News in jet algorithms:

SISCone and anti-k_t

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G.P. Salam, G. Soyez, JHEP 05 (2007) 086 [arXiv:0704.0292]

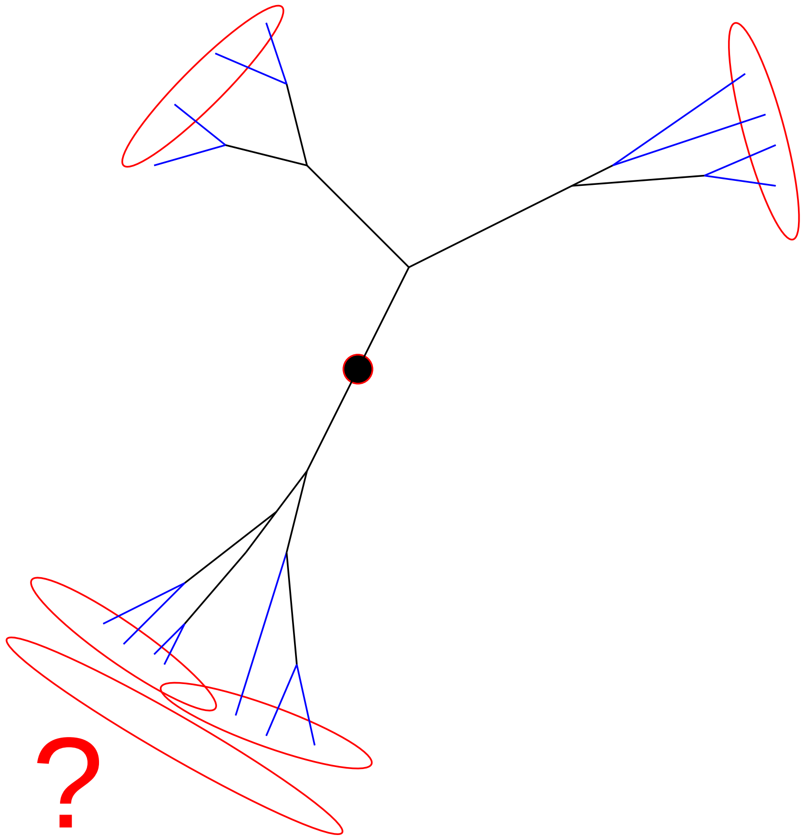
M. Cacciari, G.P. Salam, G. Soyez, JHEP 04 (2008) 063 [arXiv:0802.1189]

Jet definitions

Aim: Study hard processes

- QCD backgrounds, top quark physics
- Higgs, physics beyond the standard model

Define jets: parton $\leftrightarrow$ jet



But: partons are ambiguous

Hence: Multiple definitions of a "jet"

Two classes of algorithms

Class 1: recombination	Cass 2: cone
Successive recombinations of the "closest"$^{(a)}$ pair of particle	find directions of energy flow $\equiv$ stable cones$^{(b)}$
Nice perturbative behaviour	Small sensitivity to soft radiation (UE,PU)
Often used in $e^{\pm}e^{\pm}$, $e^{\pm}p$	Often used in pp

(a) Distance:

$$k_t: \quad d_{i,j} = \min(k_{t,i}^2, k_{t,j}^2)(\Delta\phi_{i,j}^2 + \Delta y_{i,j}^2)$$

$$\text{Aachen/Cam.:} \quad d_{i,j} = \Delta\phi_{i,j}^2 + \Delta y_{i,j}^2$$

(b) **stable cones** (radius R) such that:
the total momentum of its contents points in the direction of its centre

How the cone works...

- Seeded (iterative) approaches: iterate from an initial position until stable
  - seed = initial particle
  - seed = midpoint between stable cones found at first step
- One has to deal with overlapping stable cones: 2 subclasses

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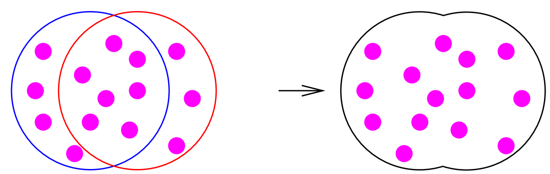
Class 2(a): cone with split-merge (ex.: JetClu, Atlas, MidPoint):

$\tilde{p}_{t,\text{shared}} > f\,\tilde{p}_{t,\text{min}}$



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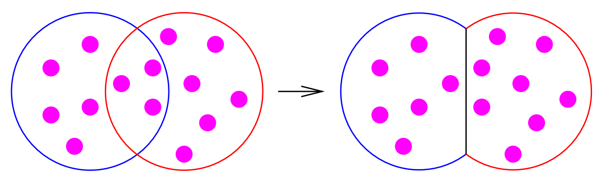

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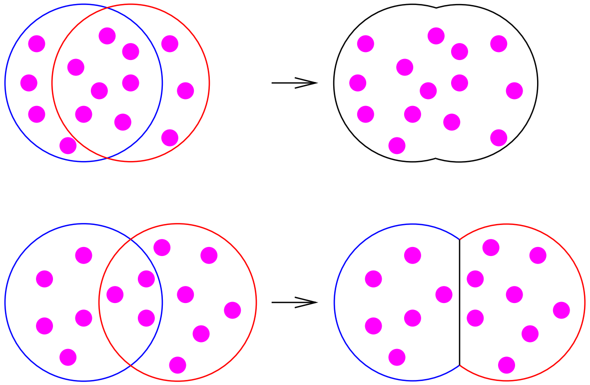


Class 2(b): cone with progressive removal (ex.: Iterative Cone)

- iterate from the hardest seed
- remove the stable cone as a jet and start again

Idea: “regular/circular” jets

SNOWMASS accords

SNOWMASS, Tevatron 1990 (i.e. old!):

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

i.e. usable by theoreticians (*e.g.* finite perturbative results)
and experimentalists (*e.g.* fast enough)

This talk:

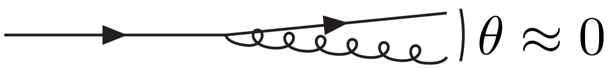
- Seeded cone algorithms miss stable cones $\Rightarrow$ theoretical problems
- That can be solved keeping experimental usefulness

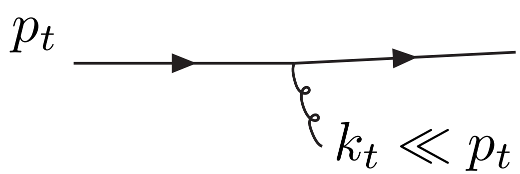
QCD divergences

QCD probability for gluon bremsstrahlung at angle θ and $\perp$-mom. k_t:

$$dP \propto \alpha_s \frac{d\theta}{\theta} \frac{dk_t}{k_t}$$

Two divergences:





Collinear

Soft

QCD probability for gluon bremsstrahlung at angle θ and $\perp$-mom. k_t:

$$dP \propto \alpha_s \frac{d\theta}{\theta} \frac{dk_t}{k_t}$$

Two divergences:

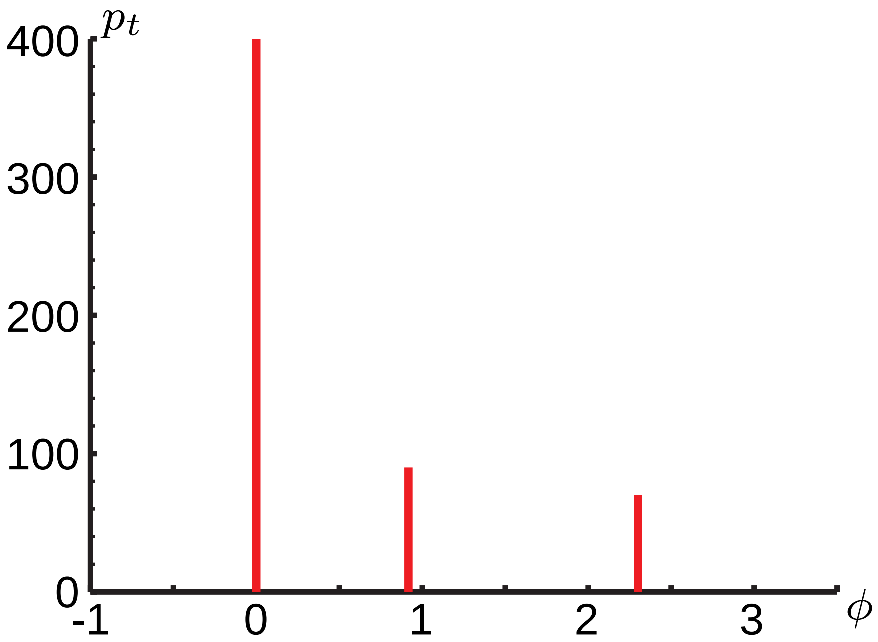
Collinear

Soft

For pQCD to make sense, the (hard) jets (or stable cones) should not change when

- one has a collinear splitting
  i.e. replaces one parton by two at the same place (η, ϕ)
- one has a soft emission *i.e.* adds a very soft gluon

IR unsafety of the Midpoint alg



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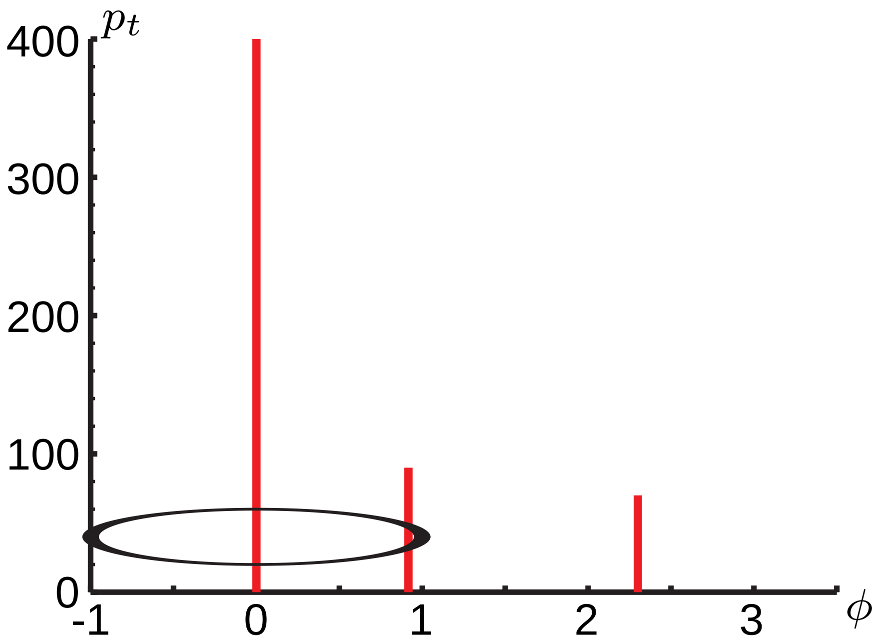

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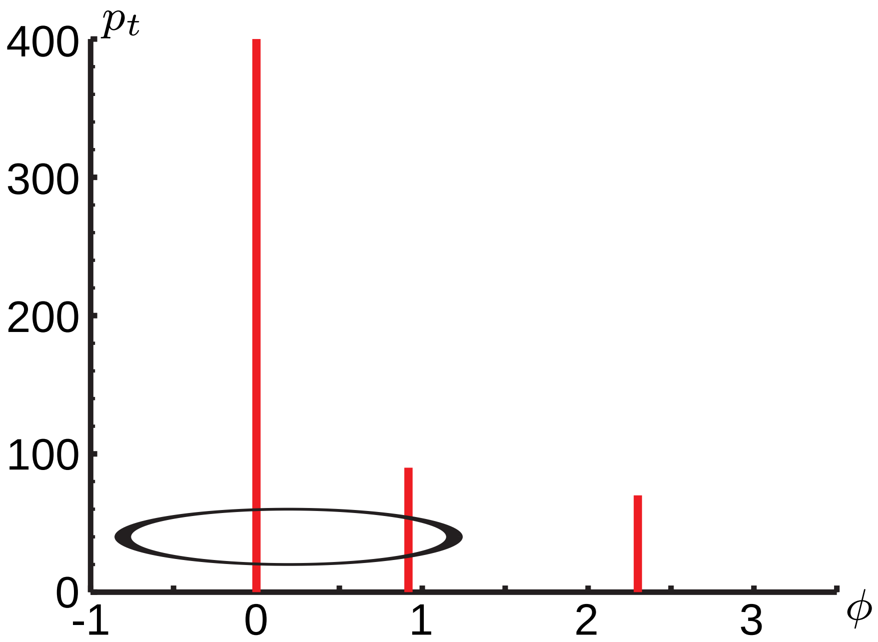

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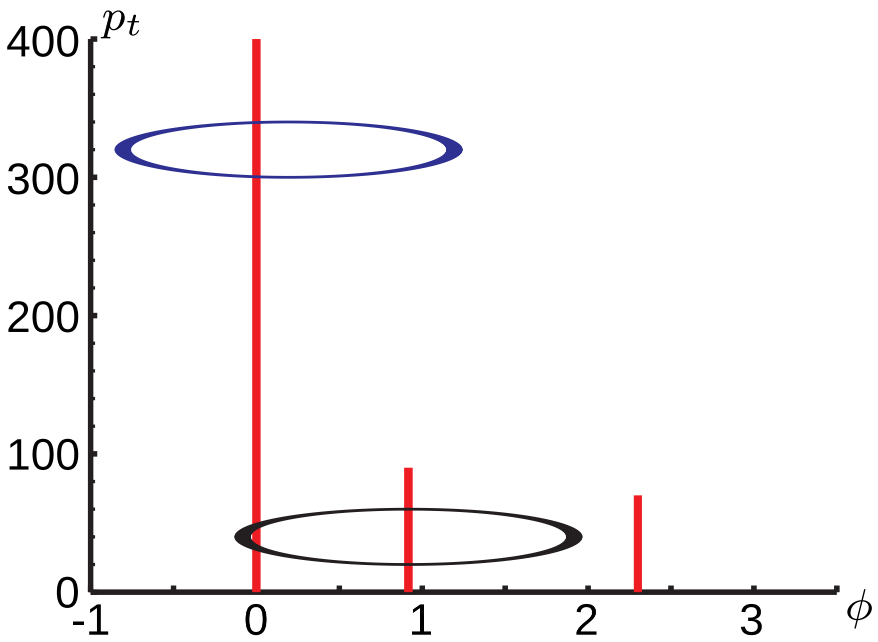

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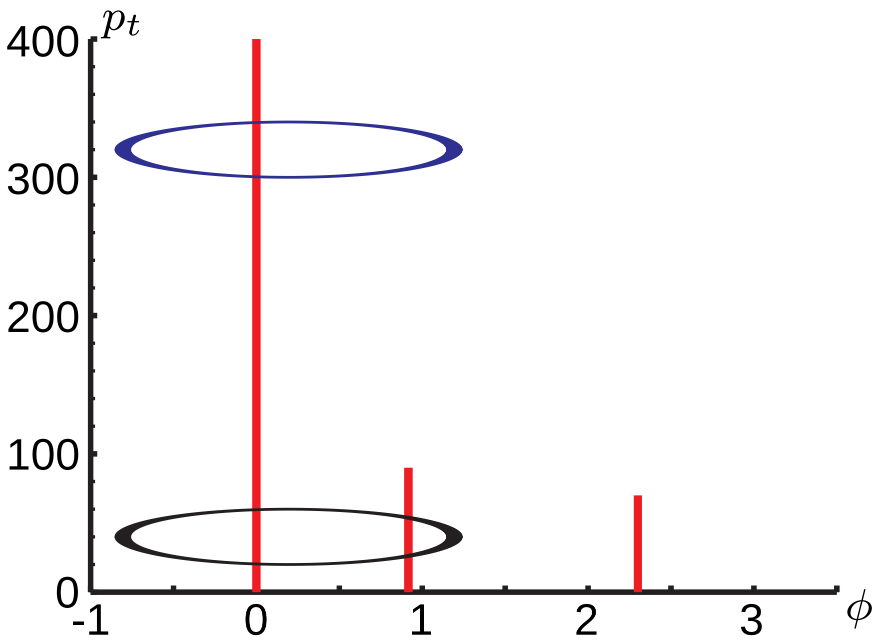

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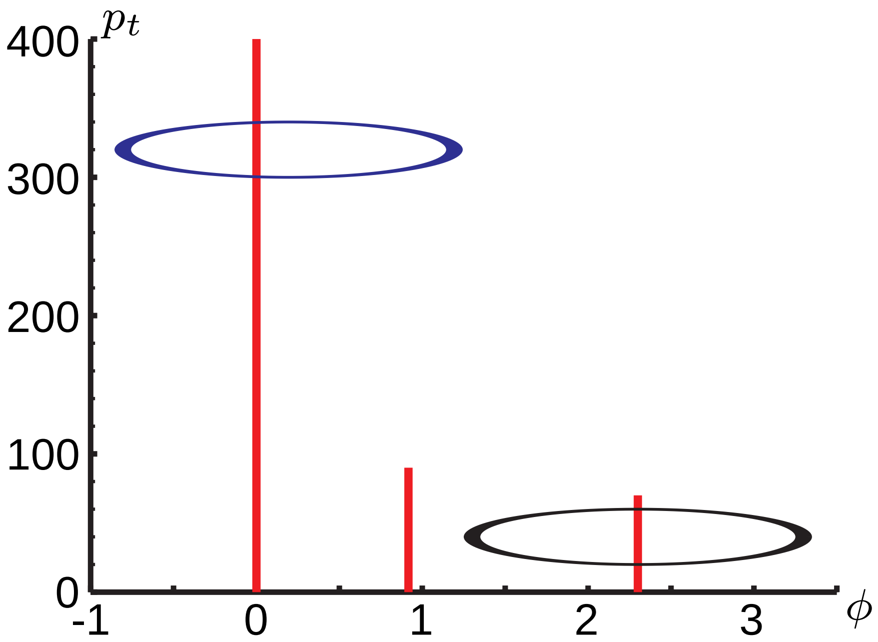

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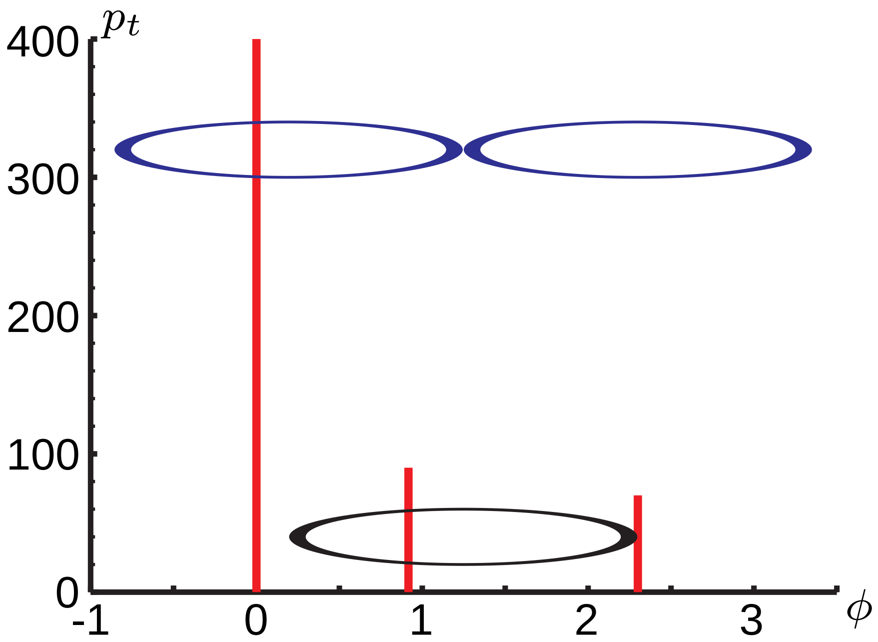
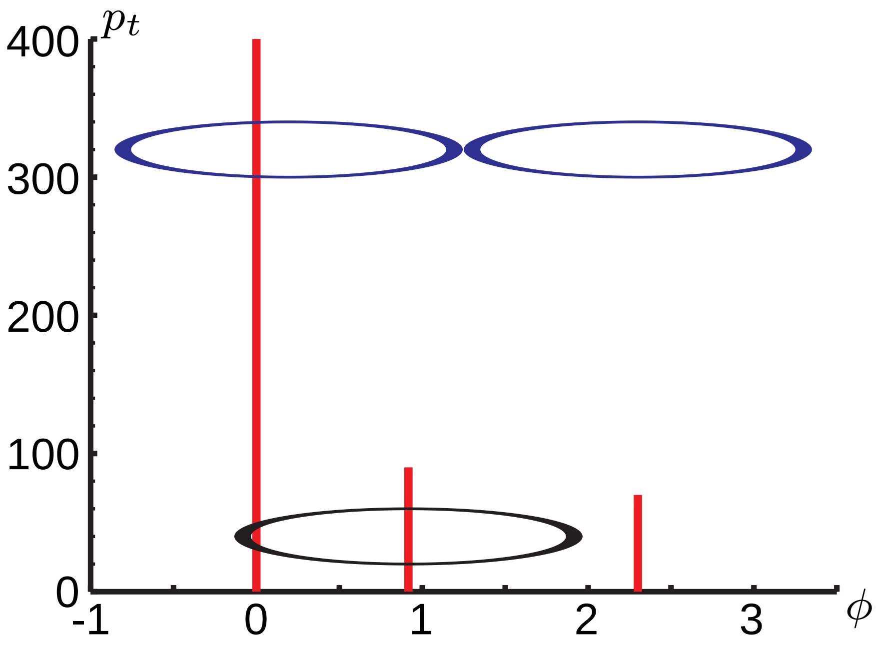
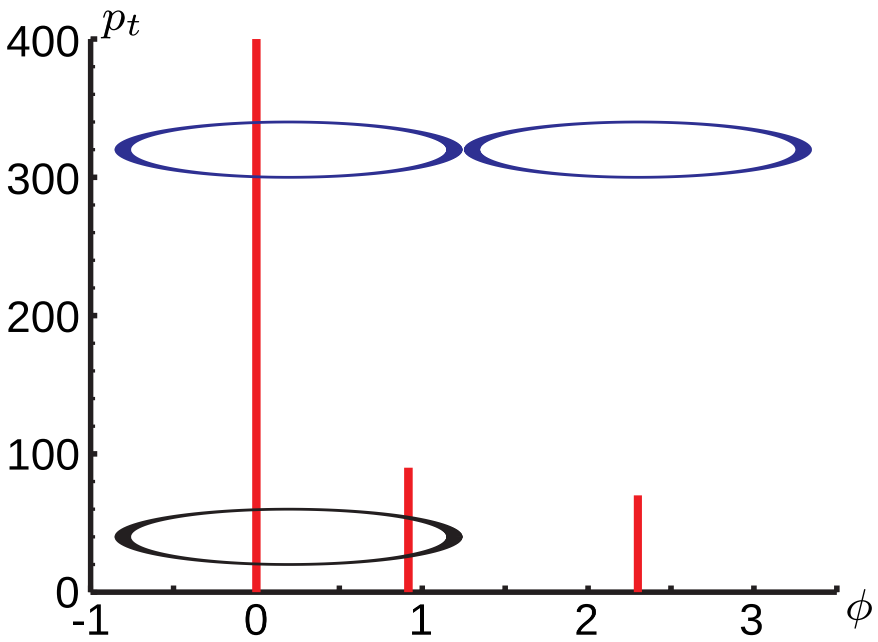



p_t
400
300
200
100
0
-1
0
1
2
3
ϕ

IR unsafety of the Midpoint alg



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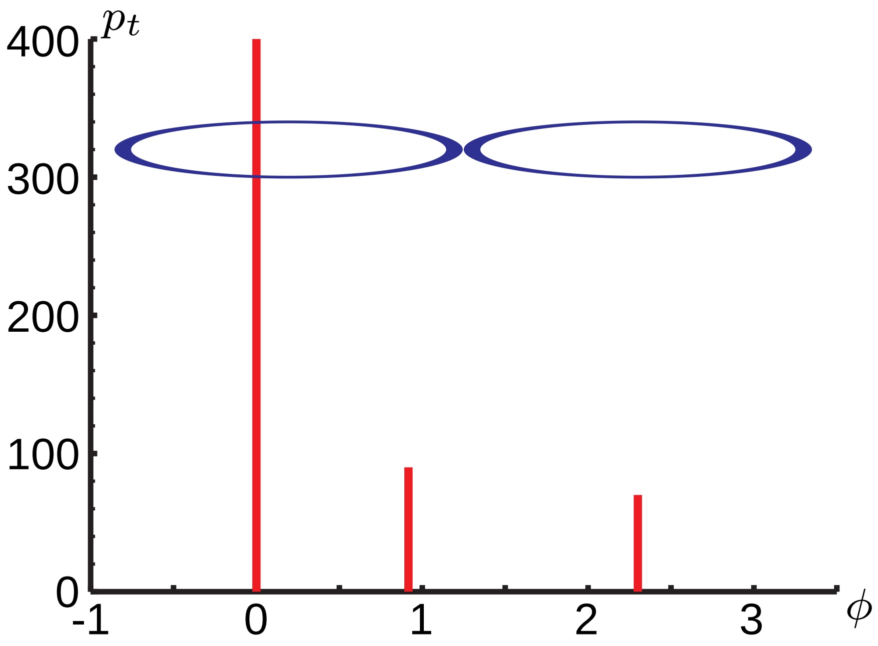

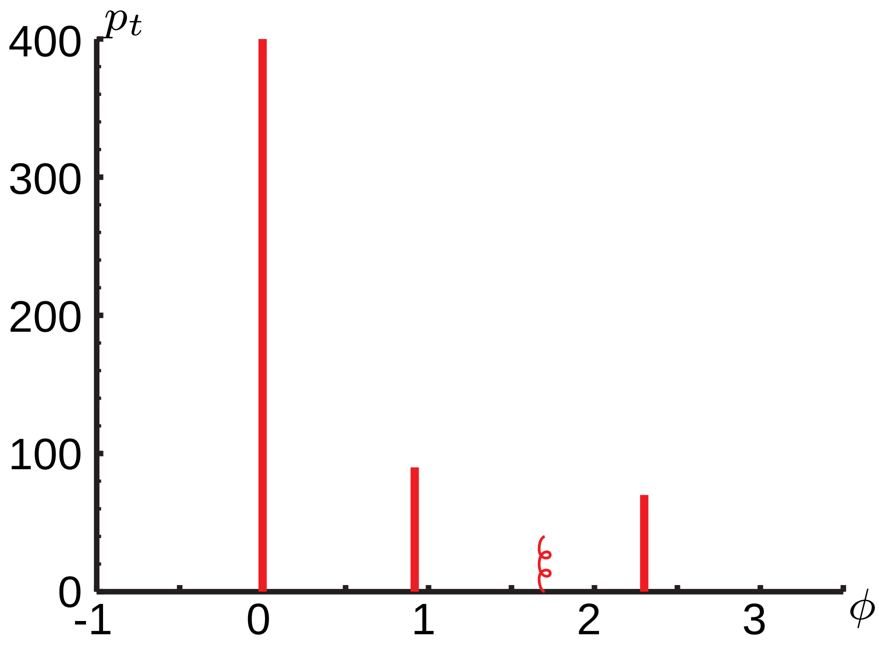

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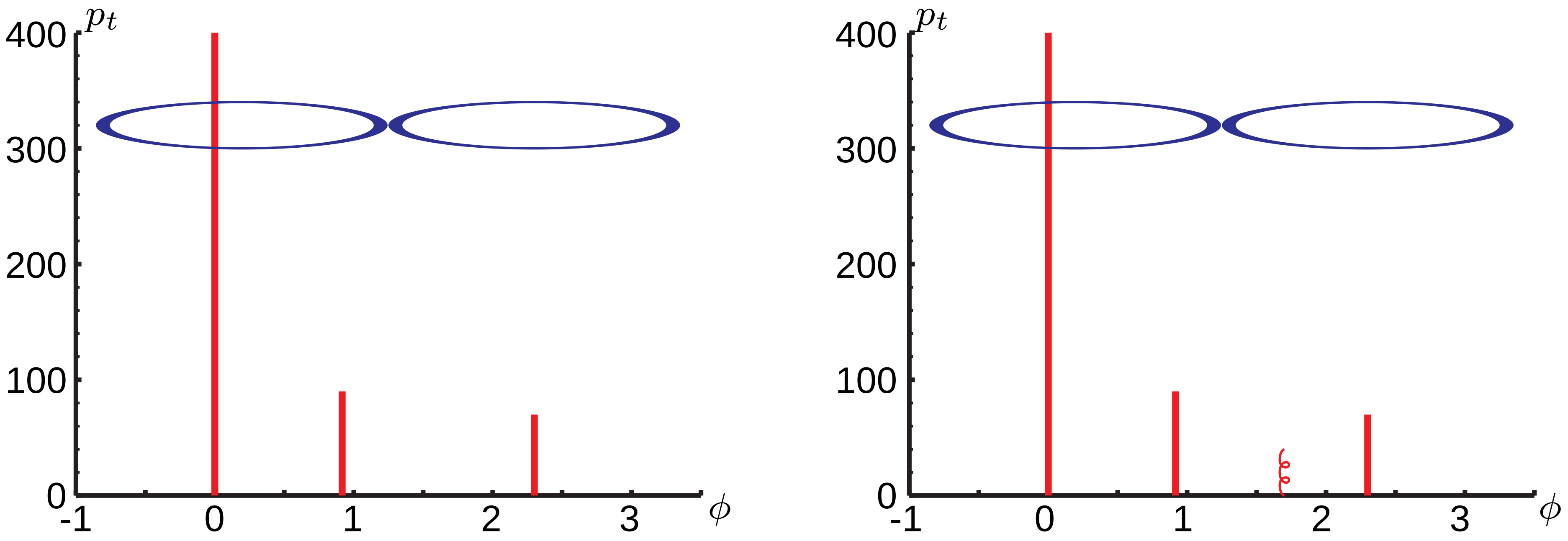

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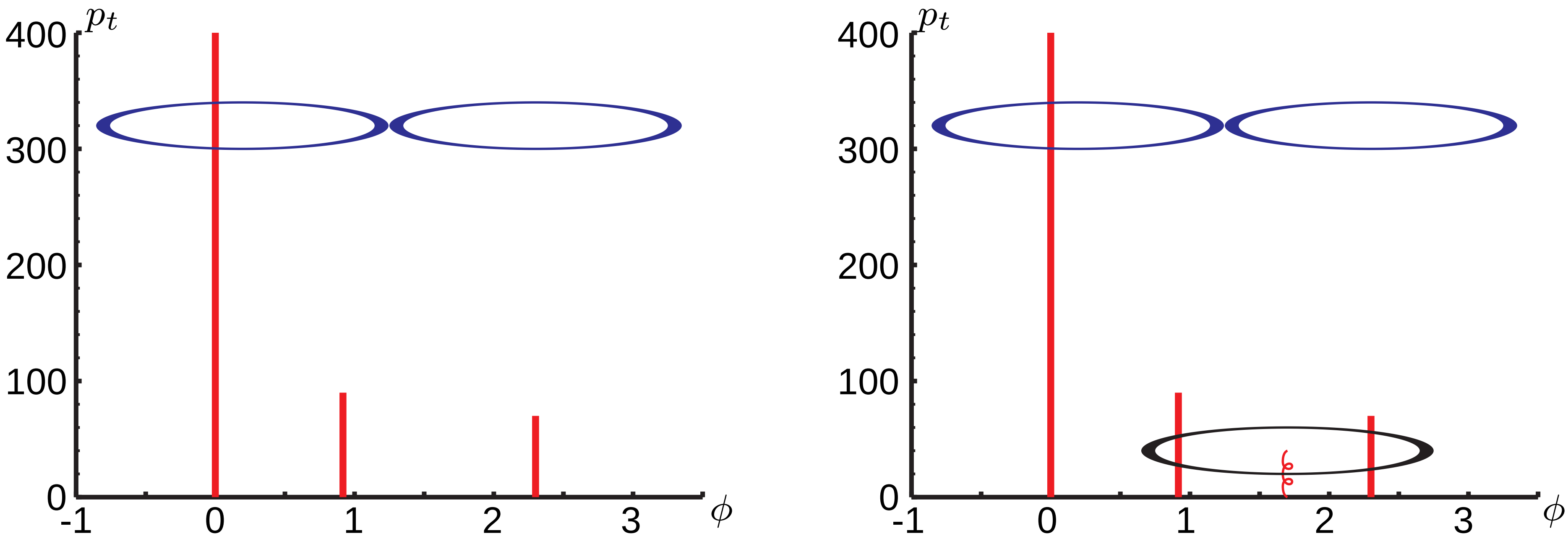

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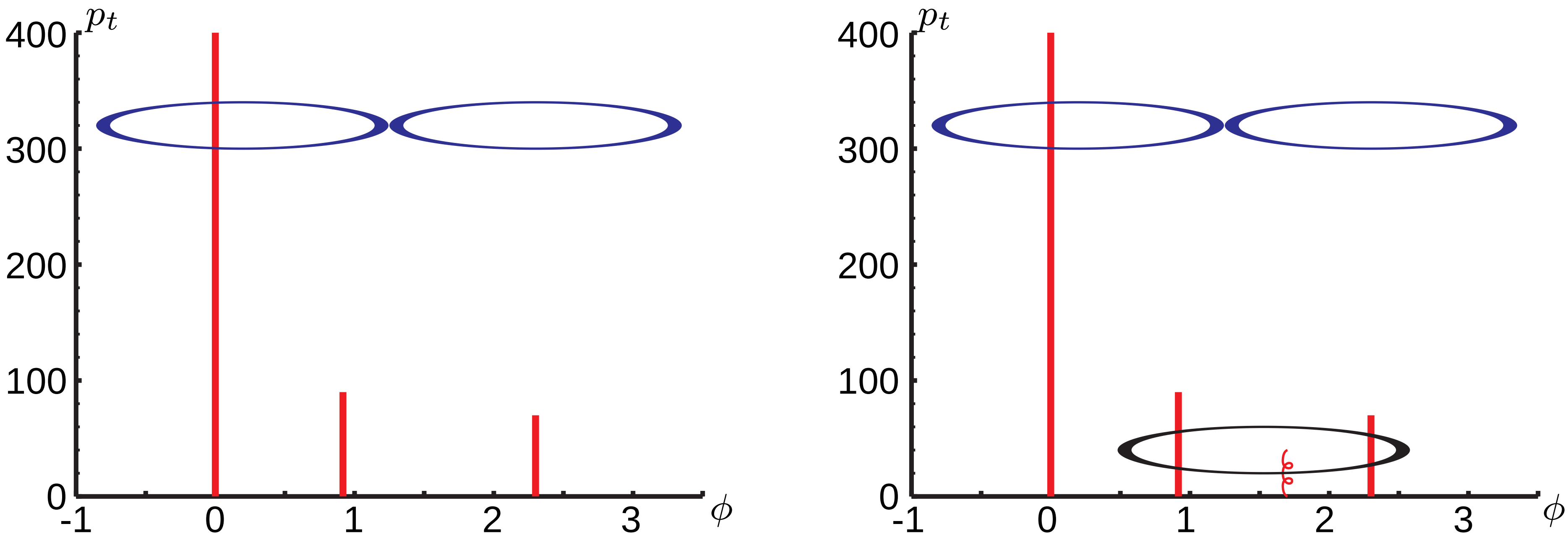

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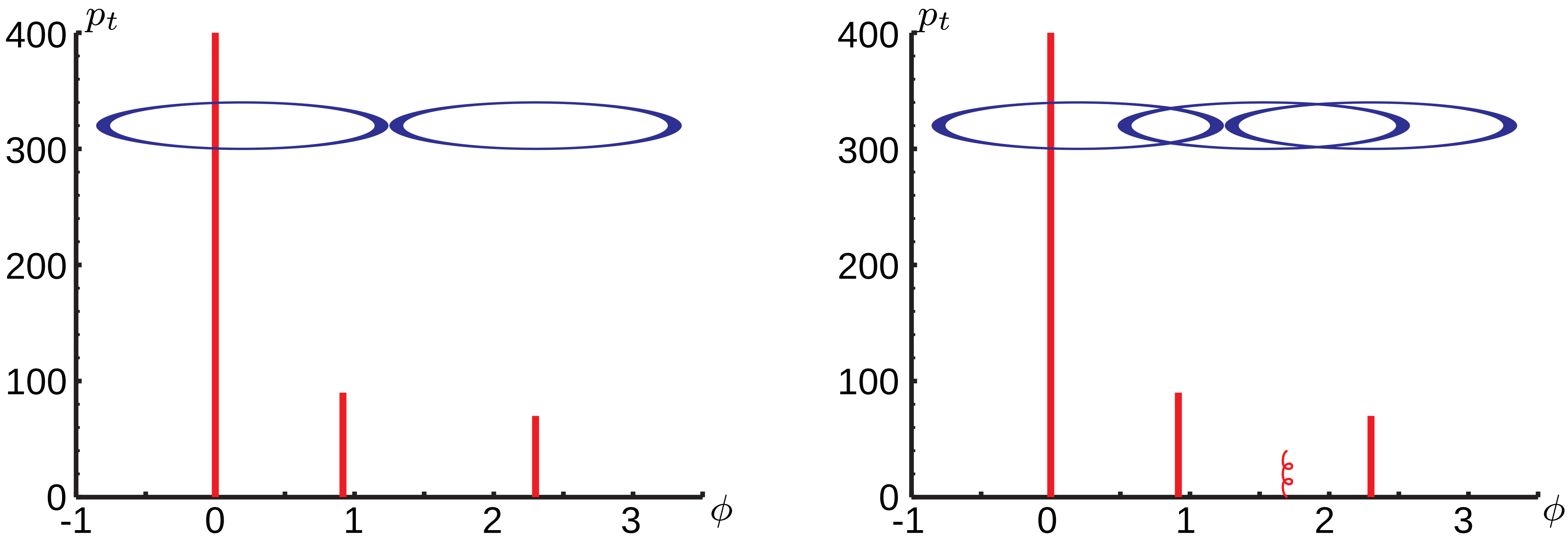
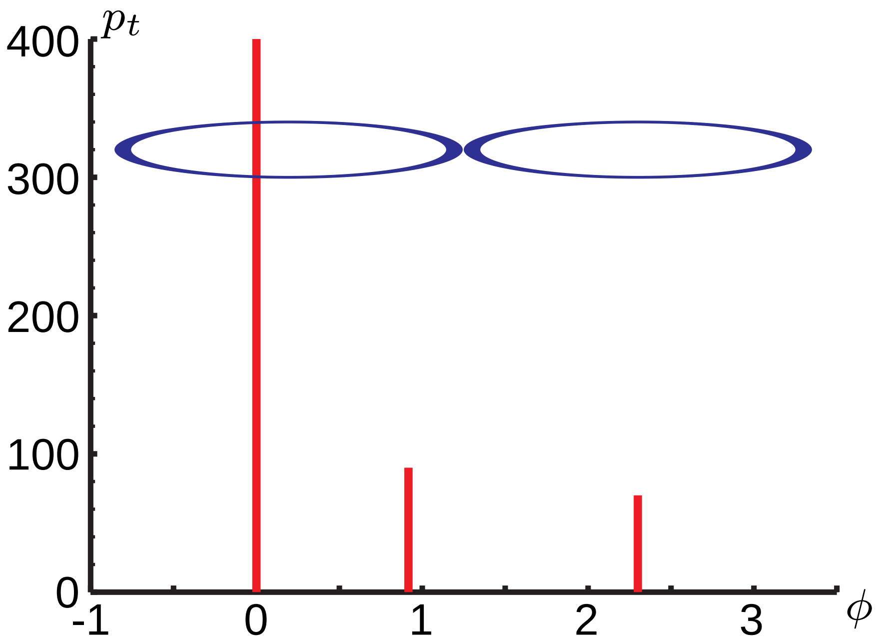
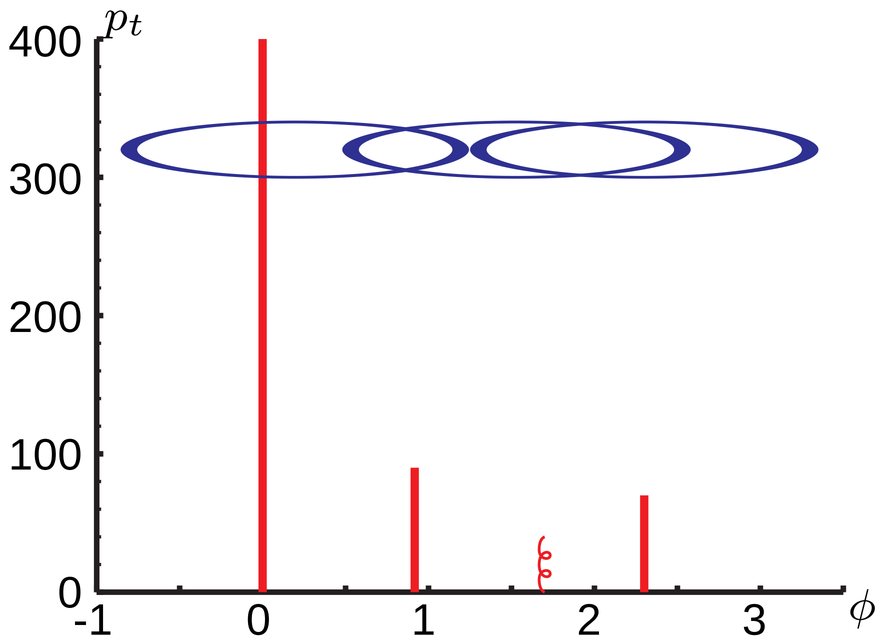


Stable cones:

Midpoint: {1,2} & {3} {1,2} & {3} & {2,3}

IR unsafety of the Midpoint alg





Stable cones:

Midpoint: {1,2} & {3} | {1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint: {1,2} & {3} | {1,2,3}

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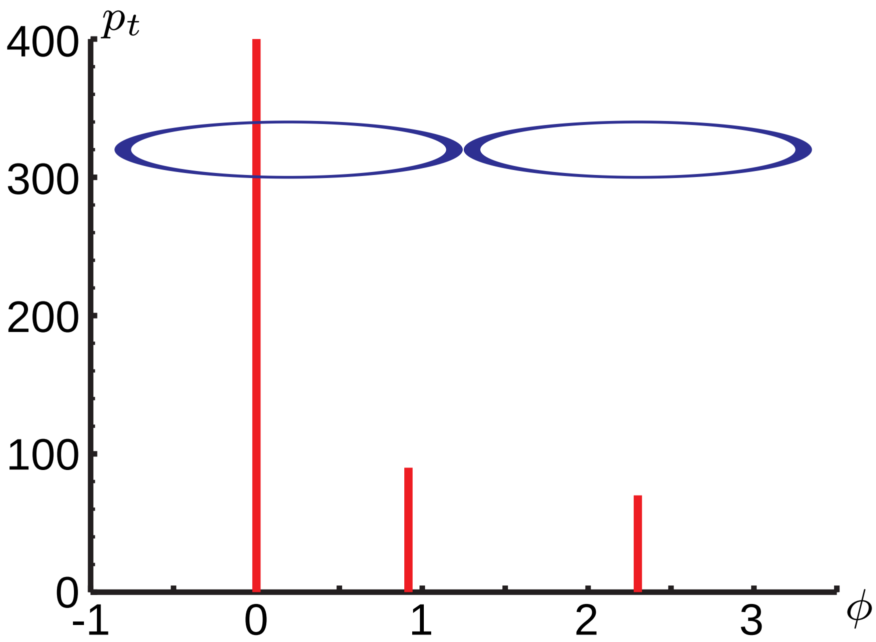

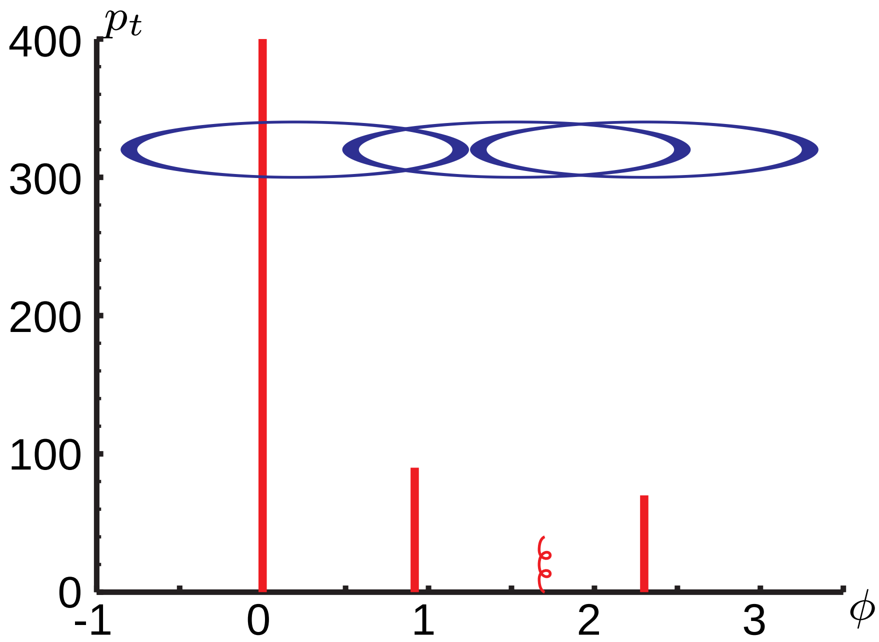

Stable cones:

Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Seedless:	{1,2} & {3} & {2,3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

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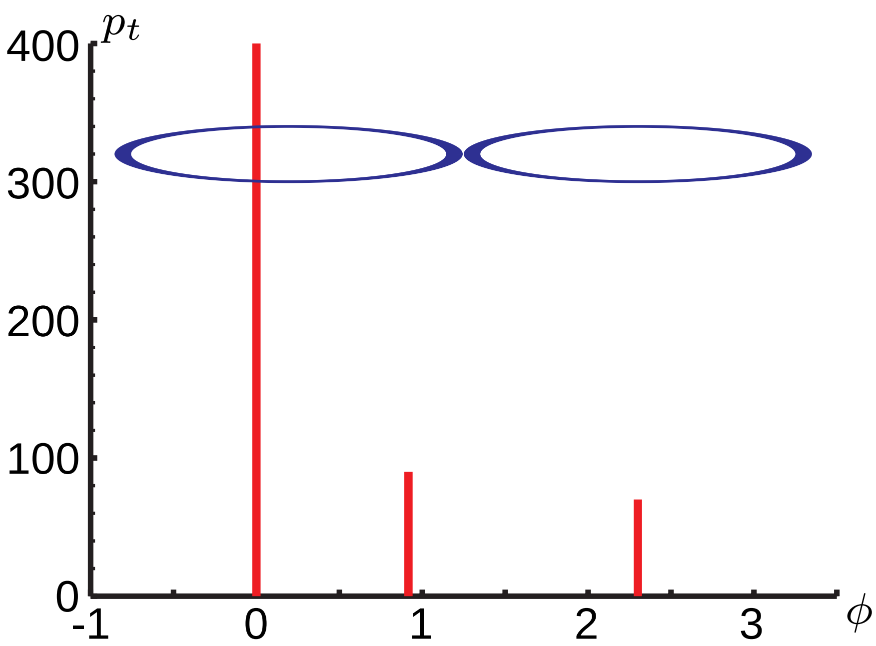
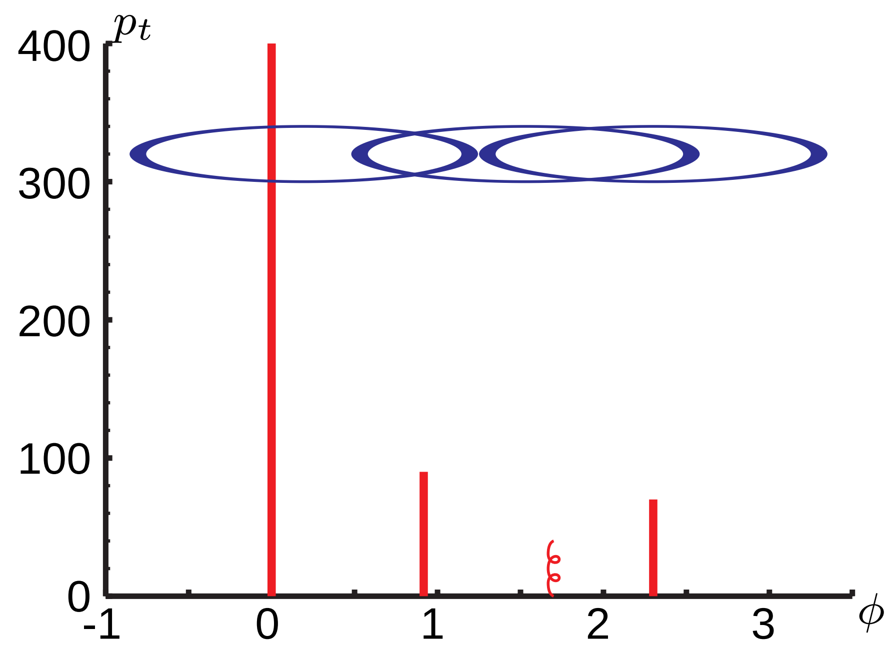
Seedless:	{1,2,3}	{1,2,3}





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Stable cone missed $\longrightarrow$ IR unsafety of the midpoint algorithm

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
- Naive approach: check stability of each subset of particle

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- Naive approach: check stability of each subset of particle
  Complexity is $\mathcal{O}\left(N2^N\right)$
  $\Rightarrow$ definitely unrealistic: 10^{17} years for $N = 100$
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

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Idea: use geometric arguments

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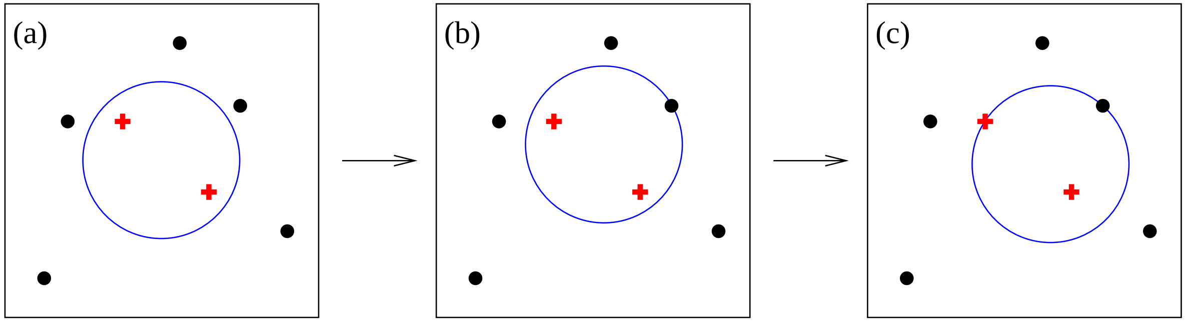

- Each enclosure can be moved (in any direction) until it touches a point
- ... then rotated until it touches a second one

Solution: SISCone

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- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

Idea: use geometric arguments

$\Rightarrow$ Enumerate all pairs of particles
with 2 circle orientations and 4 possible inclusion/exclusion
$\longrightarrow$ find all enclosures

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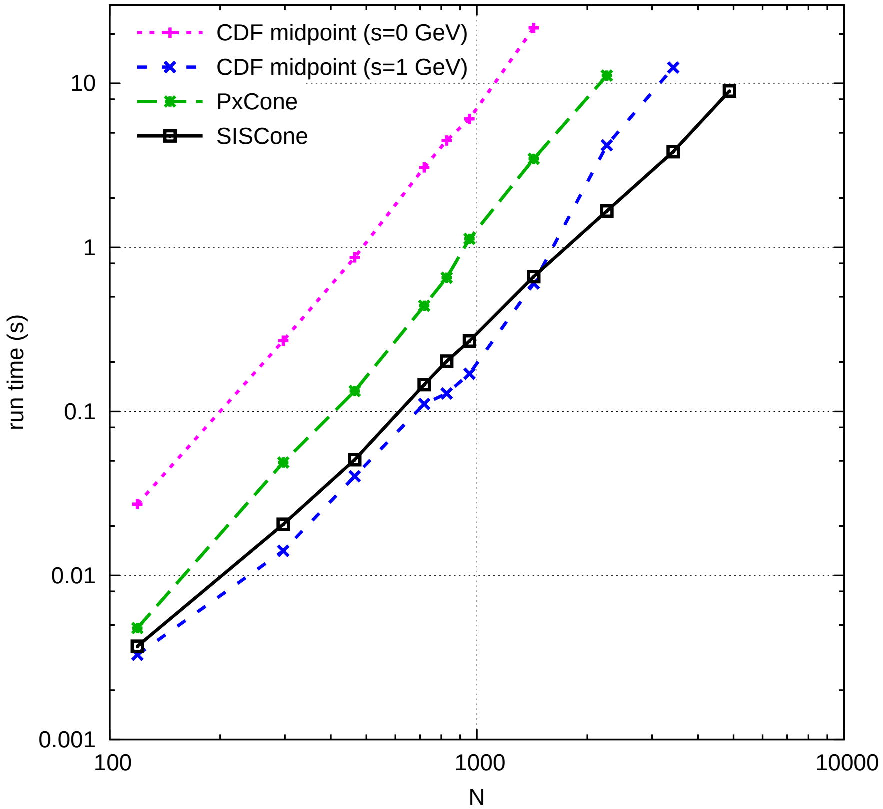
- Complexity: $\mathcal{O}\left(N^3\right)$, with improvements: $\mathcal{O}\left(N^2\log(N)\right)$

$\longrightarrow$ C++ implementation: Seedless Infrared-Safe Cone algorithm (SISCone)
G.Salam, G.S., JHEP 04 (2007) 086; http://projects.hepforge.org/siscone

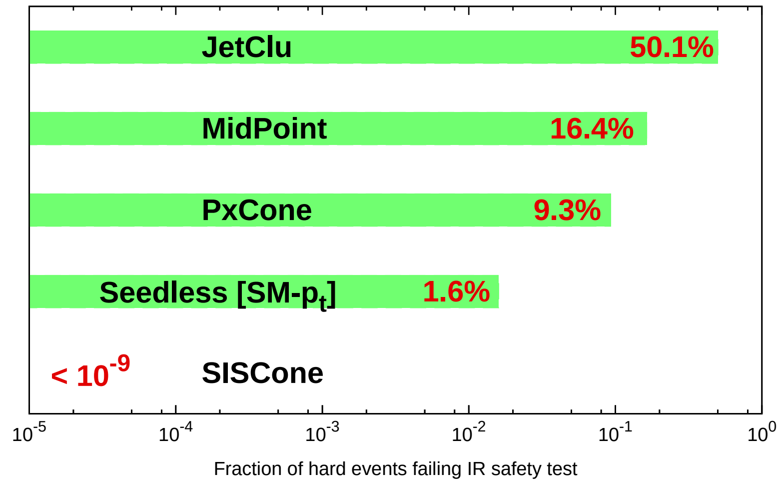
NB.: also available from FastJet
[M.Cacciari, G.Salam, G.S.]; http://www.lpthe.jussieu.fr/∼salam/fastjet

Physical impact

Execution timings:



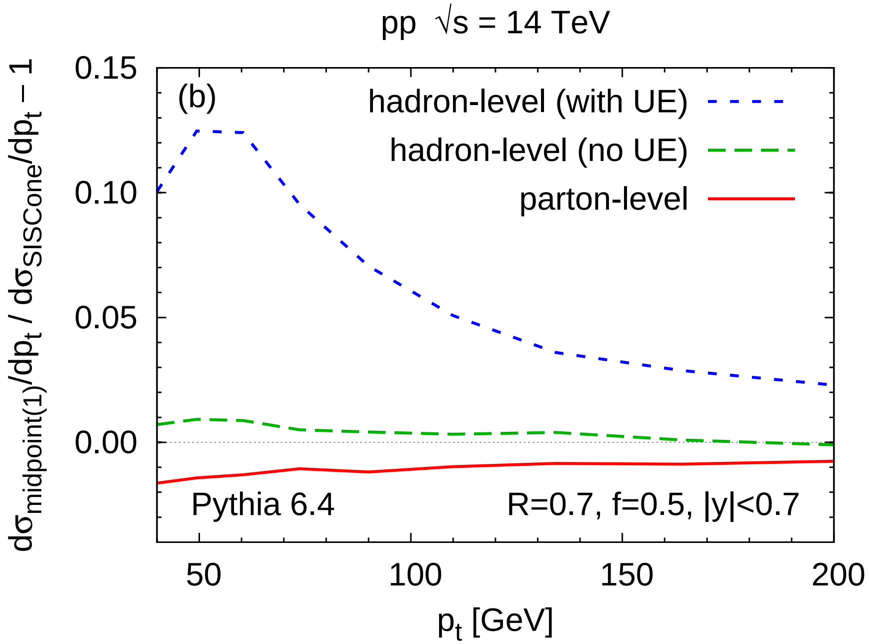
Random hard & soft partons fraction with "hard $\neq$ hard+soft"



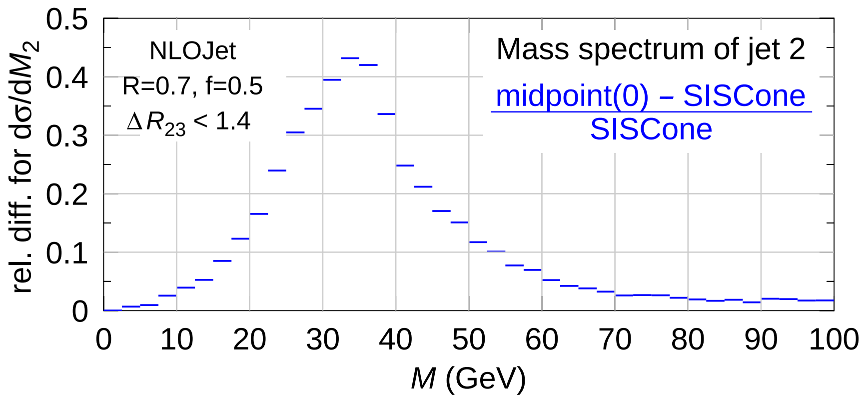
- at least as fast as midpoint cones
- IR safe
  - JetClu,ATLAS cone: 50% failure
  - MidPoint: 15% failure

Physical impact (2)

Inclusive (midpoint/SISCone-1)

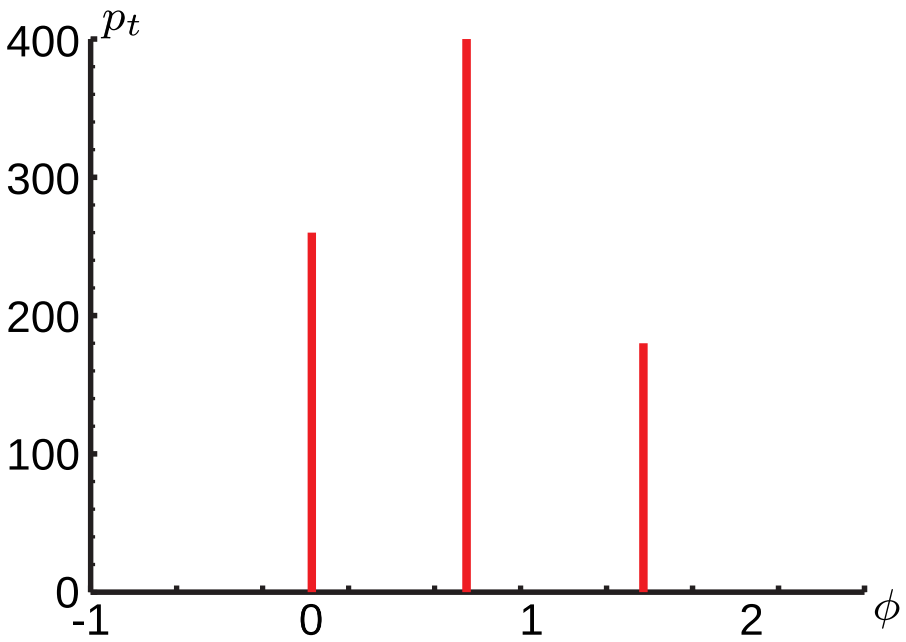


Masses in 3-jet events



- Inclusive cross-section:
  - Effect of a few percents
  - Less sensiticity to the UE
- More exclusive processes: effects $\sim 45\%$ (Important for LHC!)

Coll. unsafety of the iterative cone



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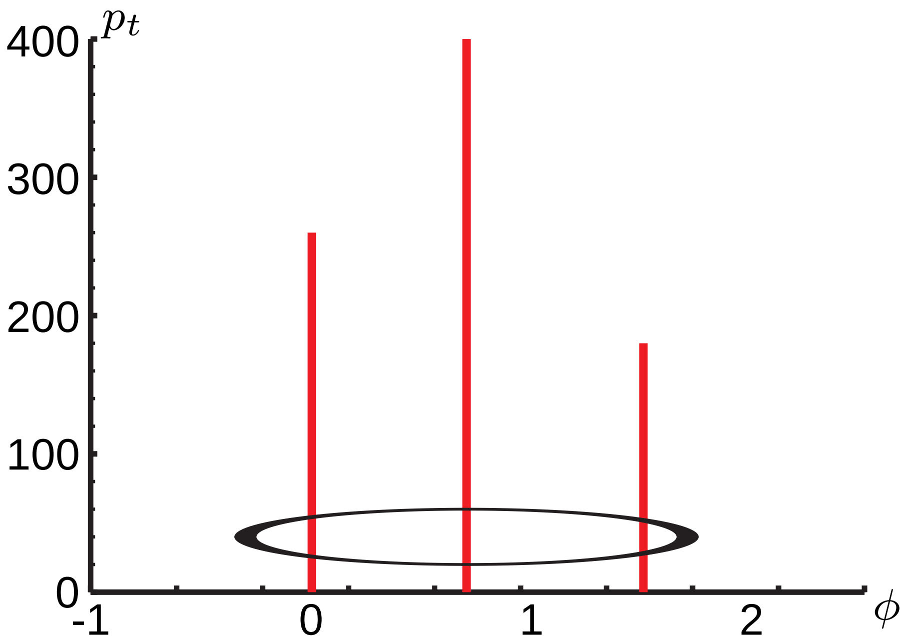

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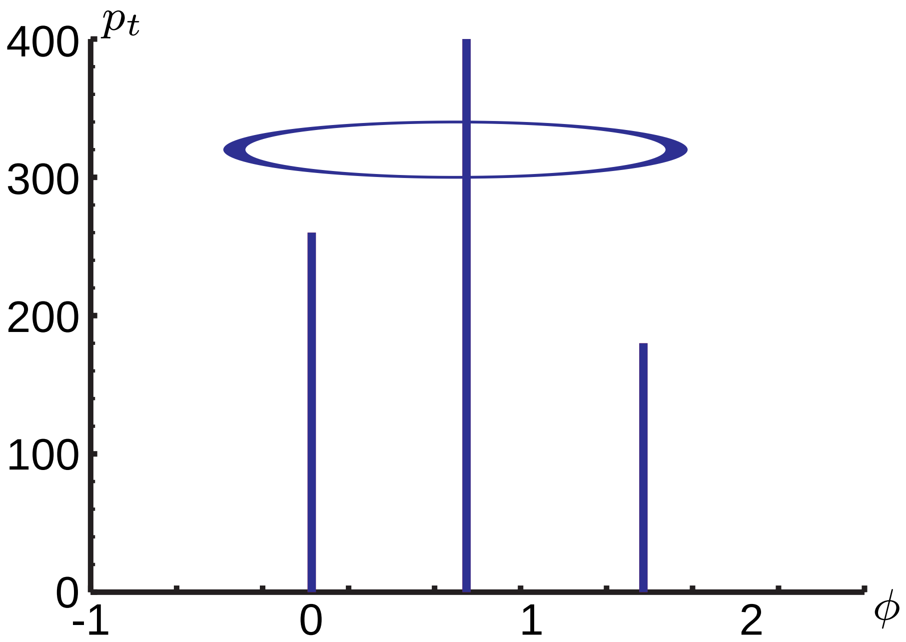

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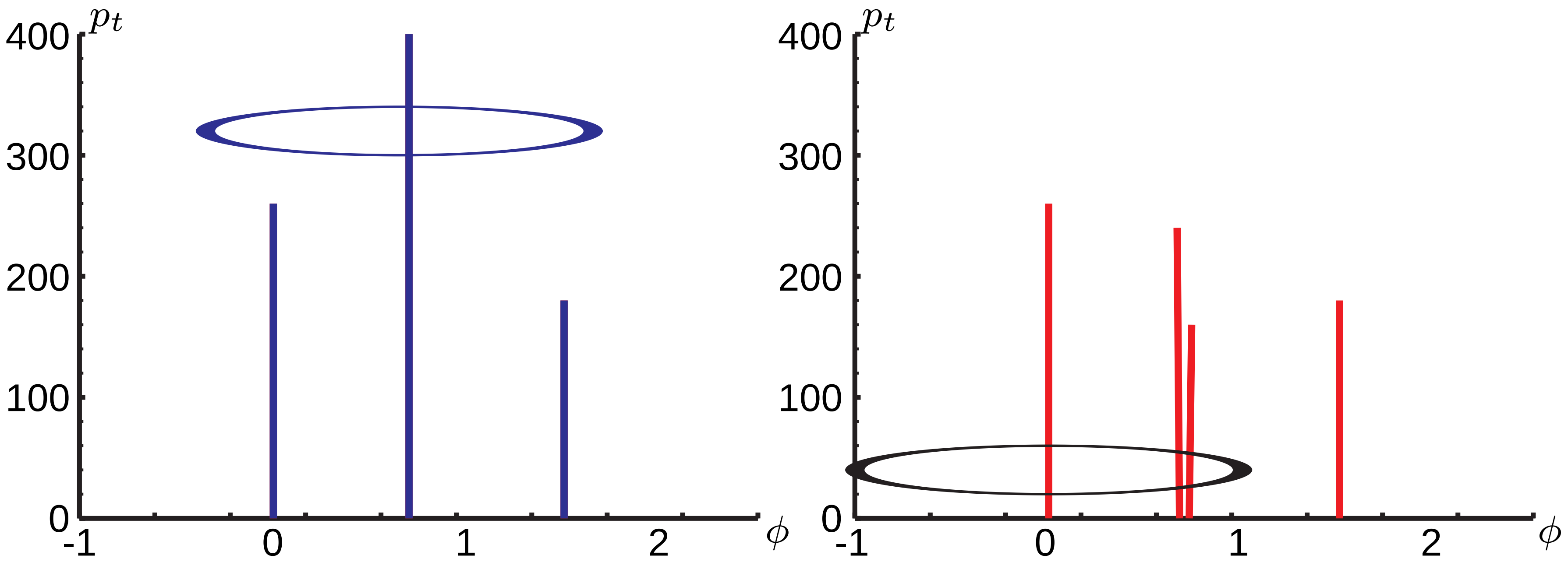

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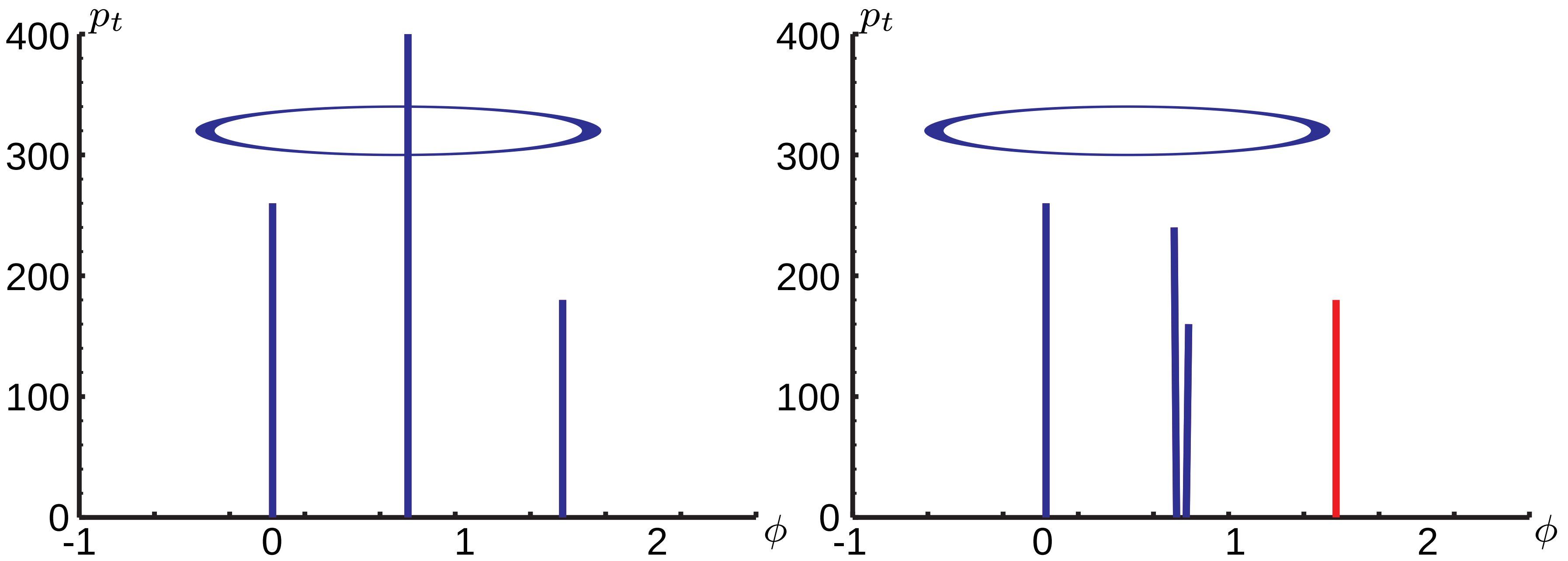

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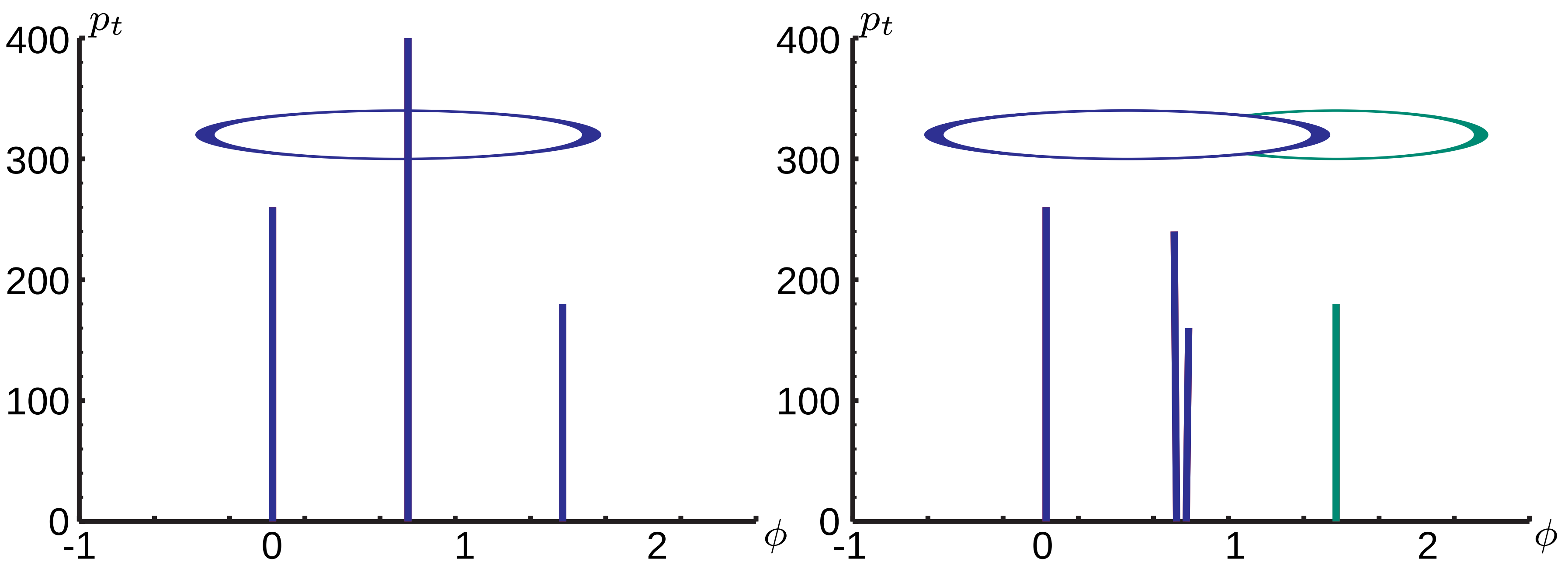

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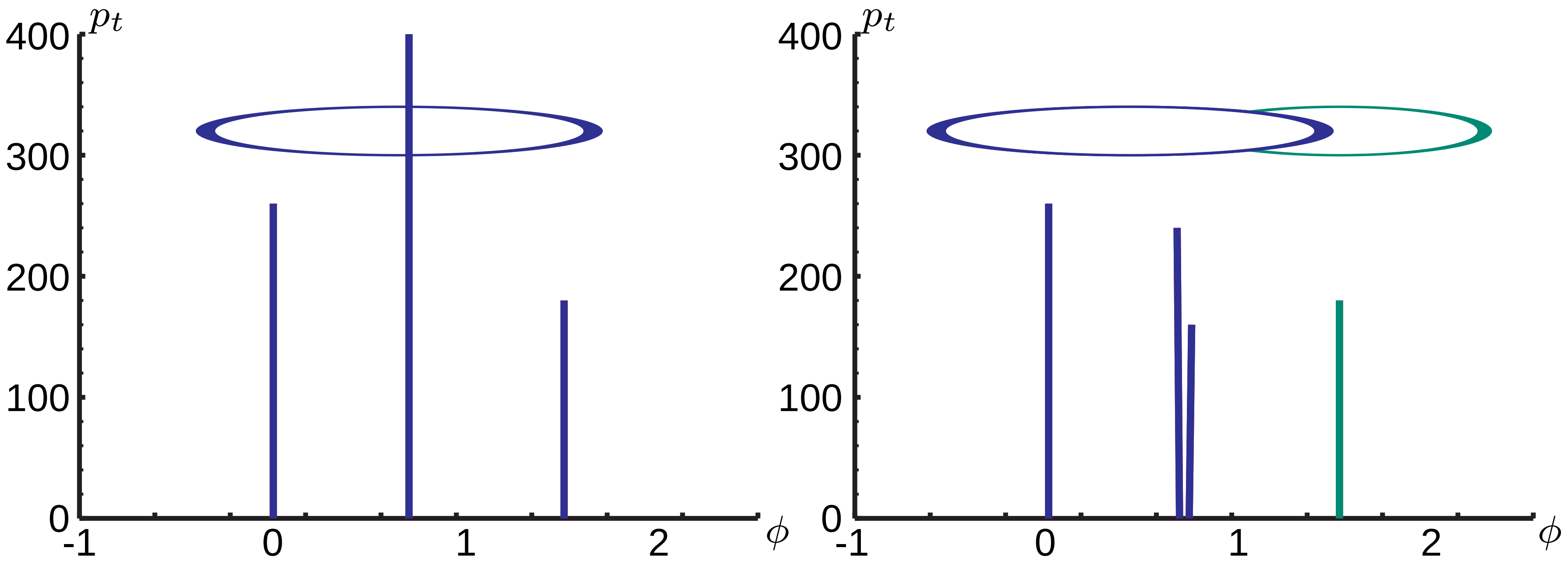

- Before collinear spliting: 1 jet
- After collinear spliting: 2 jets

$\longrightarrow$ **collinear unsafety of the iterative cone algorithm**

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p})\ \left(\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2\right)$$

- $p = 1$: k_t algorithm
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Anti-k_t

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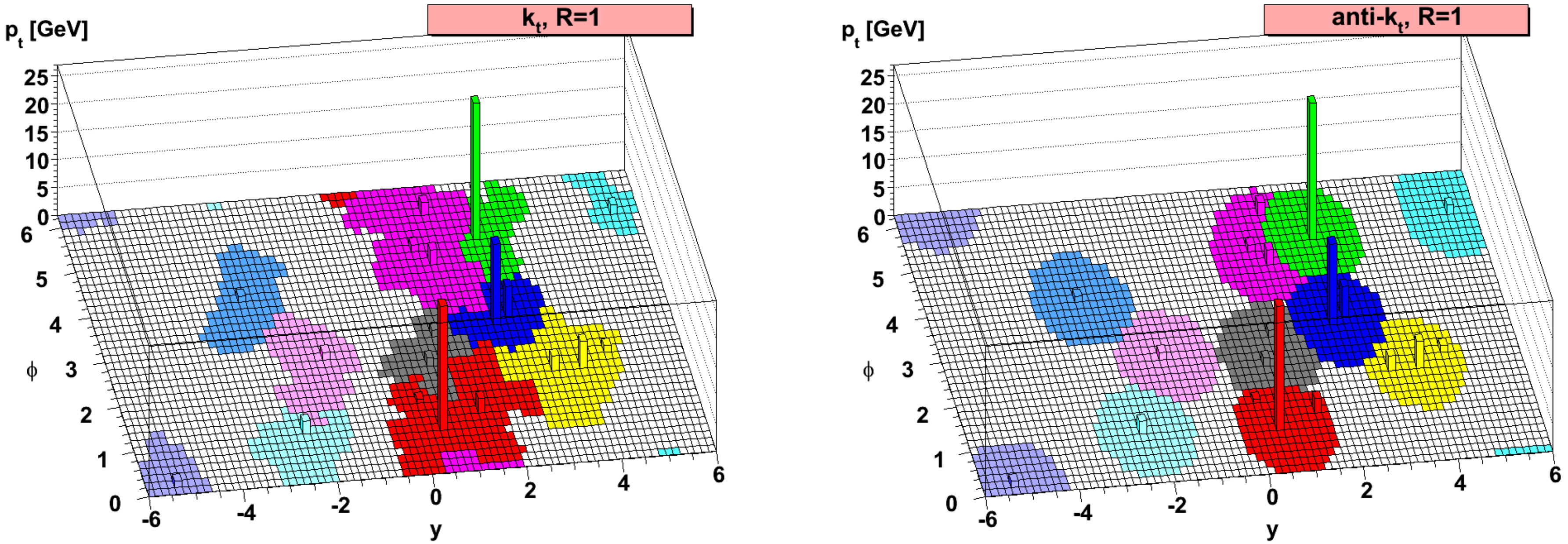
Why should that be related to the iterative cone ?!?

- "large $k_t \Rightarrow$ small distance"
  i.e. hard partons "eat" everything up to a distance R
  i.e. circular/regular jets, jet borders unmodified by soft radiation
- infrared and collinear safe

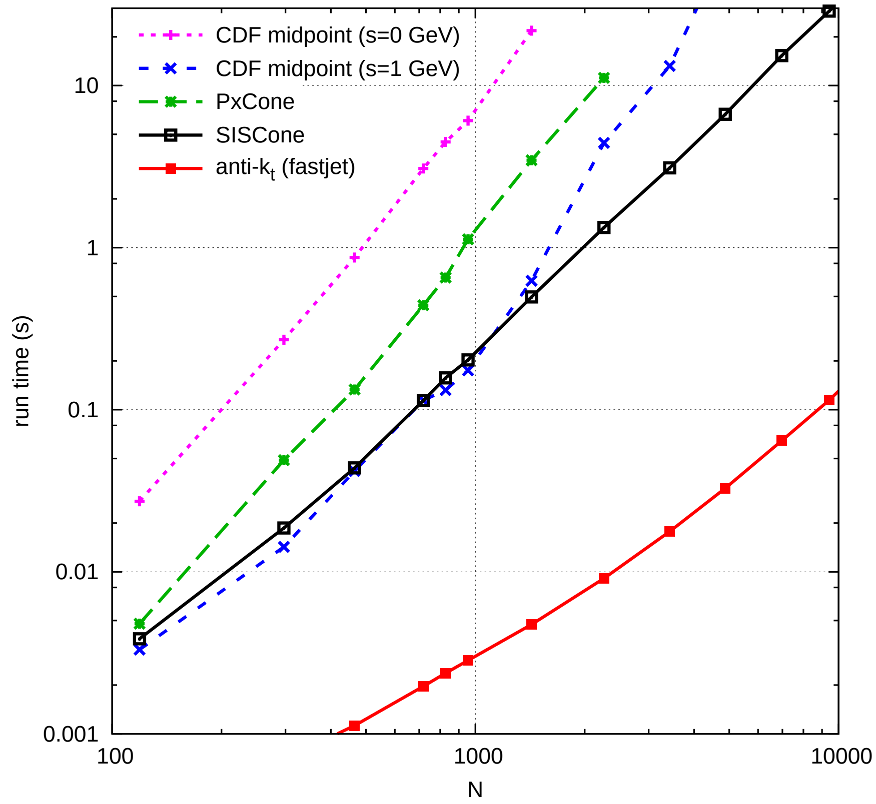
Hard event + homogeneous soft background



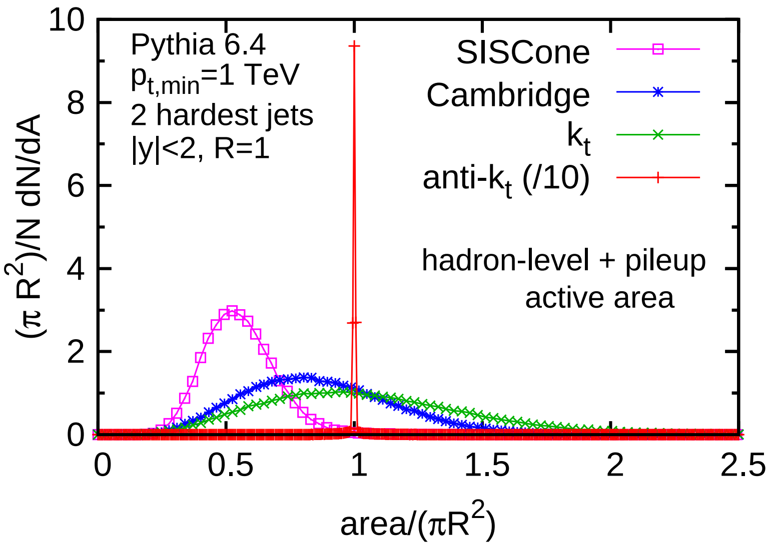
anti-k_t is soft-resilient

more in Matteo Cacciari's talk...

Execution timings:



Distribution of hard jets area



- As fast as the (fast) k_t ([M. Cacciari, G. Salam, 06])
- Regular hard jets of area πR^2

Conclusions

Midpoint and the iterative cone IR or Collinear unsafe (at $\mathcal{O}(\alpha_s^4)$)

Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
3 jet cross section	NLO	LO (NLO in NLOJet)
$W/Z/H$ + 2 jet cross sect.	NLO	LO (NLO in MCFM)
jet masses in 3 jets	LO	none (LO in NLOJet)

$\Rightarrow$ The IR-unsafety issue will matter at LHC

+ **We do not want the theoretical efforts to be wasted**

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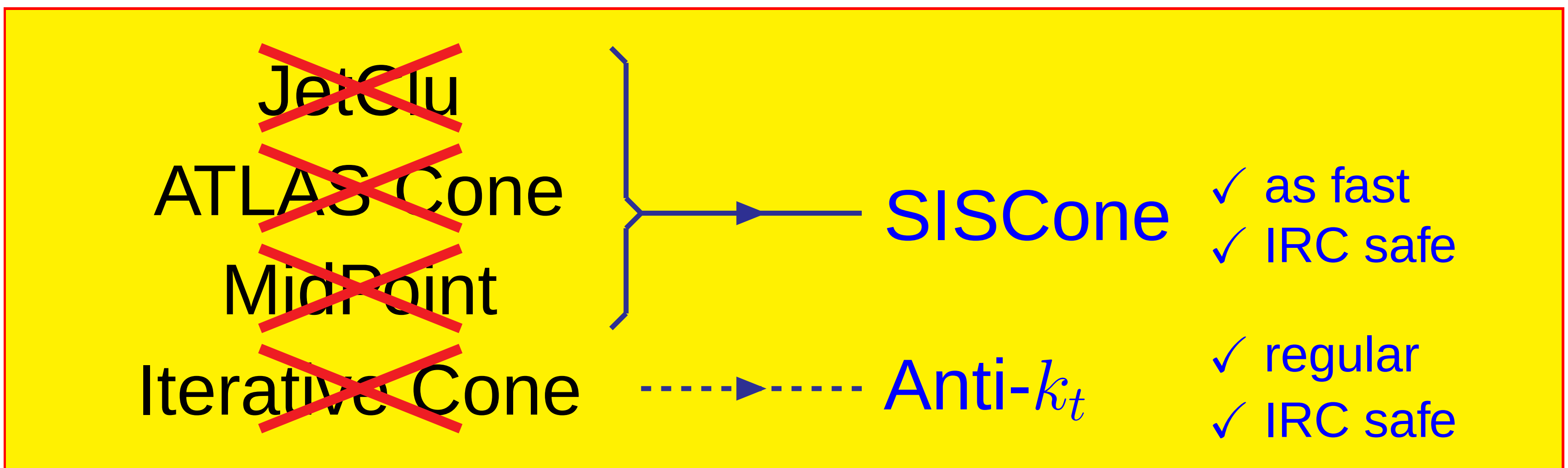

Both available from FastJet (http://www.lpthe.jussieu.fr/∼salam/fastjet)