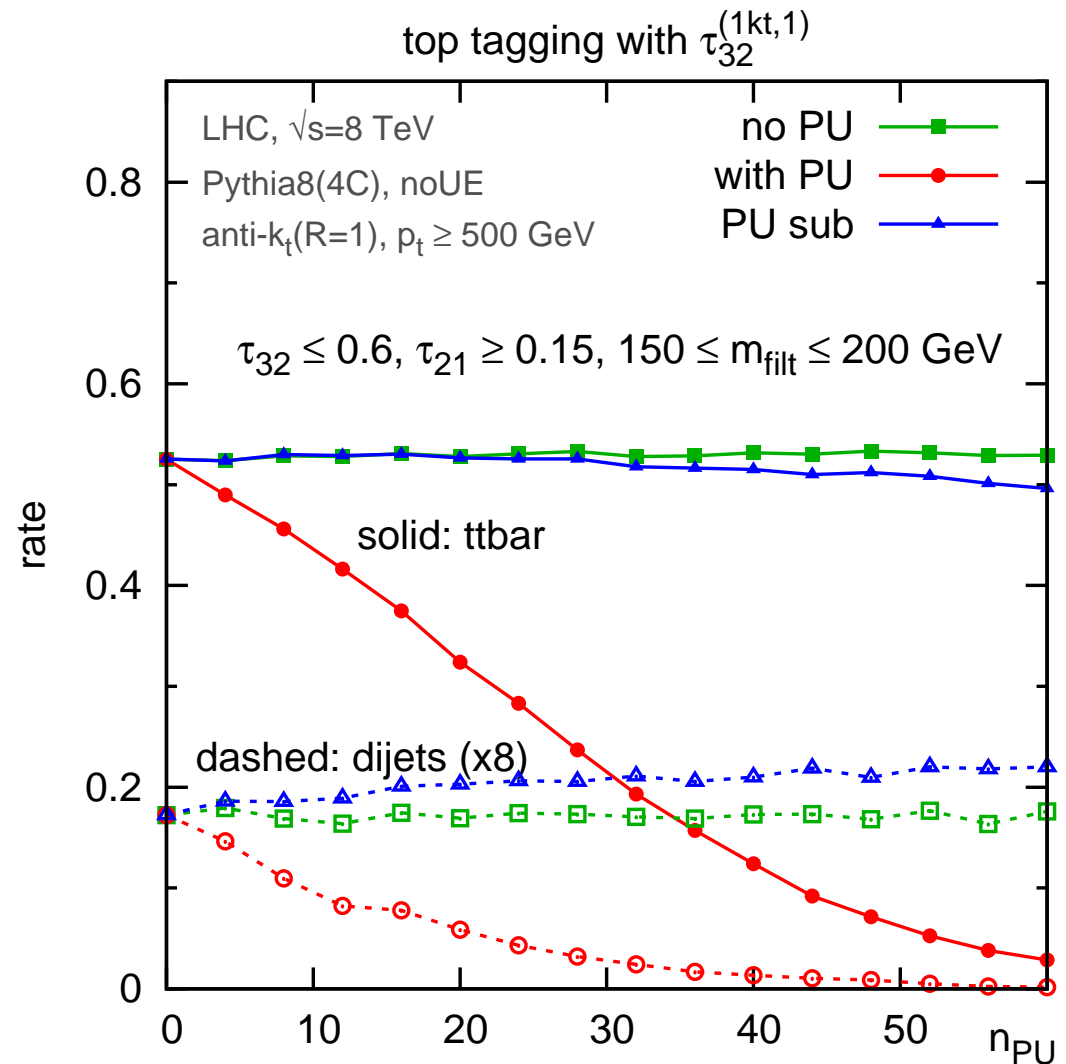
$$\tau_N^{(\text{axes}, \beta)}(\text{jet}) = \frac{1}{\tilde{p}_t} \sum_{i \in \text{jet}} p_{t,i} \min_{a \in \text{axes}} (\Delta R_{i,a})^\beta$$



# Application 2: $N$ -subjettiness top tagging

Top tagging: cut on  $\tau_3/\tau_2$  and on the filtered mass

efficiencies very well recovered

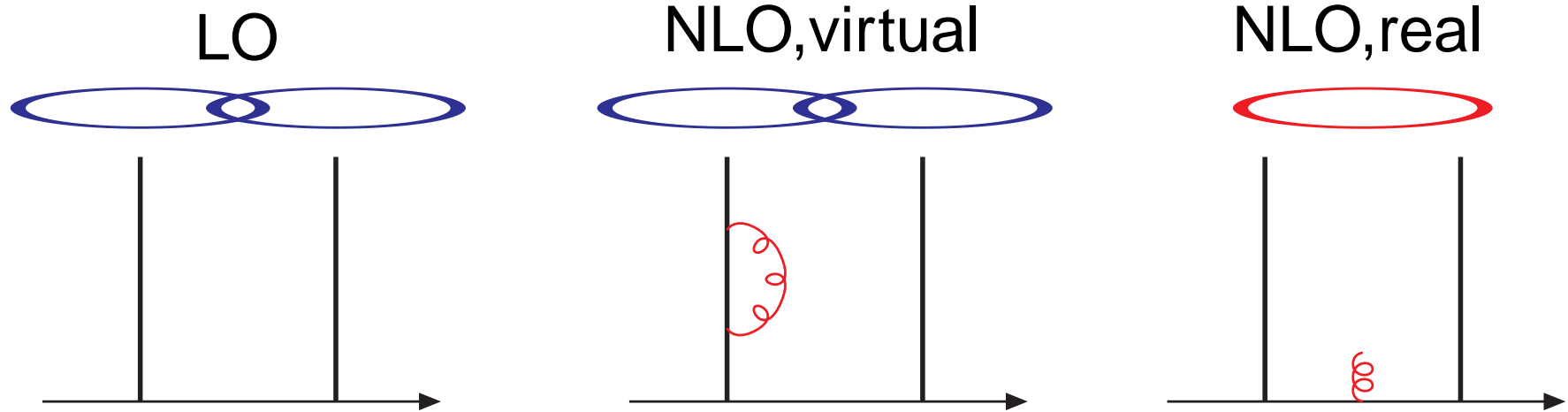


# Conclusion and perspectives

- Many recent developments in use at the LHC:
  - robust and finite jet algs. In particular: **anti- $k_t$**
  - **FastJet**: fast and standard interface for jets
  - efficient and generic **PU subtraction** method
- **Future**
  - **FastJet** keeps improving (**3.1 to be released**)
  - PU: reduce noise sensitivity (e.g. **dynamic filter**)
  - Boosted tags: **analytic understanding needed**  
→ **design better taggers**

# IR safety

## Example: JetClu



**cancellation between real and virtual spoiled**

JetClu, ATLASCone:	IR unsafety with 2+1 particles	(NLO for inclusive jet x-sect)
CDF/D0MidPoint:	IR unsafety with 2+1 particles	(NNLO for inclusive jet x-sect)
CMSIterativeCone:	collinear unsafety with 2+1 particles	(NNLO for inclusive jet x-sect)

Origin: incomplete determination of the stable (*i.e.* self-consistent) cones

# The FastJet lemma

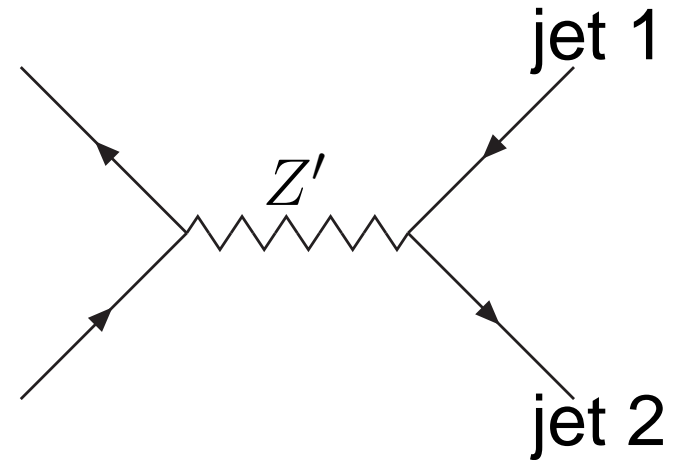
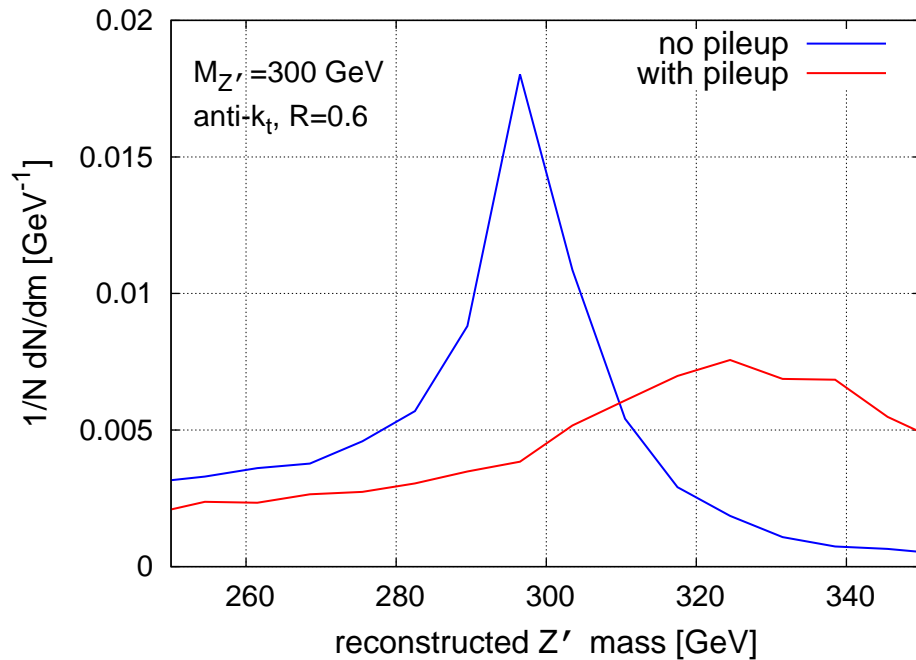
It  $(i, j)$  is the pair that minimize the  $k_t$  distance and  $k_{t,i} < k_{t,j}$ , then  $j$  is  $i$ 's nearest neighbour

Proof: assume it is not, then  $\exists k$  s.t.  $\Delta R_{ik} < \Delta R_{ij}$  and

$$\begin{aligned} \min(k_{t,i}^2, k_{t,l}^2) \Delta R_{il}^2 &\leq k_{t,i}^2 \Delta R_{il}^2 \\ &\leq \min(k_{t,i}^2, k_{t,j}^2) \Delta R_{il}^2 \\ &< \min(k_{t,i}^2, k_{t,j}^2) \Delta R_{ij}^2 \end{aligned}$$



# Illustration of PU effects



- Shift due to the “ $\rho A$ ” term
- Smearing due to the “ $\sigma_{\rho A}$ ” and “ $\sigma\sqrt{A}$ ” terms

# Heavy ions

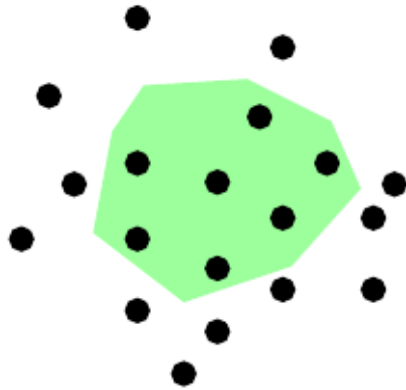
Note: same considerations for “spectator  $p$  and  $n$ ”  
in heavy ion collisions

Typical case: anti- $k_t$   $R = 0.4$ , 20 PU or 0–10% centrality

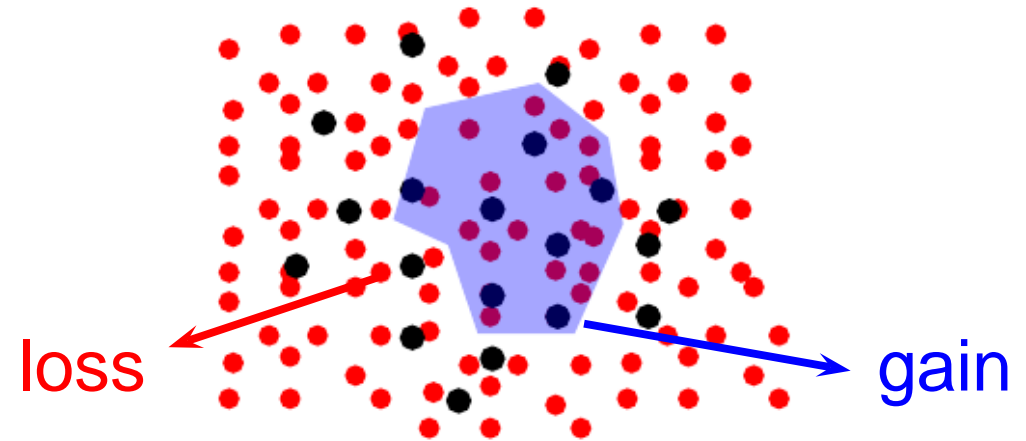
<i>Estimates</i>	LHC, $pp$	LHC, $PbPb$
$\rho$	15 GeV	200 GeV
$\sigma_\rho$	4 GeV	40 GeV
$\sigma$	5 GeV	20 GeV
$A_{\text{jet}}$	0.5	0.5
$\delta p_{t,\text{jet}}$	7.5 GeV	100 GeV
$\sigma_{\text{jet}}$	3.5 GeV	16 GeV

# Back reaction

No background



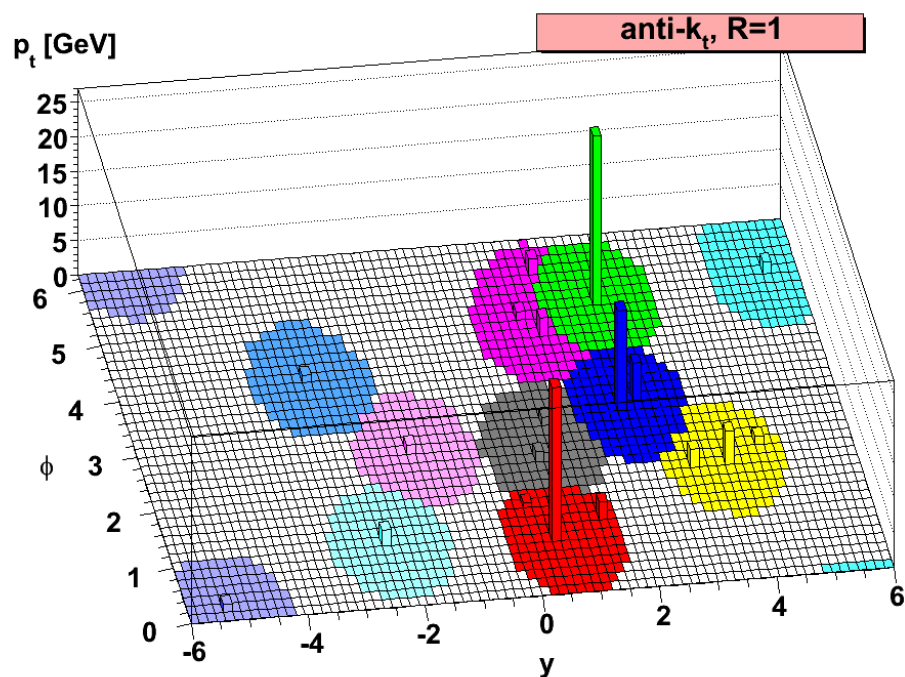
With background



Negligible for anti- $k_t$   
(a nice consequence of its soft resilience)

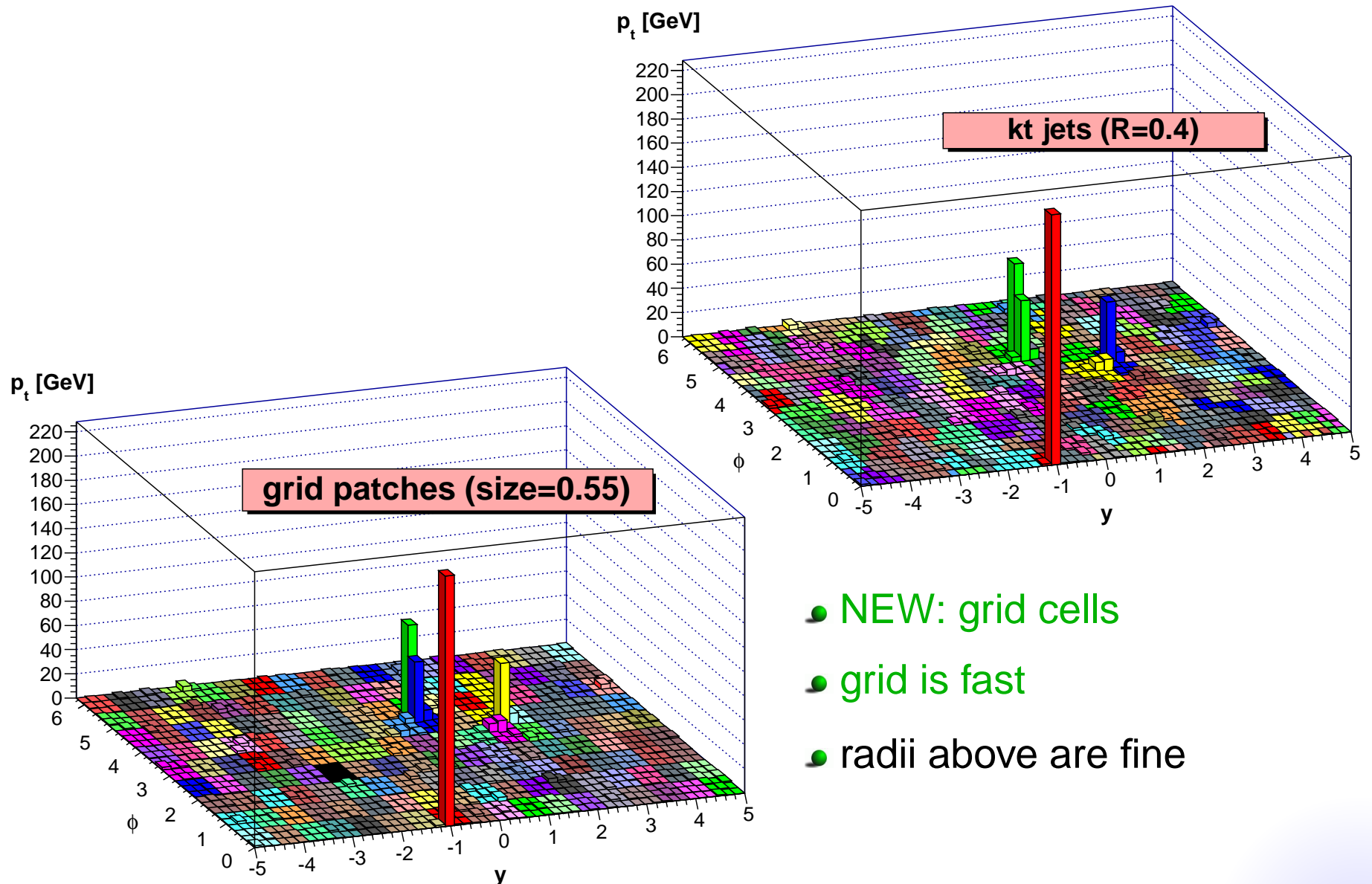
# Determining jet area

- **jet area**: available with jet clustering
  - add a dense coverage of particles with tiny  $p_t$  ( $\equiv$  area quanta)
  - jet area  $\propto$  number of these “ghosts” in the jet



# Patches of similar size

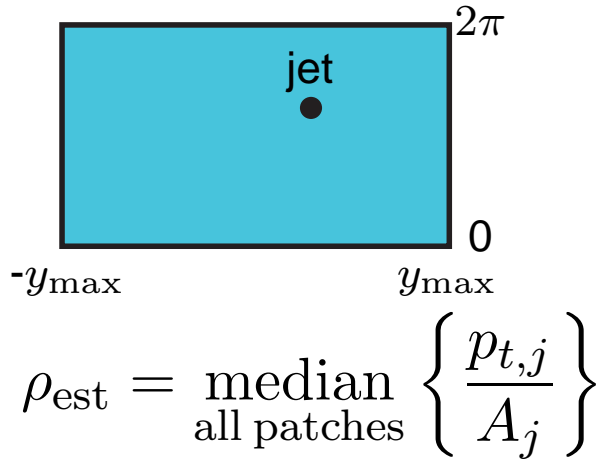
Patched are typically one of the 2 following types



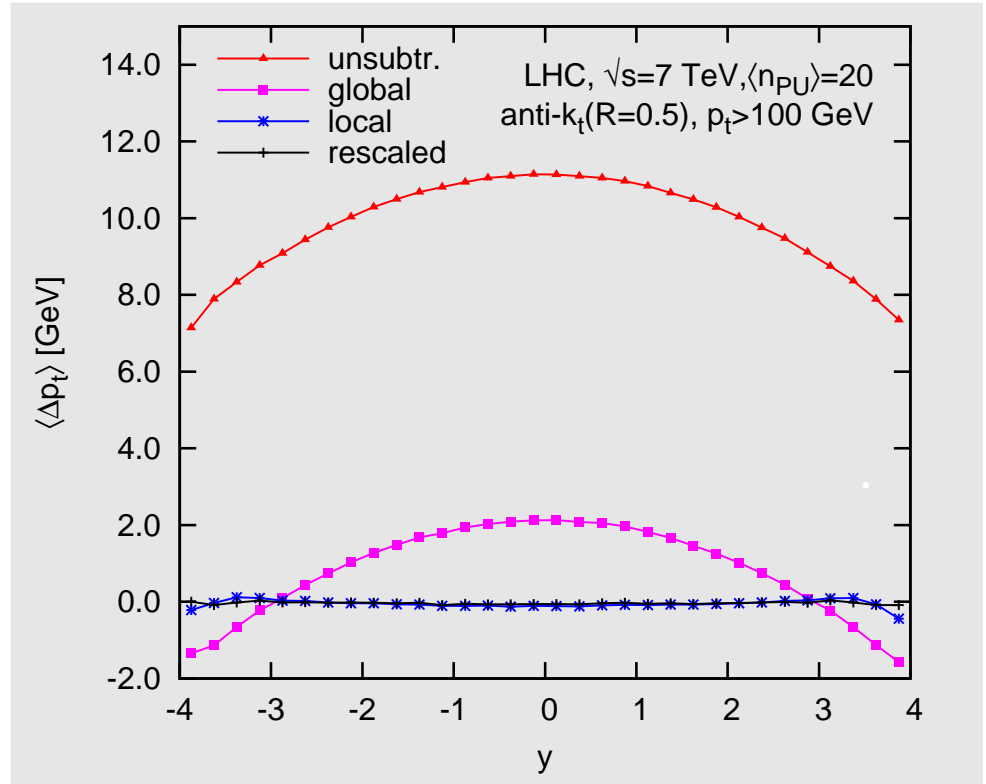
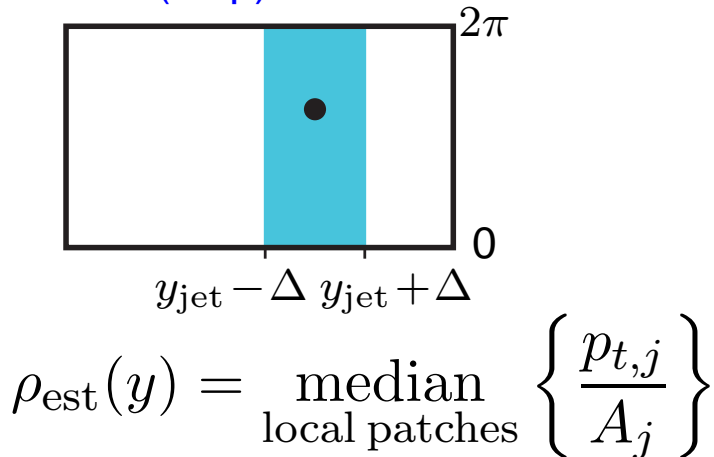
- NEW: grid cells
- grid is fast
- radii above are fine

# Rapidity/positional dependence

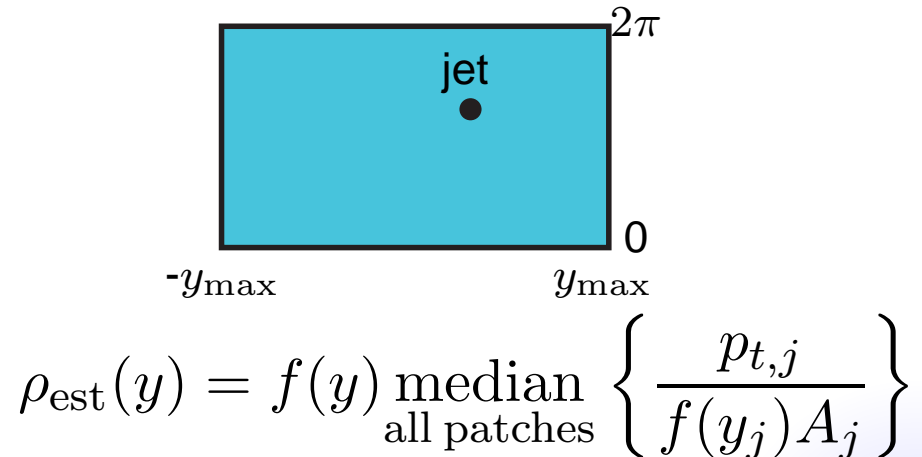
Global



Local (strip)



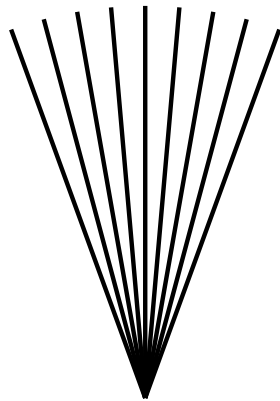
Rescaled



# Computing derivatives: back to the concepts

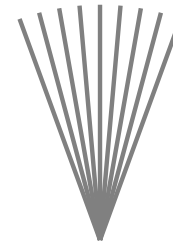
Idea of area-median subtraction:

pile-up



$$\propto \rho \times$$

ghosts



# Computing derivatives: back to the concepts

Idea of **area-median subtraction**:

pile-up

ghosts

$$\{p_i^\mu\} \propto \rho \times \{g_i^\mu\}$$



# Computing derivatives: back to the concepts

Idea of area-median subtraction:

pile-up

ghosts

$$\{p_i^\mu\} \sim \rho \times \{g_i^\mu\} \frac{a_{\text{ghost}}}{\bar{g}_t}$$

The set of ghosts mimics the uniform background  
→ use that to compute the derivatives numerically

$$\bar{g}_t = \text{ghost } p_t = \text{ghostscale}$$

# Jet shape subtraction

No pileup:  $s_{\text{noPU}}(\text{jet}) = s(\{p_{t,i}\}_{\text{hard}})$

With pileup:  $s_{\text{wPU}}(\text{jet}) = s(\{p_{t,i}\}_{\text{hard}}, \{p_{t,i}\}_{\text{PU}})$

- Assume  $\rho \ll p_t$  and expand in series of  $\rho$
- $\text{PU} \propto \text{ghosts} \Rightarrow \partial_\rho \propto \partial_{\text{ghostscale}}$
- numerically: vary the ghost scale to compute the derivatives

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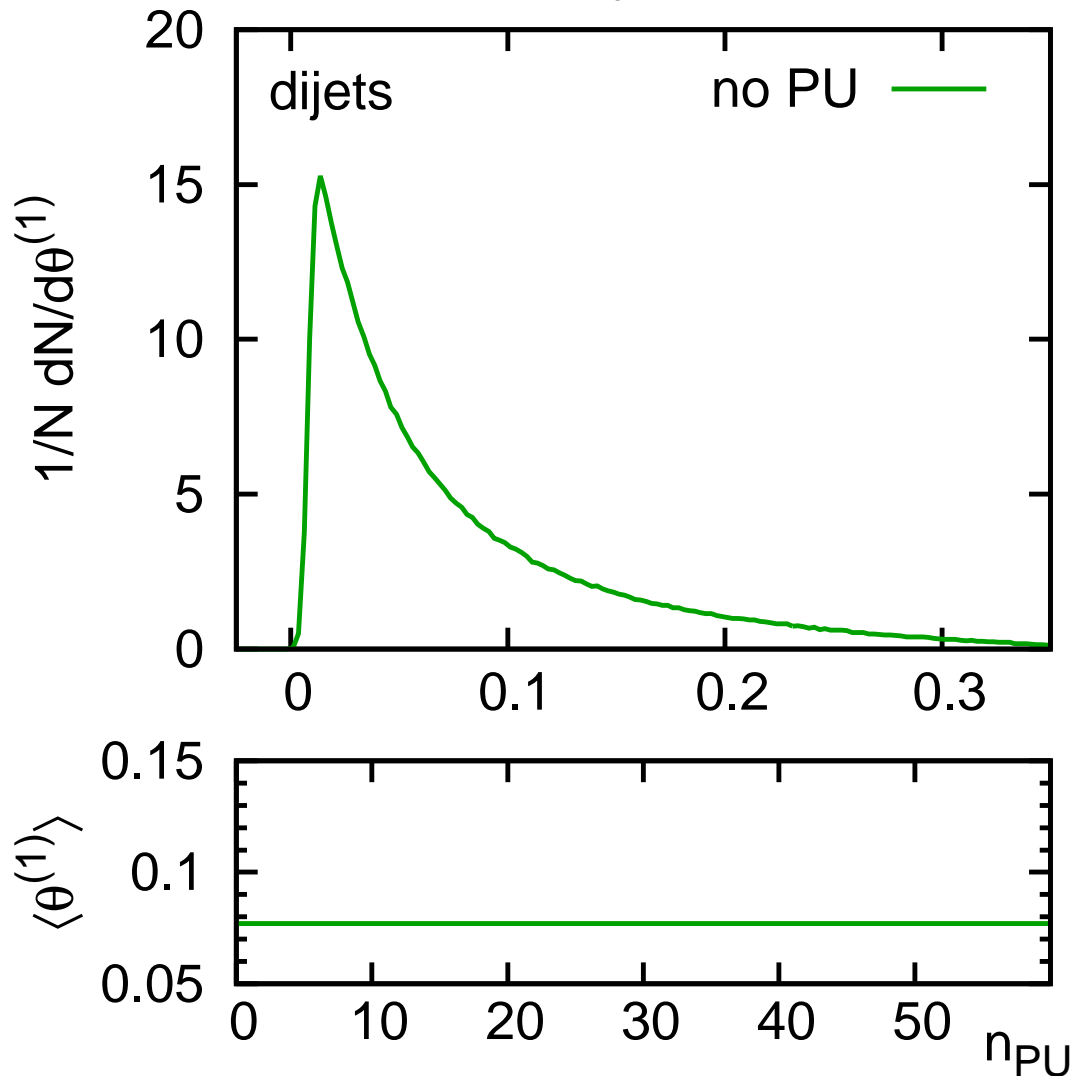
- Assume  $\rho \ll p_t$  and expand in series of  $\rho$
- $\text{PU} \propto \text{ghosts} \Rightarrow \partial_\rho \propto \partial_{\text{ghostscale}}$
- numerically: vary the ghost scale to compute the derivatives

Subtraction:

$$\begin{aligned} s_{\text{sub}}(\text{jet}) &= s_{\text{wPU}}(\text{jet}) - \rho a_{\text{ghost}} \partial_{\text{ghostscale}} s_{\text{wPU}}(\text{jet}) \\ &+ \frac{1}{2} (\rho a_{\text{ghost}})^2 \partial_{\text{ghostscale}}^2 s_{\text{wPU}}(\text{jet}) + \dots \end{aligned}$$

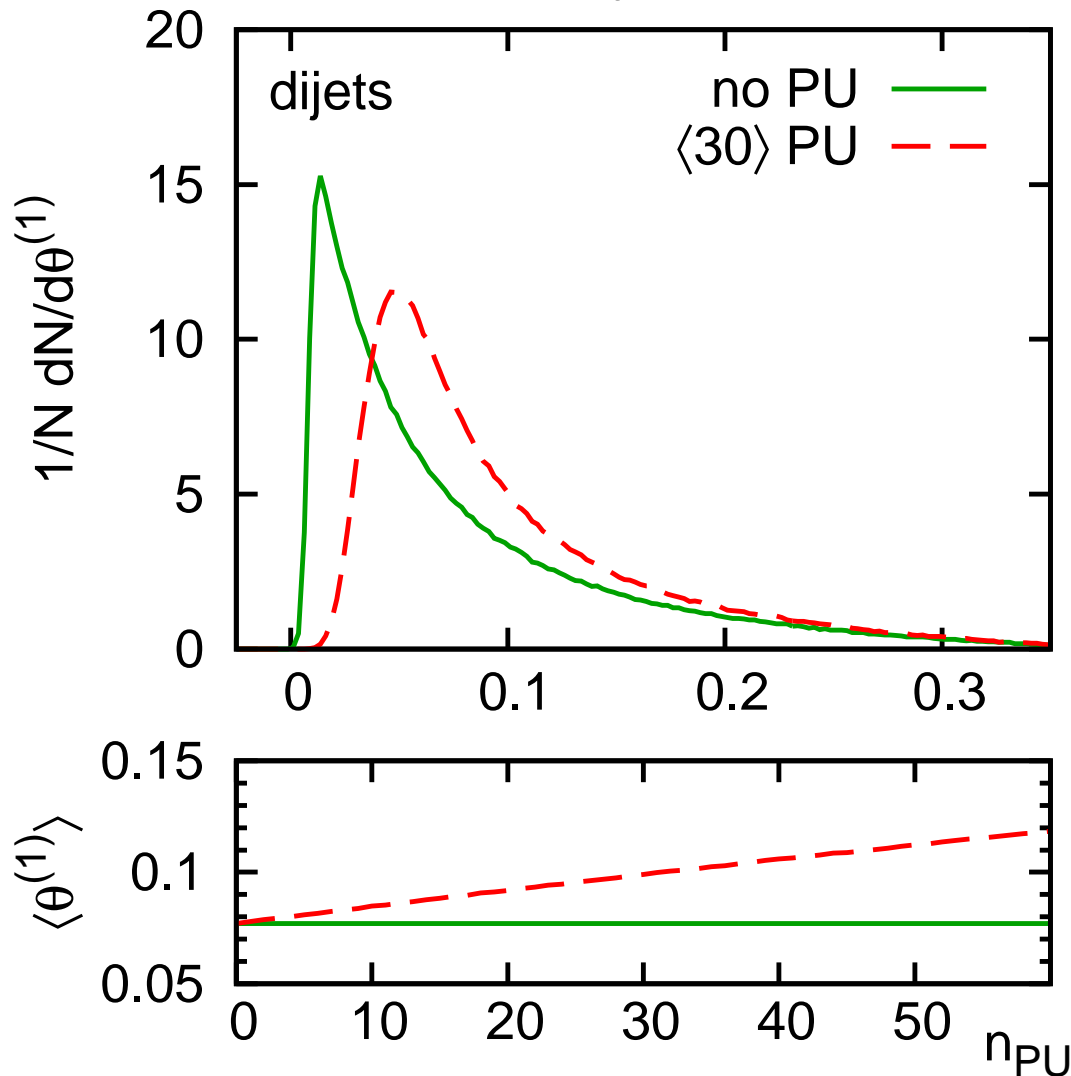
# detailed example: angularities

$$\theta^{(\alpha)} = \frac{\sum_{i \in \text{constits}} p_{t,i} \Delta R_{i,\text{jet}}^\alpha}{\sum_{i \in \text{constits}} p_{t,i}}$$



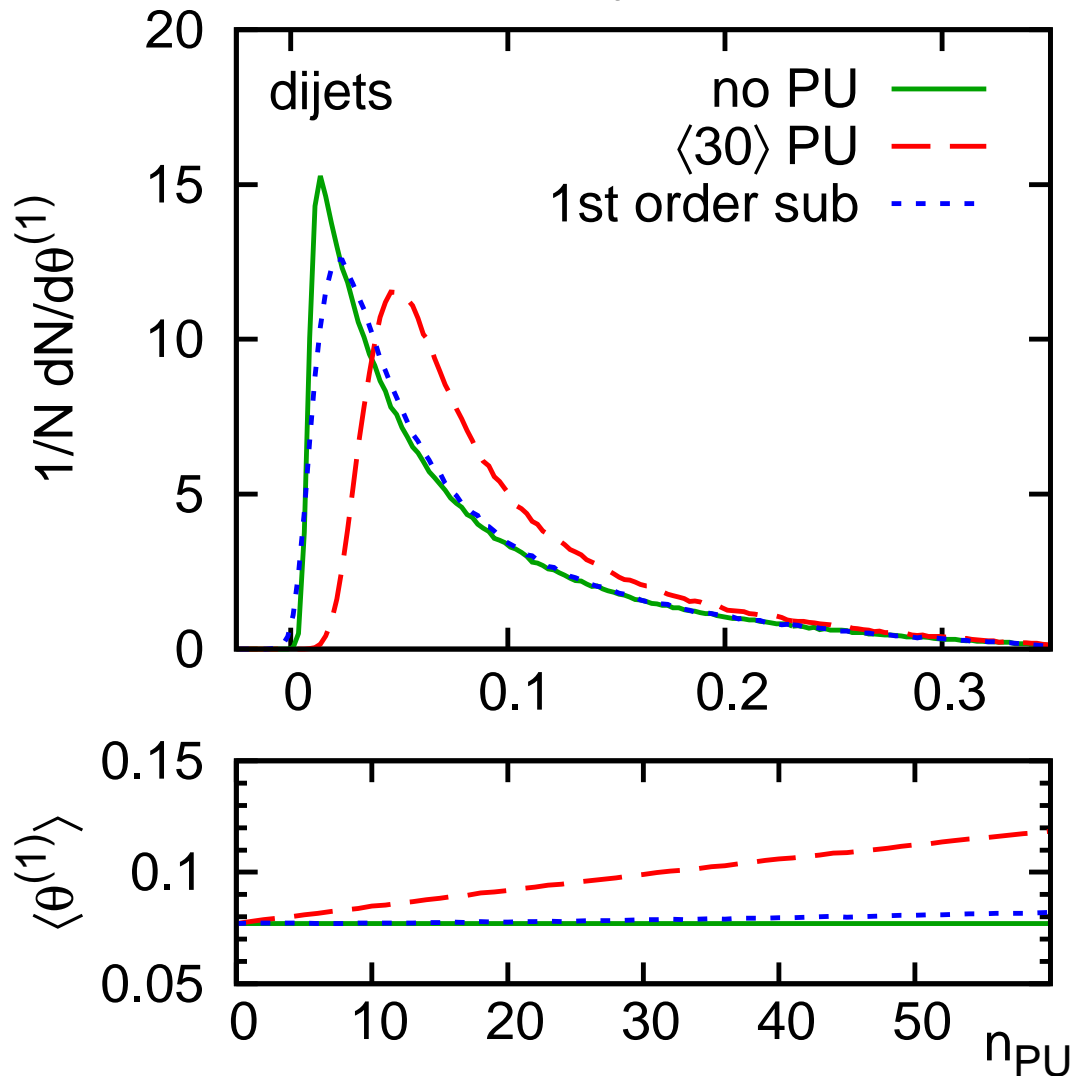
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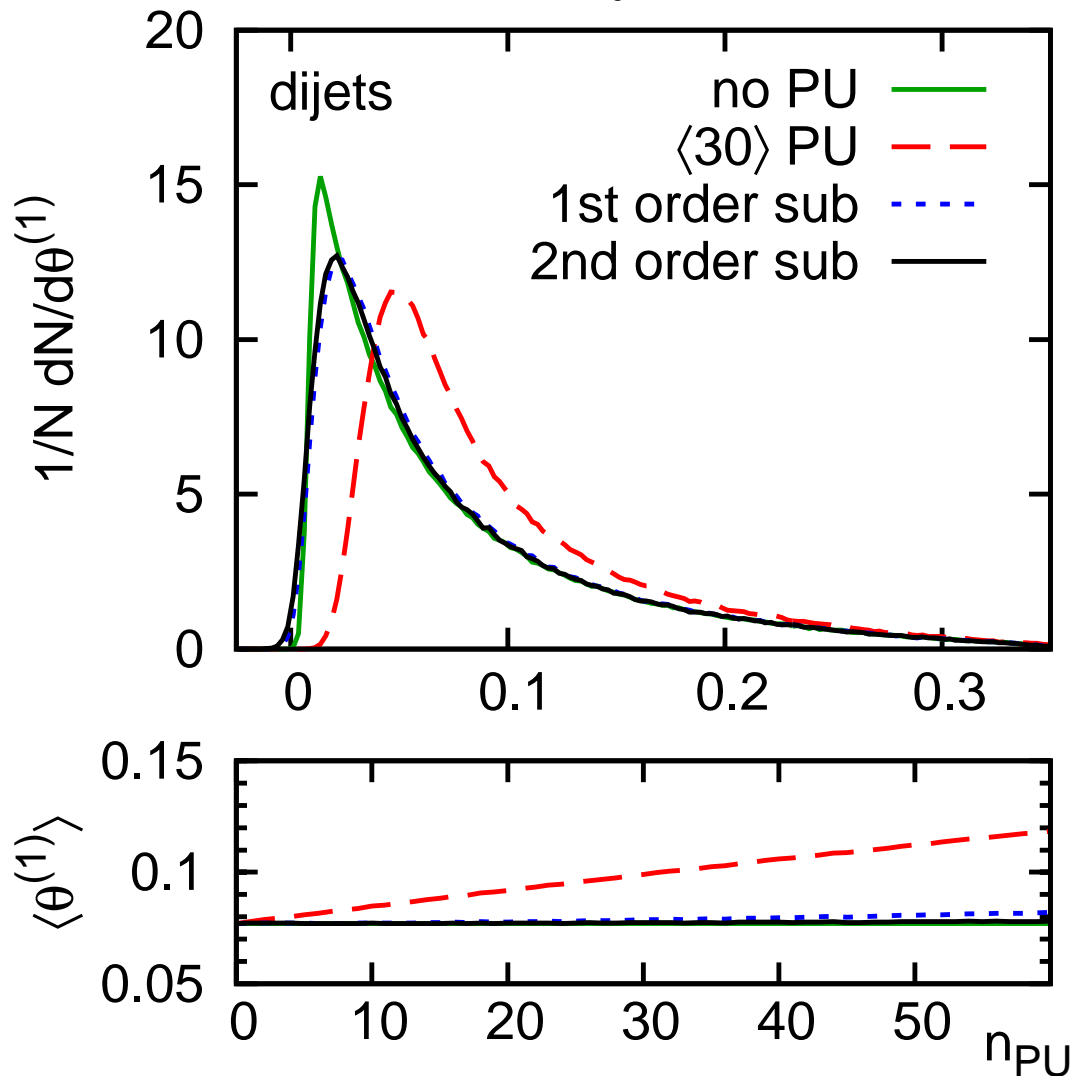
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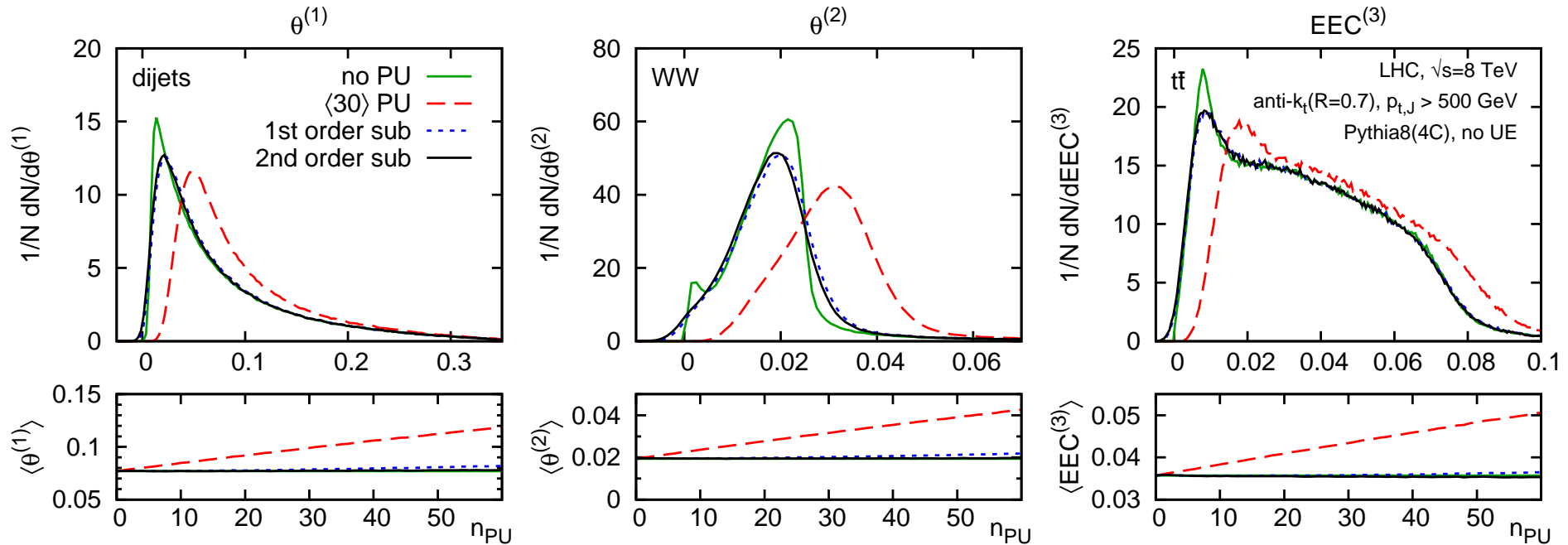


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$$\theta^{(\alpha)} = \frac{\sum_{i \in \text{constits}} p_{t,i} \Delta R_{i,\text{jet}}^\alpha}{\sum_{i \in \text{constits}} p_{t,i}}$$



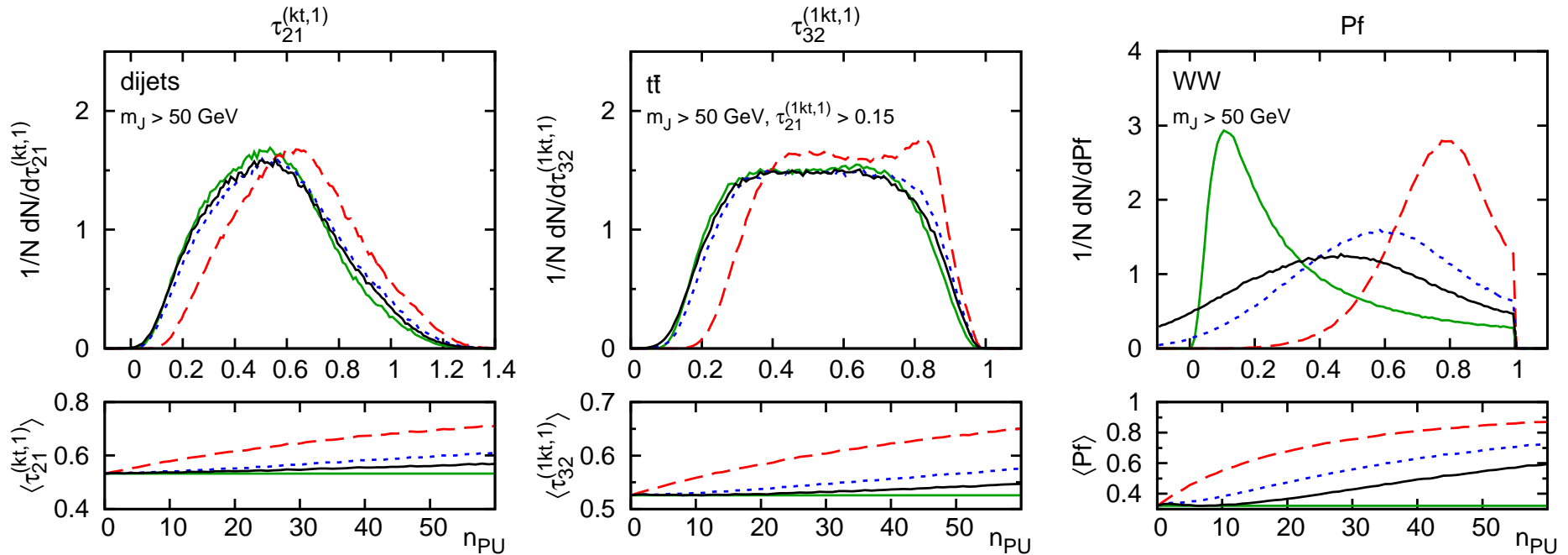
# Shape subtraction examples (1/2)



- PU has a 50-100% effect (5-10% for  $p_t$ )
- Subtraction works (very) well generically
- Broadening of sharp peaks (PU fluctuations!)



# Shape subtraction examples (2/2)



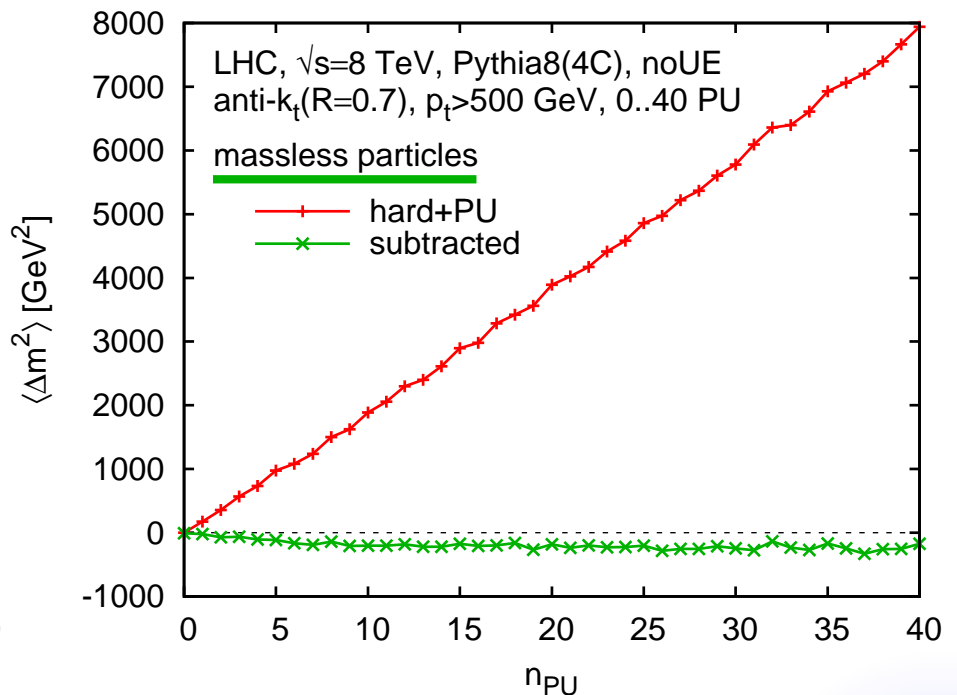
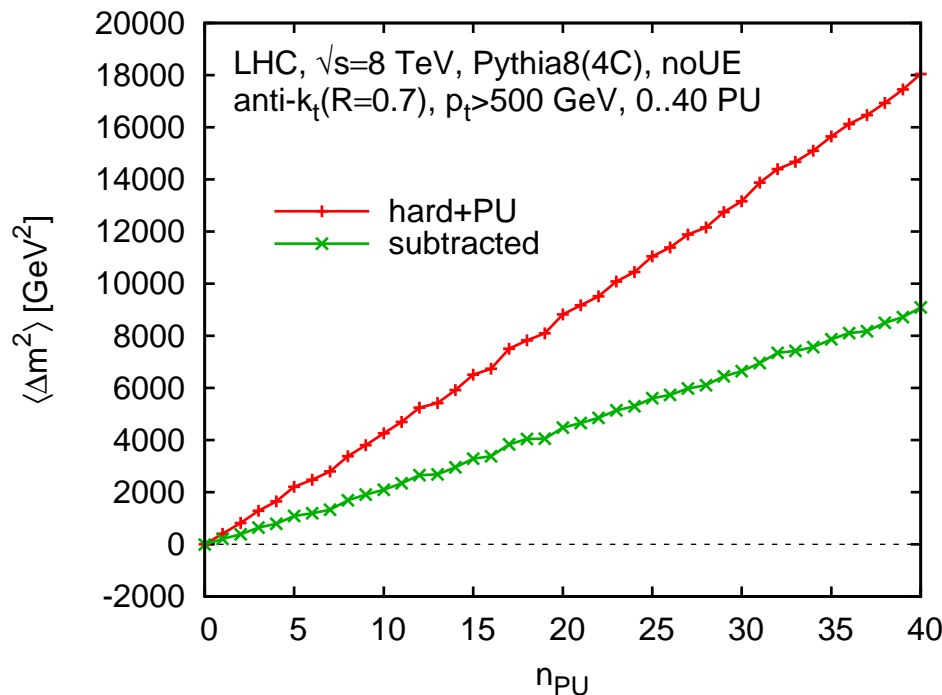
- 2nd derivative sometimes helpful
- Planar flow very poor
  - very PU-sensitive! (from  $Pf = 0$  to  $Pf = 1$ )
  - series expansion breaks for  $n_{PU} \gtrsim 15$

# Jet mass: the problem

$$p_{\text{jet}}^{\mu,(\text{sub})} = p_{\text{jet}}^{\mu} - \rho_{\text{est}} A_{\text{jet}}^{\mu}$$

applies to the jet 4-momentum

How do we do for the jet mass?



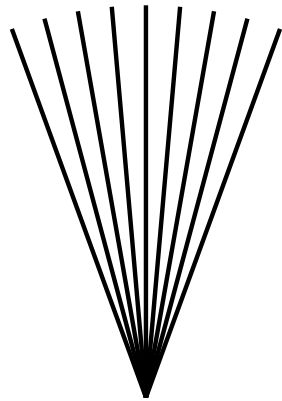
OK for massless particles... but if one want massive ones...

# Back to the concepts in pictures

area-median subtraction:

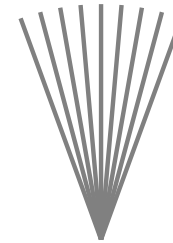
$$m_t = \sqrt{p_t^2 + m^2}$$

pile-up



$$\propto \rho \times$$

ghosts



$$\sum_i p_i^\mu$$

$$(p_t \cos(\phi_i), p_t \sin(\phi_i), \\ m_t \sinh(y_i), m_t \cosh(y_i))$$

$$\sum_i g_i^\mu$$

$$(g_t \cos(\phi_i), g_t \sin(\phi_i), \\ \sinh(y_i), \cosh(y_i))$$

Need an extra term to account for  $m_t \neq p_t$

# Back to the concepts: subtraction

New 4-vector subtraction formula:

$$p_{\text{sub}}^{\mu} = p^{\mu} - \rho A^{\mu} - \rho_m A_m^{\mu}$$

with

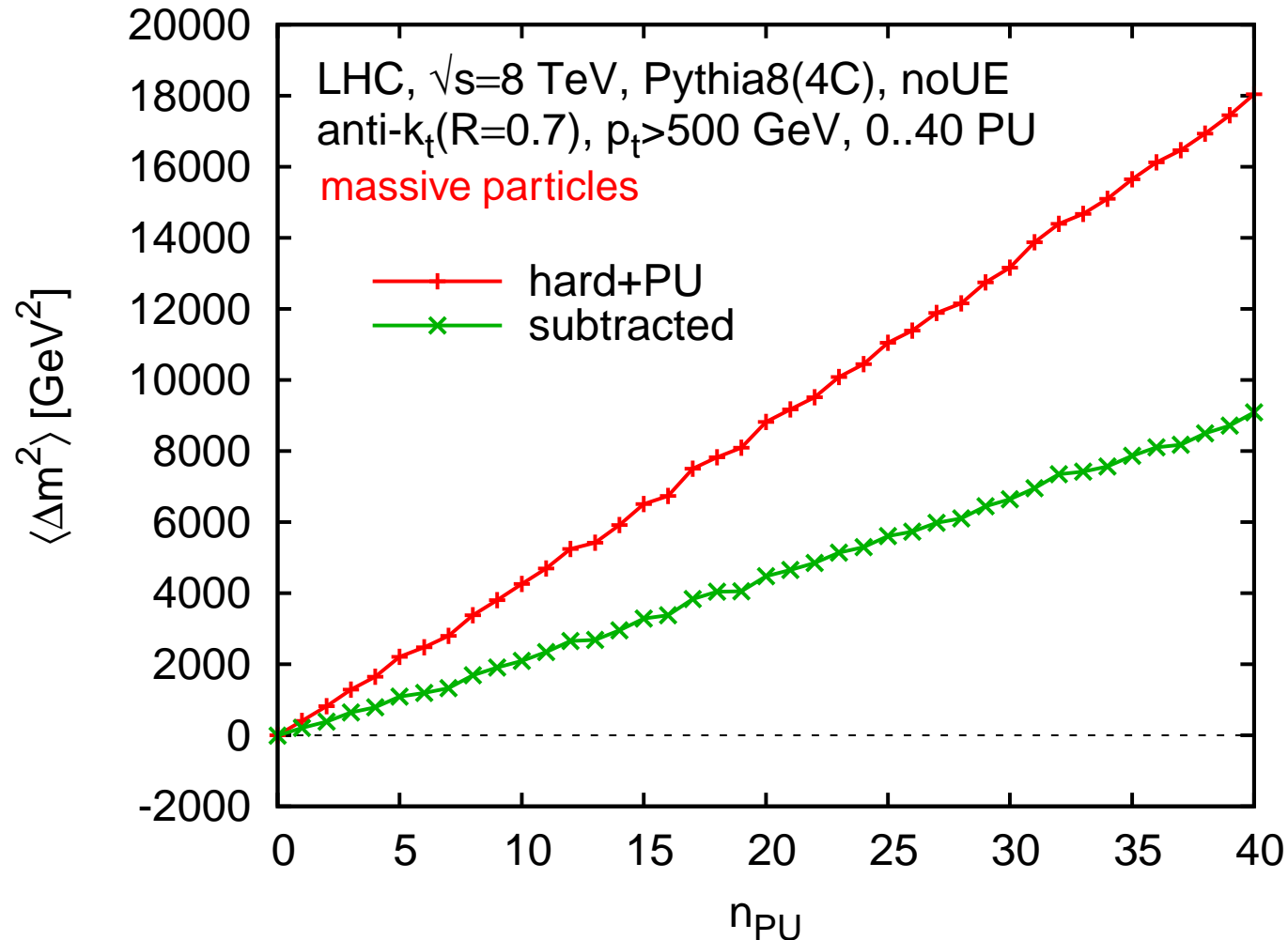
$$\rho = \text{median}_{j \in \text{patches}} \left\{ \frac{p_{t,j}}{A_t} \right\}$$

$$\rho_m = \text{median}_{j \in \text{patches}} \left\{ \frac{\sum_{i \in j} m_{t,i} - p_{t,i}}{A} \right\}$$

$$A_m^{\mu} \equiv (0, 0, A_z, A_E)$$

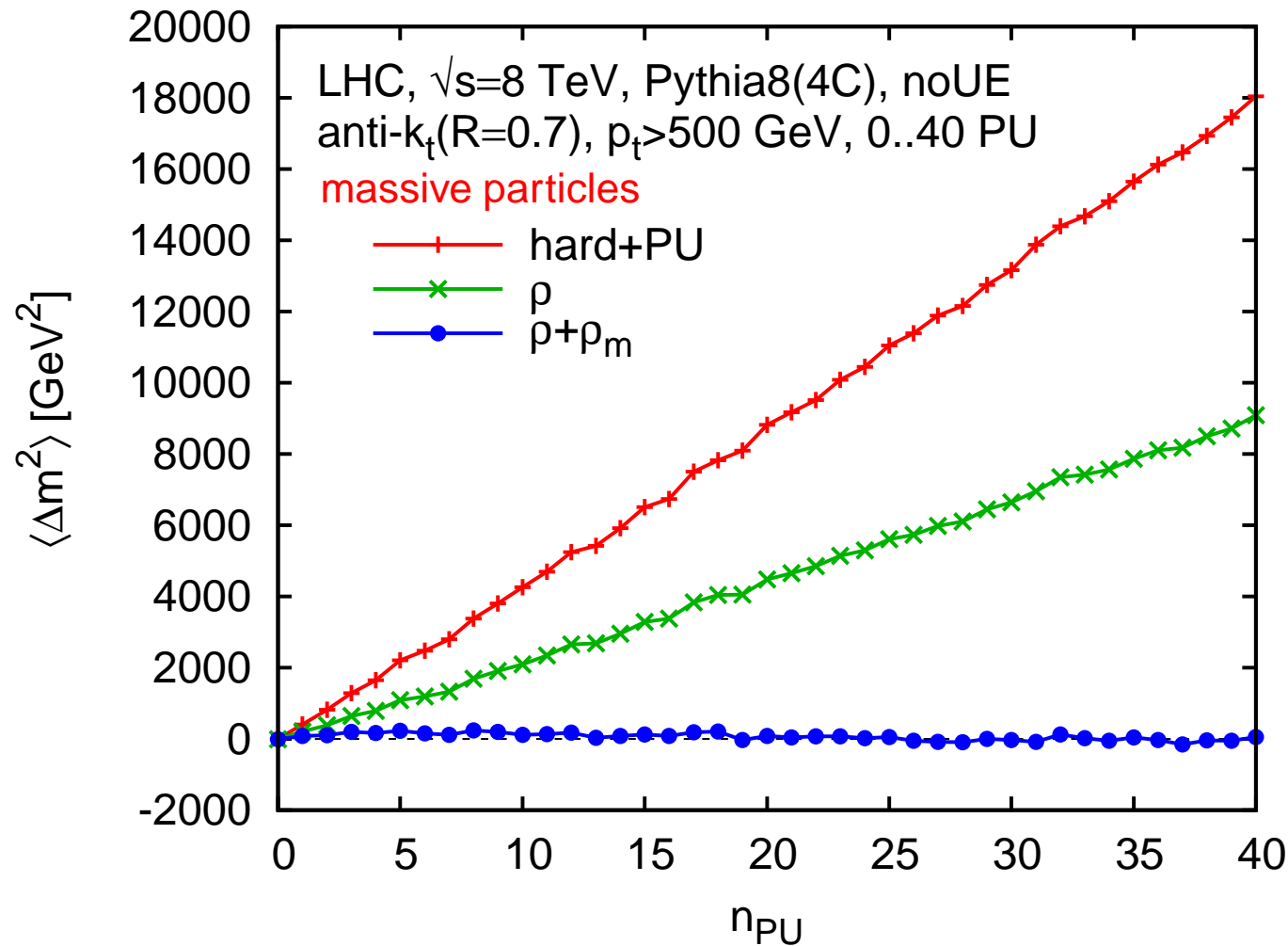
# Jet mass subtraction revisited

$$p_{\text{sub}}^{\mu} = p^{\mu} - \rho A^{\mu}$$



# Jet mass subtraction revisited

$$p_{\text{sub}}^{\mu} = p^{\mu} - \rho A^{\mu} - \rho_m A_m^{\mu}$$



## Subtraction can be used internally in tools

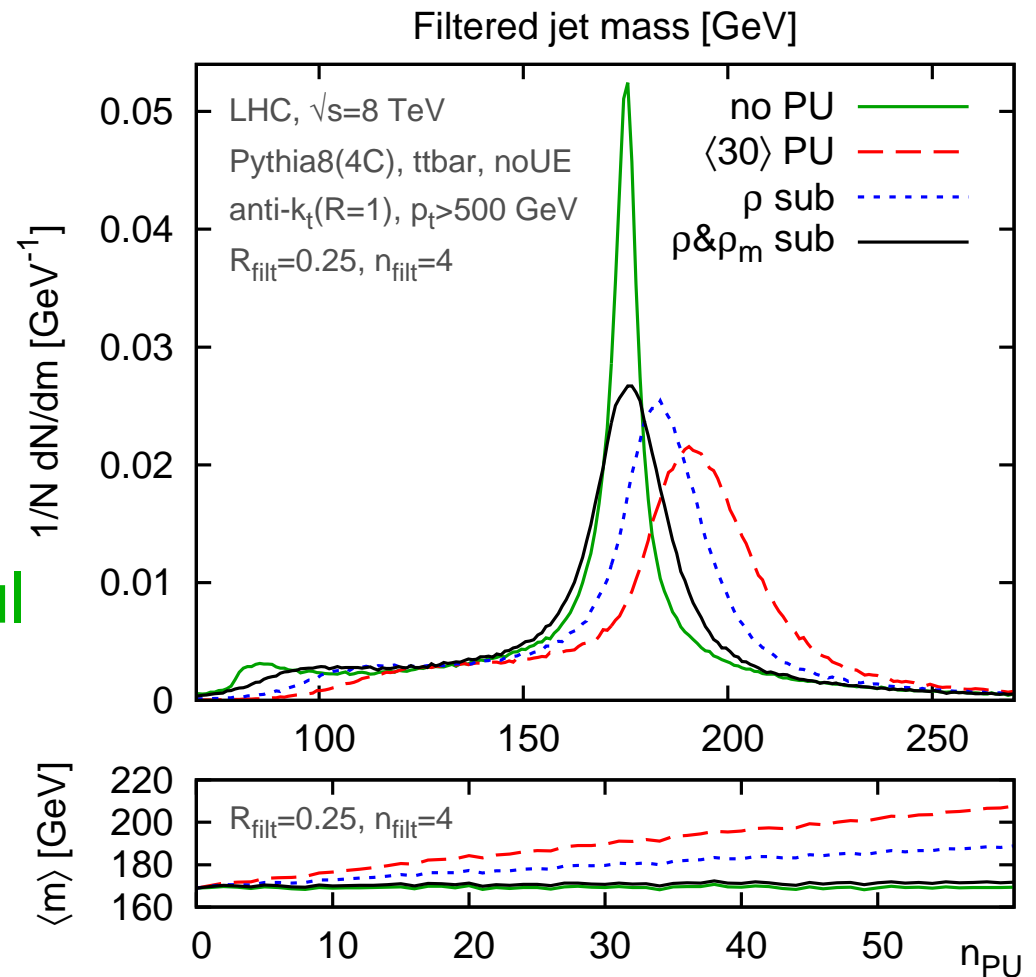
### Examples:

- subtract each subject before selecting the  $n$  hardest in filtering
- subtract each subject before applying the  $p_t$  cut in trimming
- subtract at each unclustering step in pruning

# Usage in tools

## Subtraction can be used internally in tools

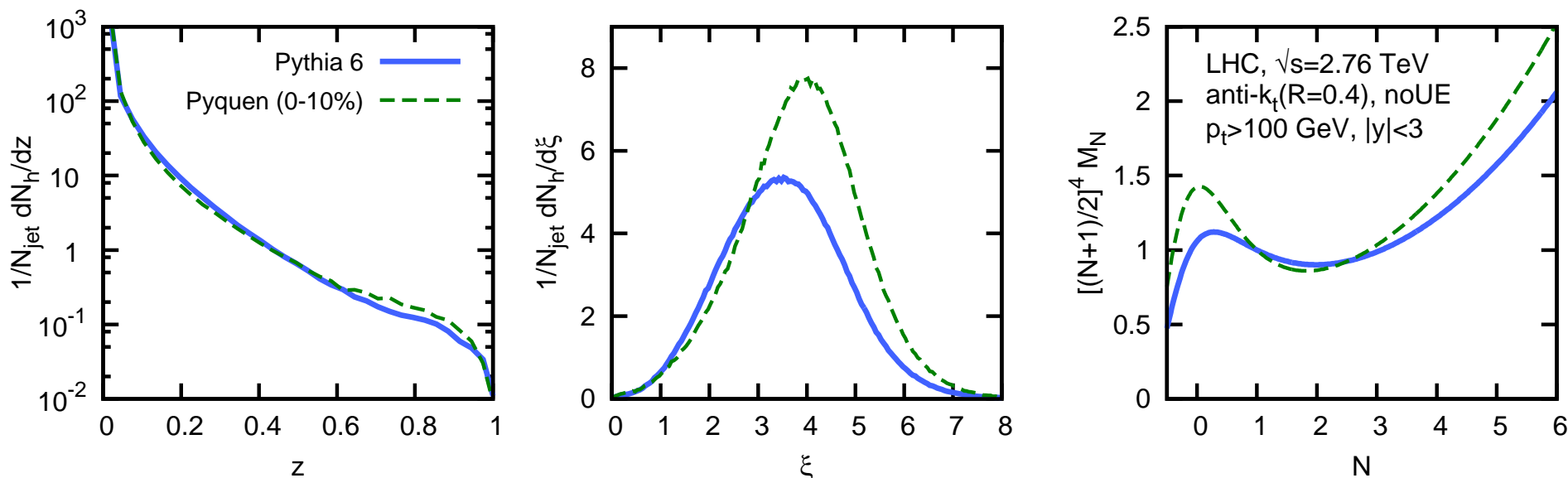
grooming+subtraction  
can prove very powerful





# Fragmentation function in HI

Example ( $\xi = \log(1/z)$ ):



Some interesting values of  $N$ :

- $N = 0$  is the particle multiplicity
- with only charged tracks  $N = 1$  is the charged fraction of momentum
- Hadron spectrum  $\propto p_t^{-n}$   
 $\Rightarrow M_{n-1}$  is the ratio of the hadron and jet spectra

# *“Standard” background subtraction*

## Underlying idea:

- measure the medium where it is not affected by the hard jets
- subtracts that from the fragmentation function

## Simple test:

region transverse to the dijet event with the same area

# Subtraction in moment space

Alternative approach:  
use jet-area-based techniques in moment space

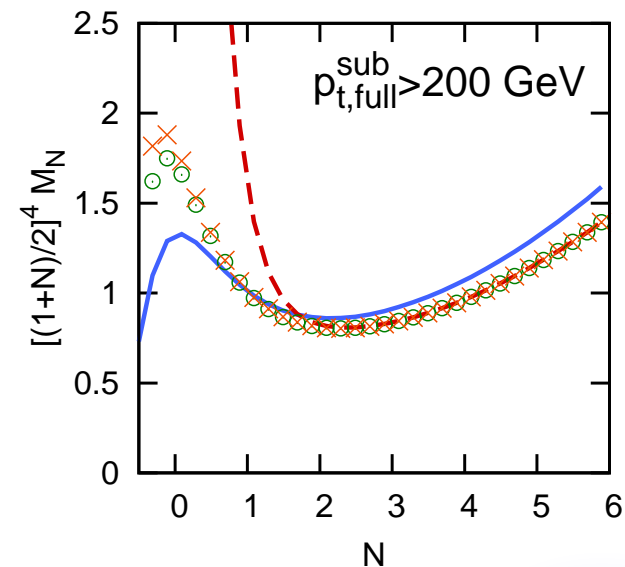
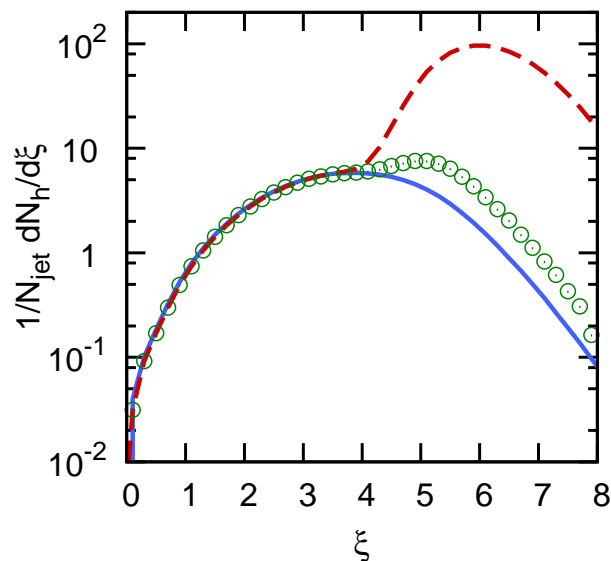
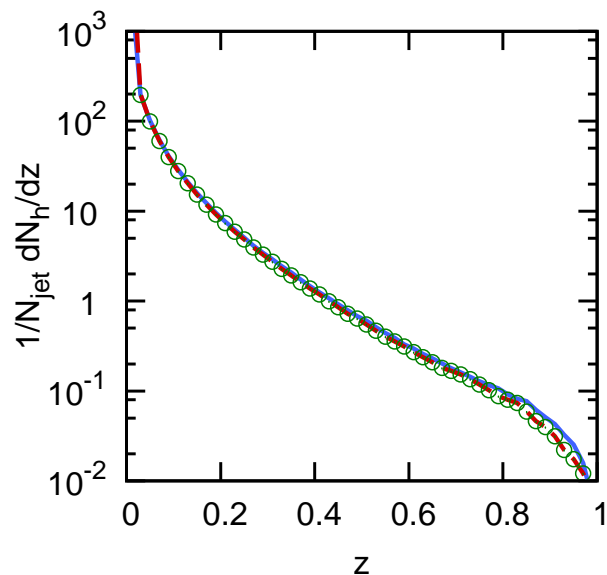
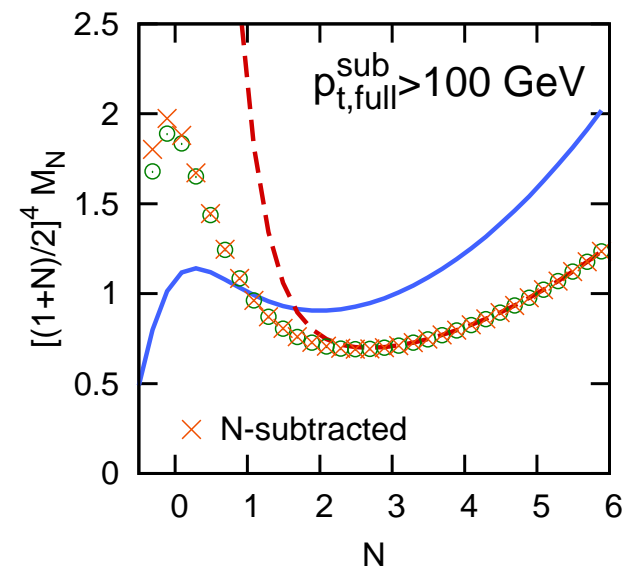
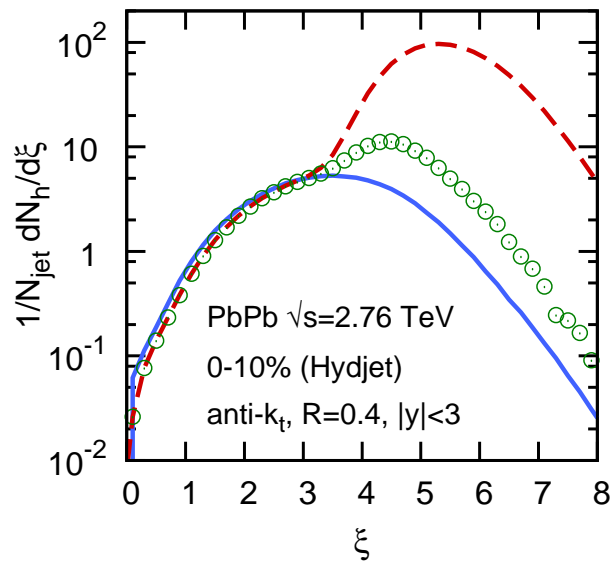
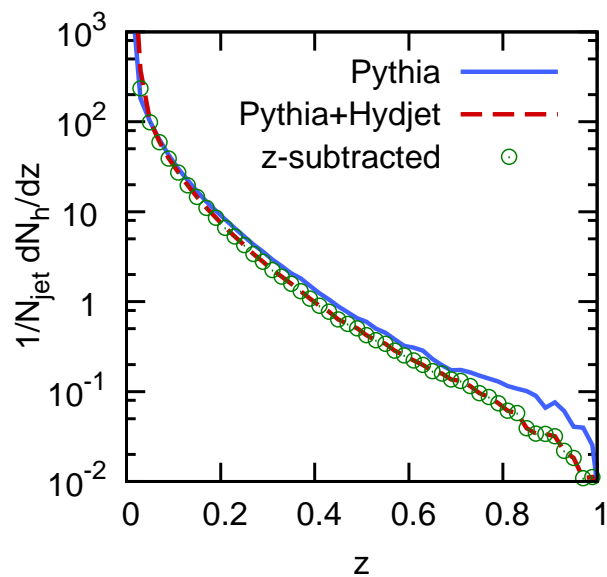
Introduce a new background property  $\rho_N$

$$\rho = \underset{\text{patches}}{\text{median}} \left\{ \frac{p_{t,\text{patch}}}{A_{\text{patch}}} \right\} \quad \rho_N = \underset{\text{patches}}{\text{median}} \left\{ \frac{\sum_{i \in \text{patch}} p_{t,i}^N}{A_{\text{patch}}} \right\}$$

and subtract using

$$M_N^{\text{sub}} = \frac{\sum_{i \in \text{jet}} p_{t,i}^N - \rho_N A}{(p_t - \rho A)^N}$$

# Fragmentation function subtraction



similar improvement for both methods but not better than a  $p_t$  cut

# Improved subtraction

## Problem:

- steeply falling jet spectrum
- cut on  $p_{t,\text{full}}^{\text{sub}}$  tends to pick smaller  $p_{t,\text{hard}}$  with upwards fluctuations

## Consequences:

- $p_{t,\text{jet}}$  overestimates i.e.  $z$  underestimated: underestimation at large  $N$
- extraneous soft particles in the medium: overestimation at small  $N$

# Improved subtraction

A simple unfolding can be analytically computed in moment space

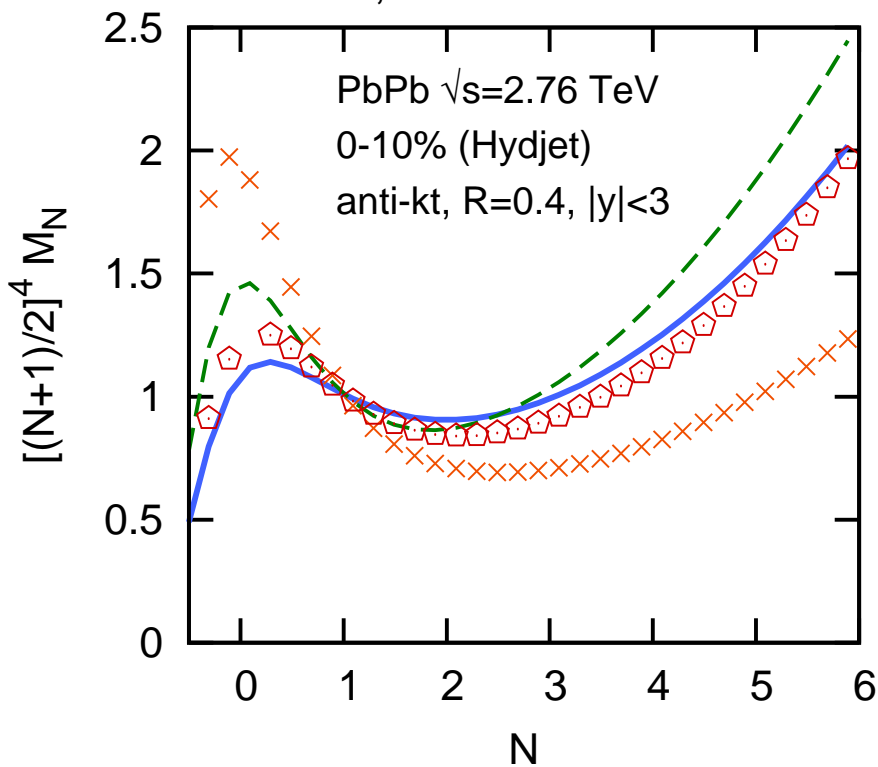
- assuming fluctuations ( $\sigma$ ) are small
- the (unfolded) inclusive jet spectrum locally:  $dN/dp_t \propto \exp(-p_t/\mu)$
- statistical info (event-by-event) on fluctuations  $\sigma$  in  $p_t$  and  $\sigma_N$  in  $\sum p_t^N$  (obtained like  $\rho$  and  $\rho_N$ )
- info on how fluctuations on  $\sum p_t^N$  are correlated to fluctuations in  $p_t$ :  $r_N$  (from patches in the event)

$$M_N^{\text{sub,imp}} = M_N^{\text{sub}} \times \left( 1 + N \frac{\sigma^2 A}{\mu p_{t,\text{jet}}} \right) - r_N \frac{\sigma \sigma_N A}{\mu p_{t,\text{jet}}^N}$$

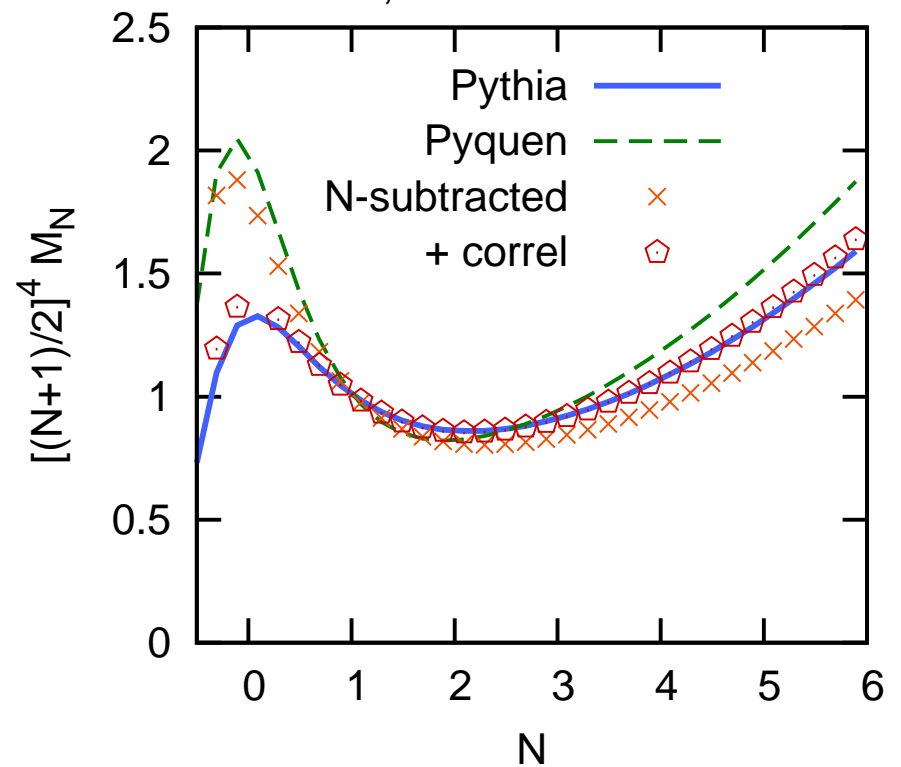
# Improved subtraction

$$M_N^{\text{sub,imp}} = M_N^{\text{sub}} \times \left( 1 + N \frac{\sigma^2 A}{\mu p_{t,\text{jet}}} \right) - r_N \frac{\sigma \sigma_N A}{\mu p_{t,\text{jet}}^N}$$

$p_{t,\text{full}}^{\text{sub}} \geq 100 \text{ GeV}$



$p_{t,\text{full}}^{\text{sub}} \geq 200 \text{ GeV}$



Much nicer and only easily done in moments!