

# **SISCone**

## ***A Seedless Infrared-Safe Cone jet algorithm***

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**University of Liège**

- In collaboration with **Gavin Salam**
- paper available as JHEP 05 (2007) 086 [arXiv:0704.0292]
- code available at `http://projects.hepforge.org/siscone`  
FastJet plugin: `http://www.lpthe.jussieu.fr/~salam/fastjet`

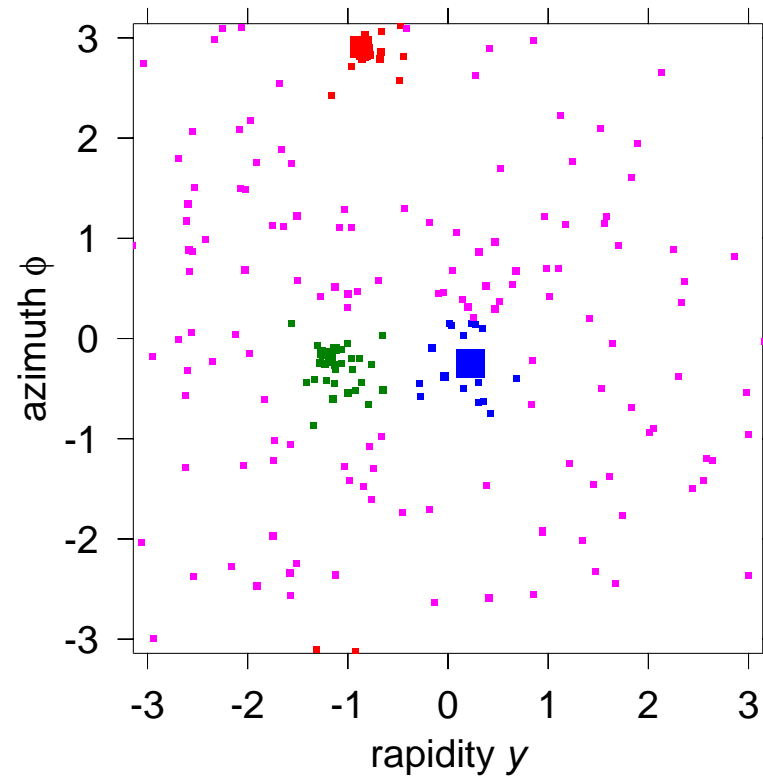
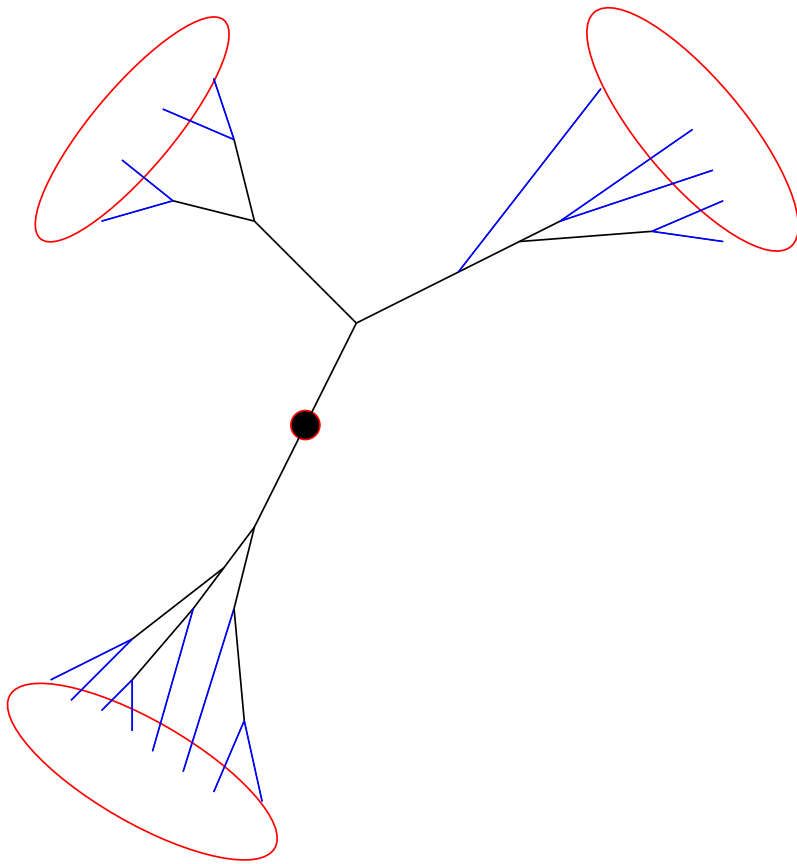
- Cone jet algorithms
- Infrared-Safety issues:
  - Why is this mandatory ?
  - IR unsafety of the midpoint algorithm
- SIScone: a practical solution
- Physical consequences:
  - Algorithm speed
  - Inclusive jet spectrum
  - Jet mass spectrum in multi-jet events
- Conclusions

# Why jet algorithms?

- Given: set of  $N$  particles with their 4-momentum

# Why jet algorithms?

- Given: set of  $N$  particles with their 4-momentum
- Quest: clustering those particles into **jets**

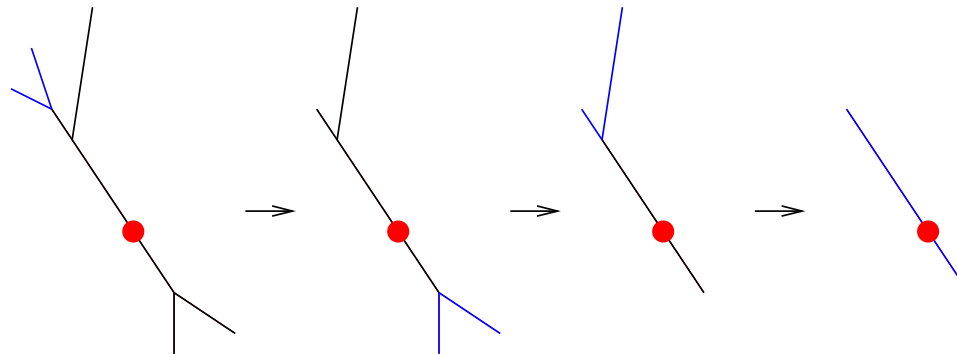


⇒ understand the original particle-level process

# Two classes of algorithms

## Class 1: recombination

Successive recombinations of the “closest” pair of particle



- Distance:

$$\underline{k_t}: d_{i,j} = \min(k_{t,i}^2, k_{t,j}^2)(\Delta\phi_{i,j}^2 + \Delta y_{i,j}^2)$$

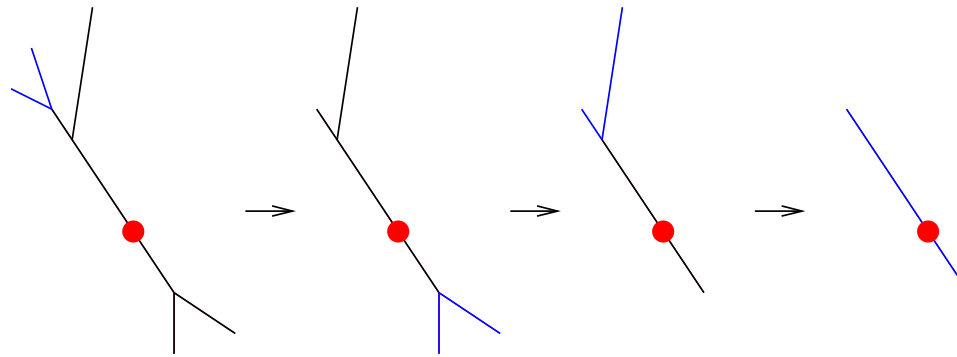
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- stop when  $d_{\min} > R$

- Often used for  $e^\pm e^\pm$  or  $e^\pm p$

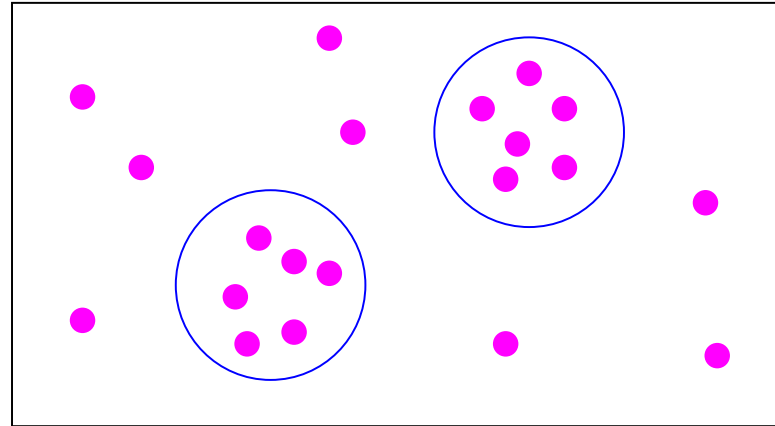
- FastJet : a fast implementation of those algorithms

[www.lpthe.jussieu.fr/~salam/fastjet/](http://www.lpthe.jussieu.fr/~salam/fastjet/) (M. Cacciari, G. Salam)

# Two classes of algorithms

## Class 2: cone

Find directions of dominant energy flow

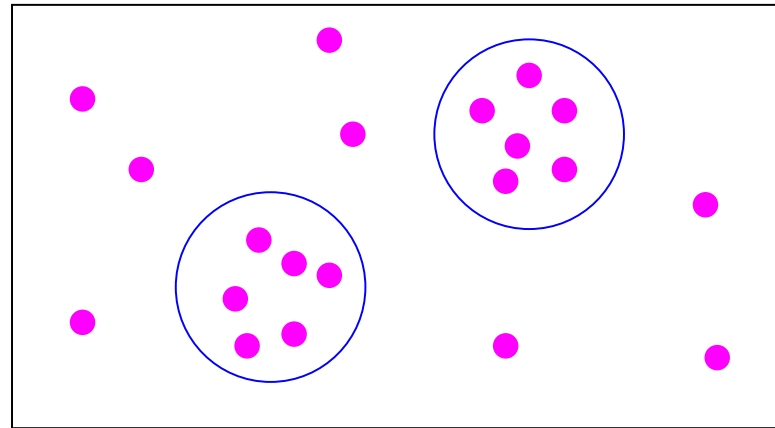


for a cone of radius  $R$  in the  $(y, \phi)$  plane, stable cones are such that:  
centre of the cone  $\equiv$  direction of the total momentum of its particle contents

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- Often used for  $pp$
- Many cone algorithms: Snowmass, JetClu, PxCone, CDF Midpoint, ...
- BUT none satisfies 1990's requirements



- Snowmass Accord (FERMILAB, 1990):  
any jet algorithm must satisfy
  1. Can be practically used in experimental analysis
  2. Can be practically used in theoretical computations
  3. Can be defined at any order of the perturbation theory
  4. Yields finite cross-sections at any order
  5. Has a small sensitivity to hadronisation corrections

# Cone requirements

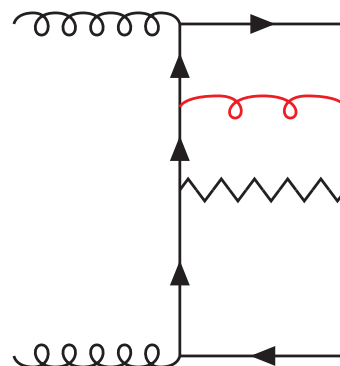
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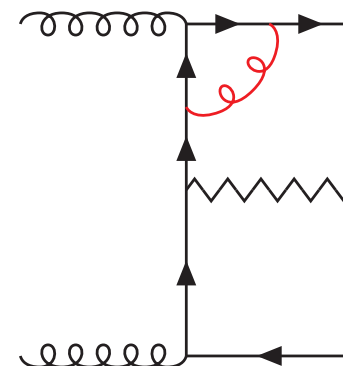
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- Previous algorithms:
  - 1, 2 and 4 never satisfied together
  - 5 is unclear (Underlying event and  $R_{\text{sep}}$  issues discussed later)
- This talk shows how to satisfy all these.

Ellipsis: IR safety, i.e. stability upon emission of soft particles, is required for perturbative computations to make sense!

Cancellation of IR divergences between real and virtual emissions of SOFT gluons in QCD



NLO, real



NLO, virtual

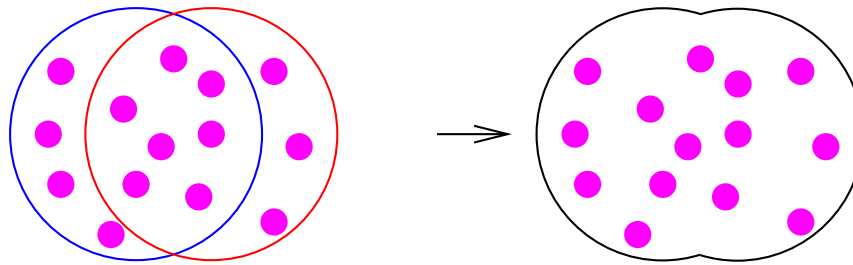
- IF Jet clustering is different in both cases, THEN the cancellation is not done and the result is not consistent with pQCD
  - ⇒ **Stable cones must not change upon addition of soft particles**
- Note: 100 GeV jet cannot change by adding a 1 GeV particle  
This would break parton/hadron correspondence

# Modern cone jet algorithm

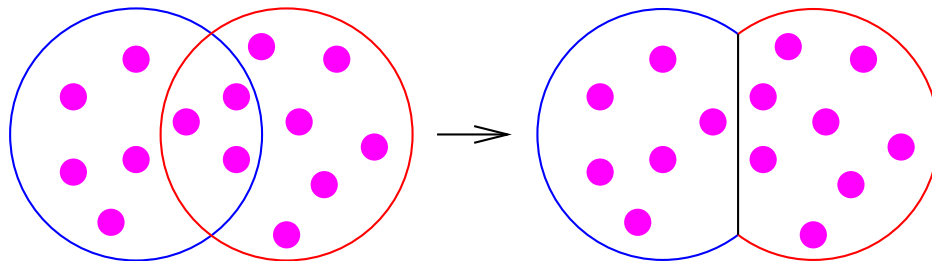
Modern cone jet algorithm (Tevatron Run II type):

- **Step 1:** find **ALL** stable cones of radius  $R$
- **Step 1'**: if some of the particles are not in stable cones, rerun Step 1 with the remaining ones.
- **Step 2:** run a split-merge procedure with overlap  $f$  to deal with overlapping stable cones

$$\tilde{p}_{t,\text{shared}} > f\tilde{p}_{t,\text{min}}$$



$$\tilde{p}_{t,\text{shared}} \leq f\tilde{p}_{t,\text{min}}$$



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This talk: **Why** finding **all** stable cones and **how**.

→ C++ implementation: Seedless Infrared-Safe Cone algorithm (SISCone)

# Typical cone: Midpoint algorithm

Usual **seeded** method to search stable cones: **midpoint cone algorithm**

- For an initial seed
  1. sum the momenta of all particles within the cone centred on the seed
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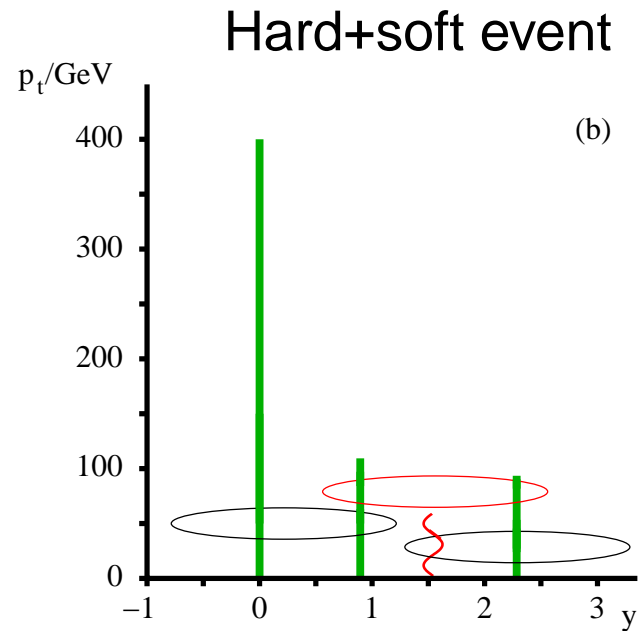
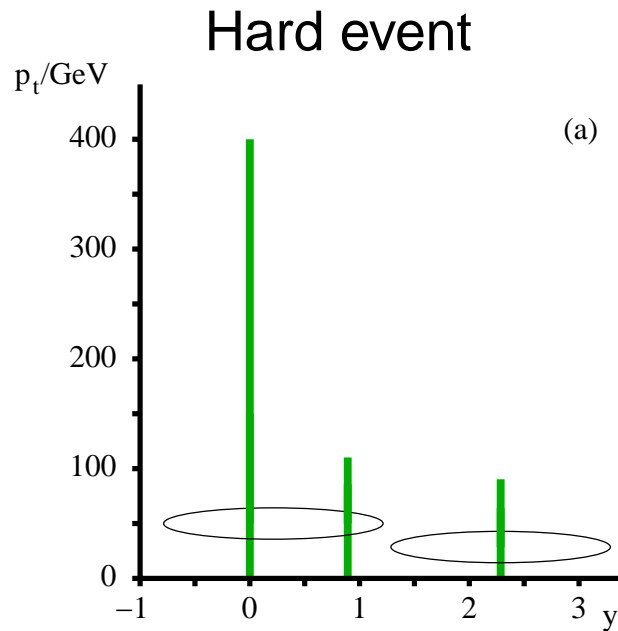
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## Problems:

- the  $p_t$  threshold  $s$  is collinear unsafe
- seeded approach  $\Rightarrow$  stable cones missed  $\Rightarrow$  infrared unsafety

# Midpoint IR Unsafety



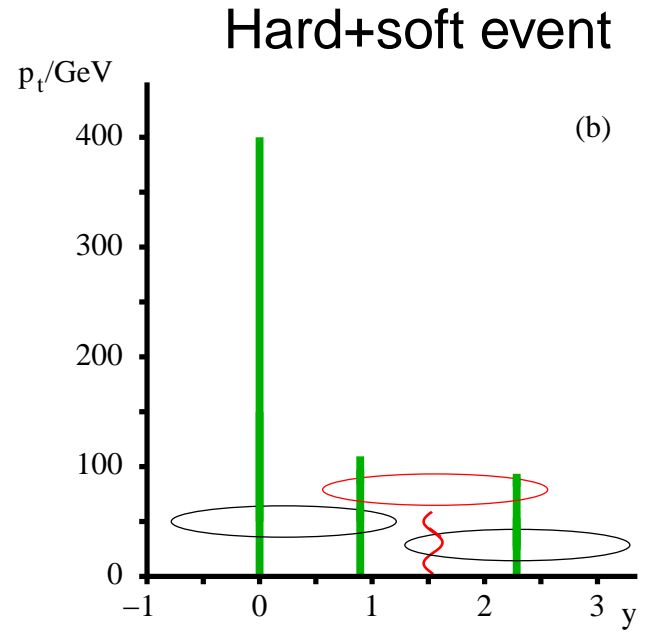
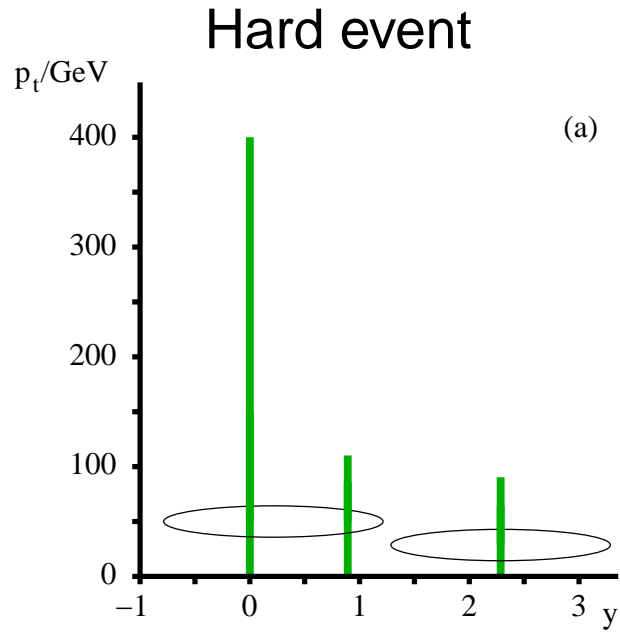
Stable cones:

Midpoint:

$\{1,2\}$  &  $\{3\}$

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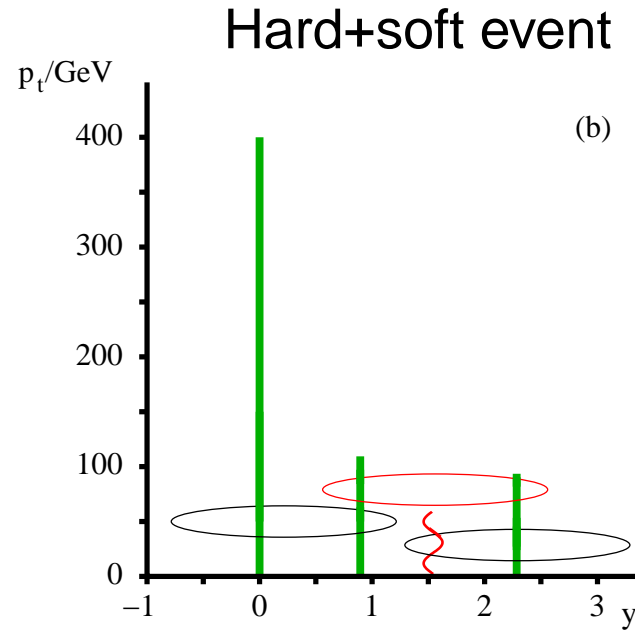
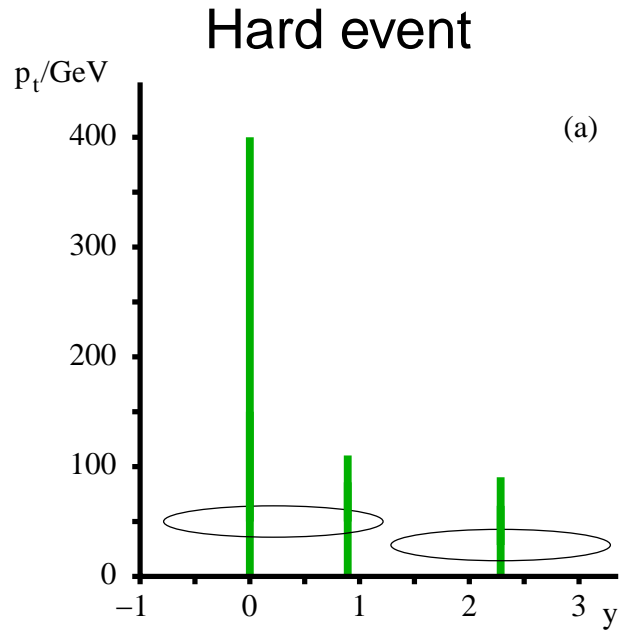
Jets: ( $f = 0.5$ )

Midpoint: {1,2} & {3}

{1,2,3}

→ IR unsafety of the midpoint algorithm

# Midpoint IR Unsafety



Stable cones:

Midpoint: {1,2} & {3}

Seedless: {1,2} & {3} & {2,3}

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# *is a seedless solution practical?*

- Solution: use a seedless approach
- Naive approach: check stability of each subset of particle

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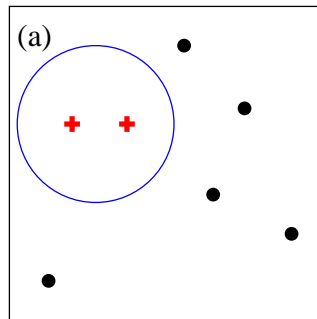
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Complexity is  $\mathcal{O}(N2^N)$   
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- Midpoint complexity:
  - For 1 seed: build and check cone content is  $\mathcal{O}(N)$
  - initially  $N$  seeds  $\Rightarrow \mathcal{O}(N)$  stable cones  
 $\Rightarrow \mathcal{O}(Nn)$  new, midpoint, seeds  
 $\Rightarrow$  midpoint complexity is  $\mathcal{O}(N^2n)$
  - with  $n \sim N$  the number of points in a circle of radius  $R$
  - Note: the number of stable cones is  $\mathcal{O}(N)$

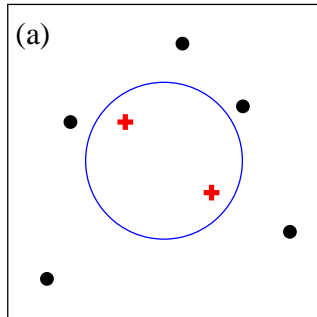


Idea: use geometric arguments



- Enumerate enclosures and check if they are stable

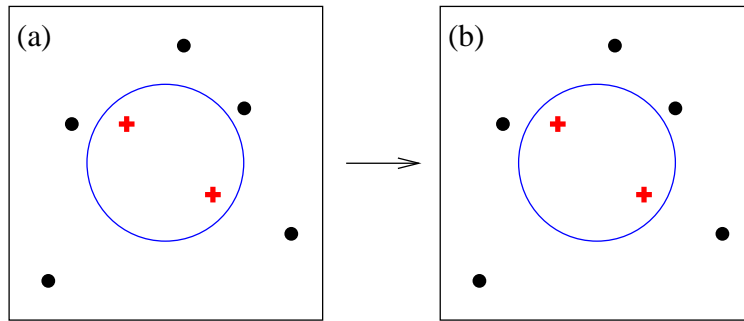
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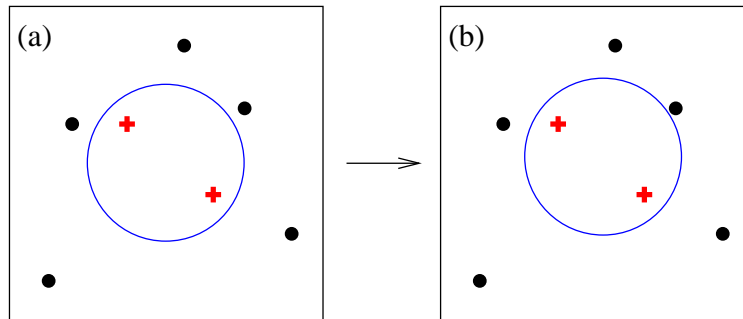
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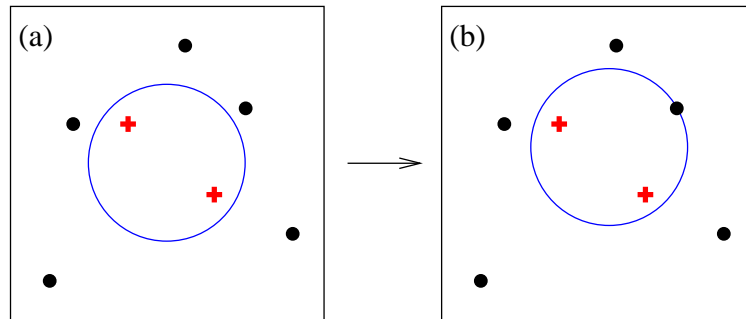
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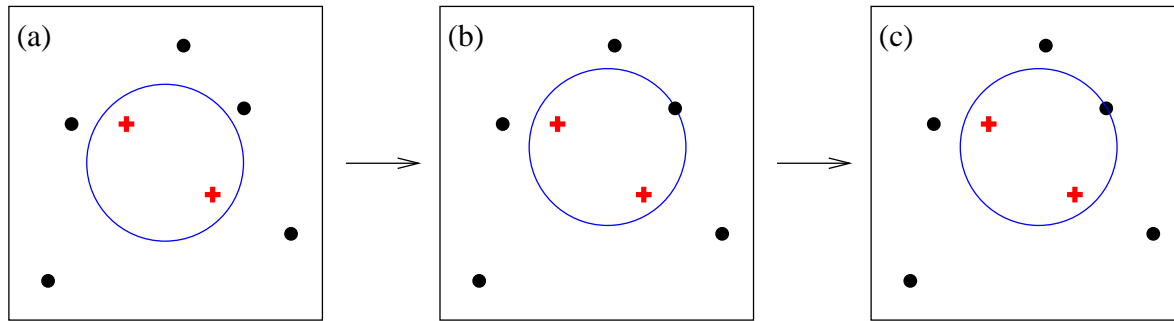
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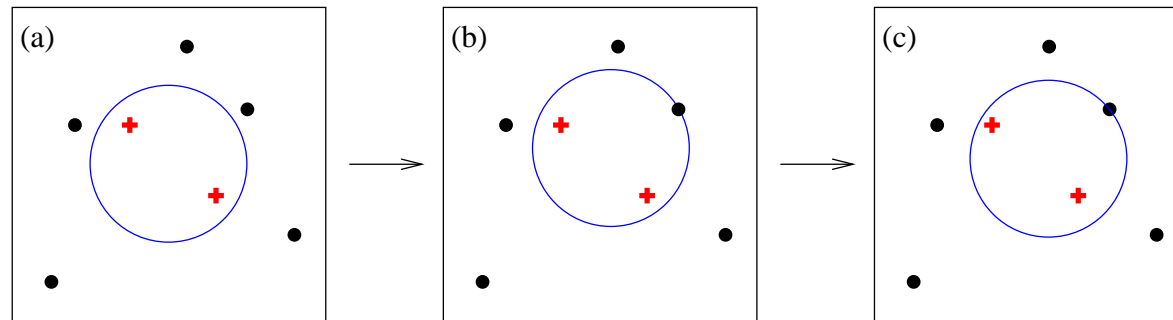
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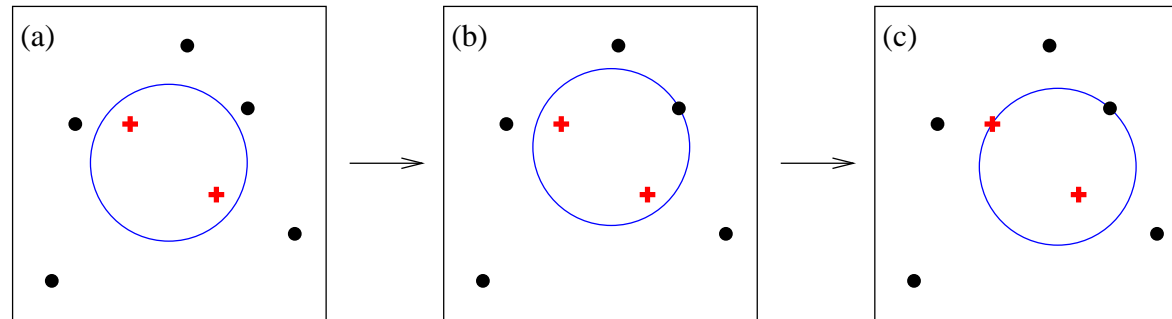
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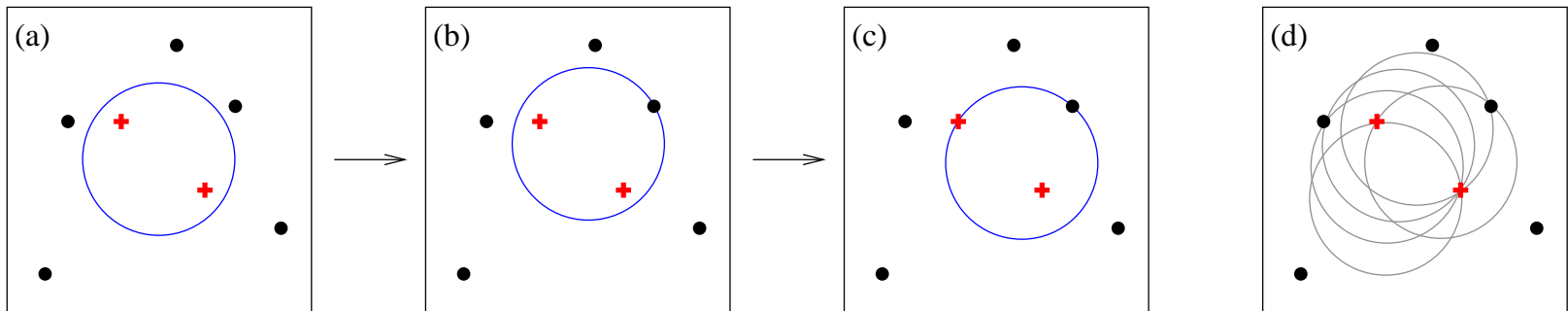


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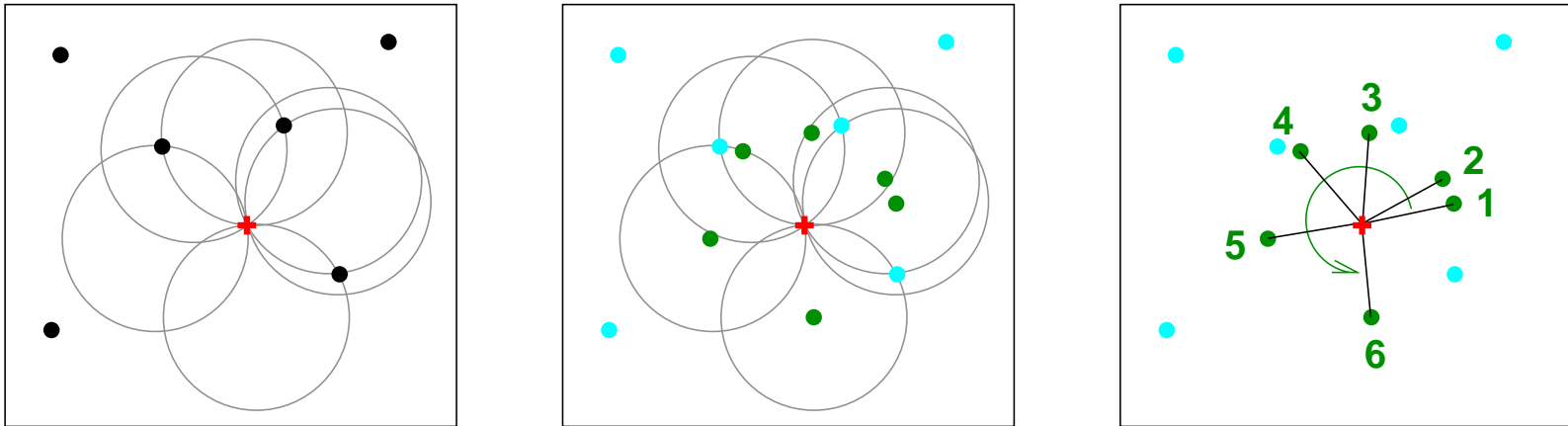
Same as midpoint... but we'll use more tricks:

- avoid systematic recomputation of cone contents
- limit complete tests of cone stability

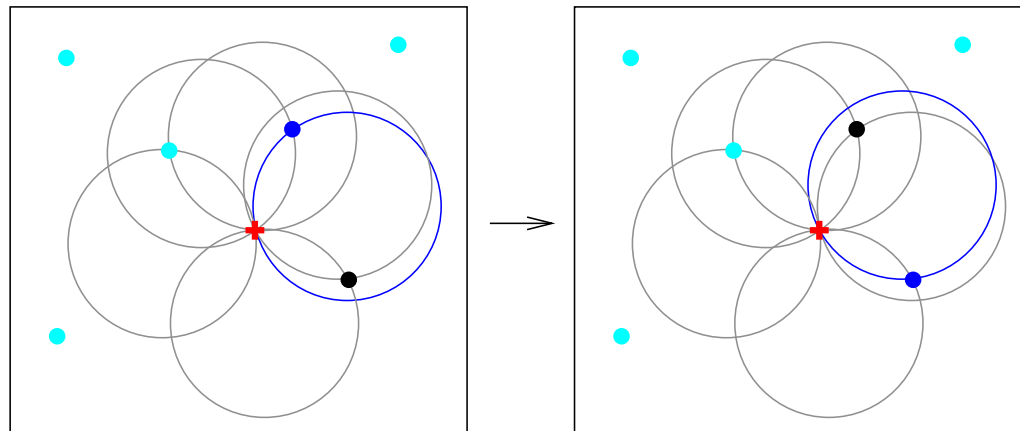
# SISCone: seedless solution

## Tricks:

- For all enclosures around a particle, introduce a **traversal order**



From one cone to the next one, contents only changed by “border” particles



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- Label the particles using a  **$q$ -bit tag**  
⇒ checkxor to identify distinct cones  
Introduces a potential “collision” problem

$$q = 96 \quad \Rightarrow \quad P(\text{collision}) = 10^{-18}$$

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- Label the particles using a  **$q$ -bit tag**  
⇒ checkxor to identify distinct cones
- Only **test “border particles” for stability (cost  $\mathcal{O}(1)$ )**  
⇒ limits the number of full stability test to  $\mathcal{O}(N)$   
checkxor → keep trace of stability tests

# The SIScone algorithm for stable-cone search

How to efficiently determine all stable cones:

- For each particle  $i$ 
  - get “partners” and associated cone centres
  - order them by angle
  - for all those candidates cones
    - check stability w.r.t. border particles
      - 4 possible  $\in$  or  $\notin$  & keep track of tested cones
    - move to the next cone
- Full stability test for the  $\mathcal{O}(N)$  not-yet-unstable candidates

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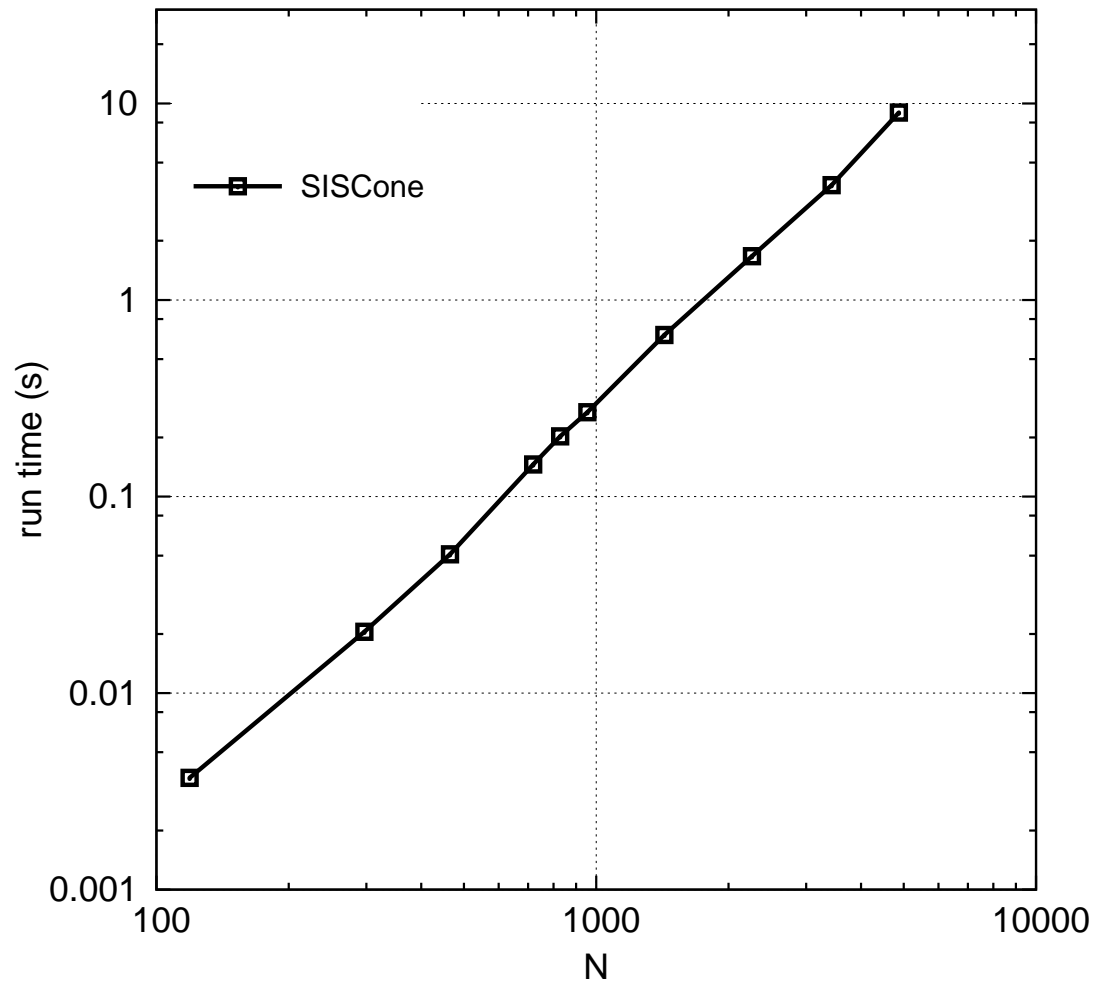
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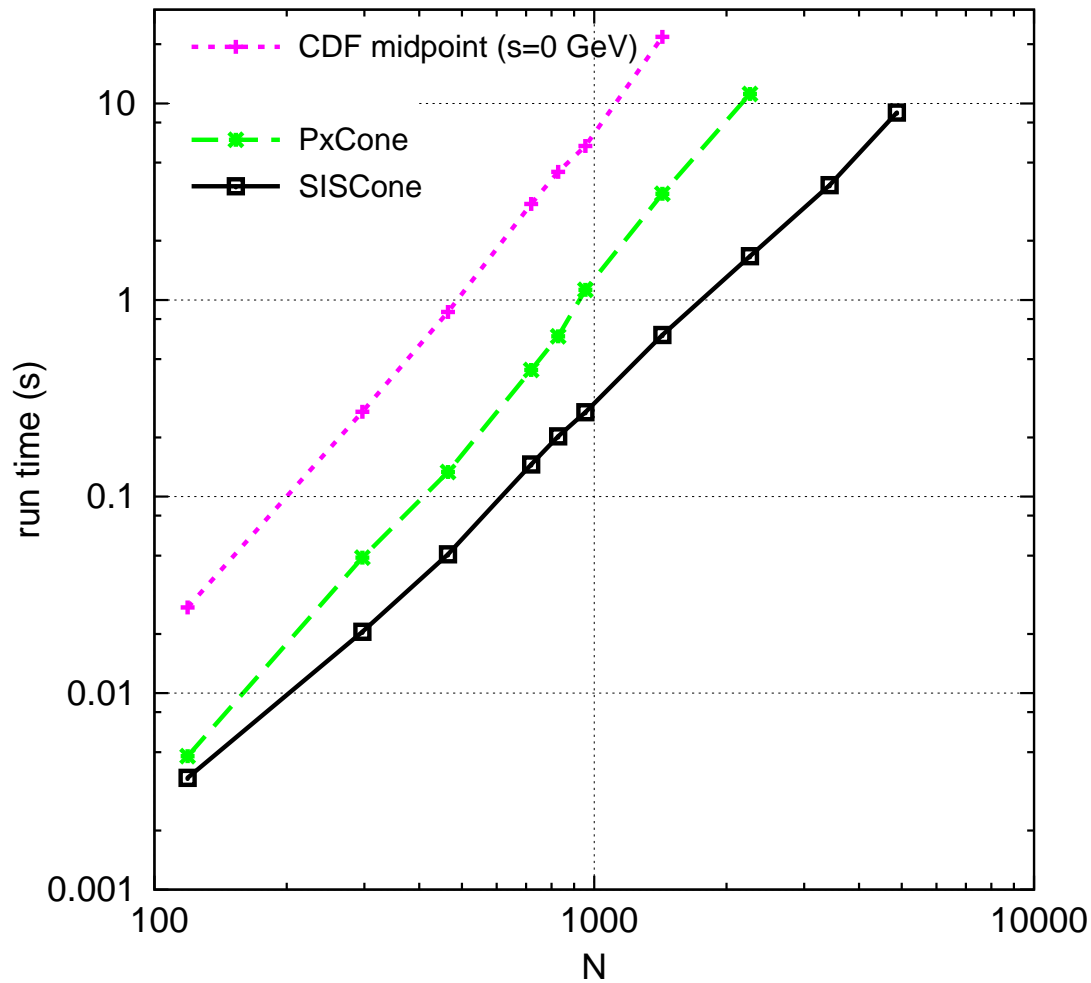
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All stable cones found in  $\mathcal{O}(Nn \log(n))$

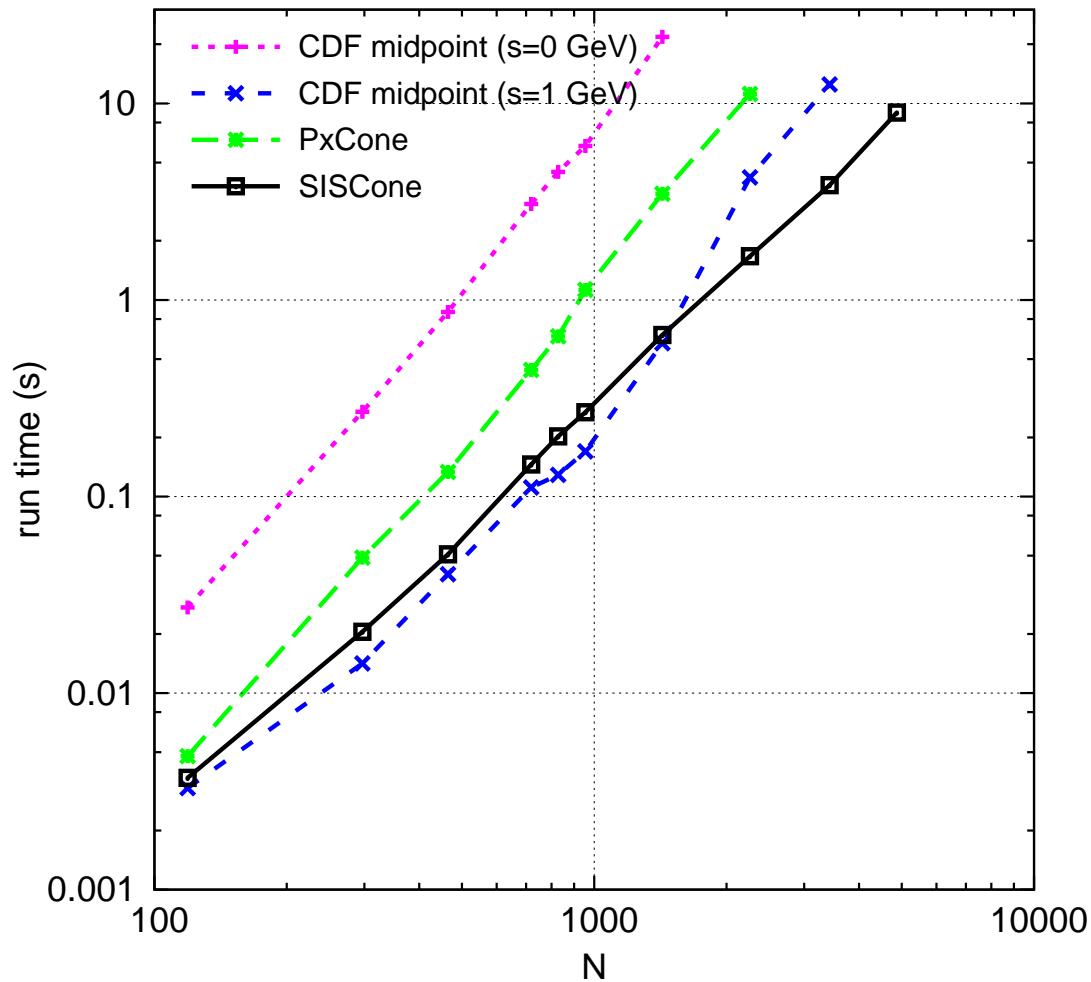
# ***SISCone vs. other cone algorithms***

***implications of a seedless cone***

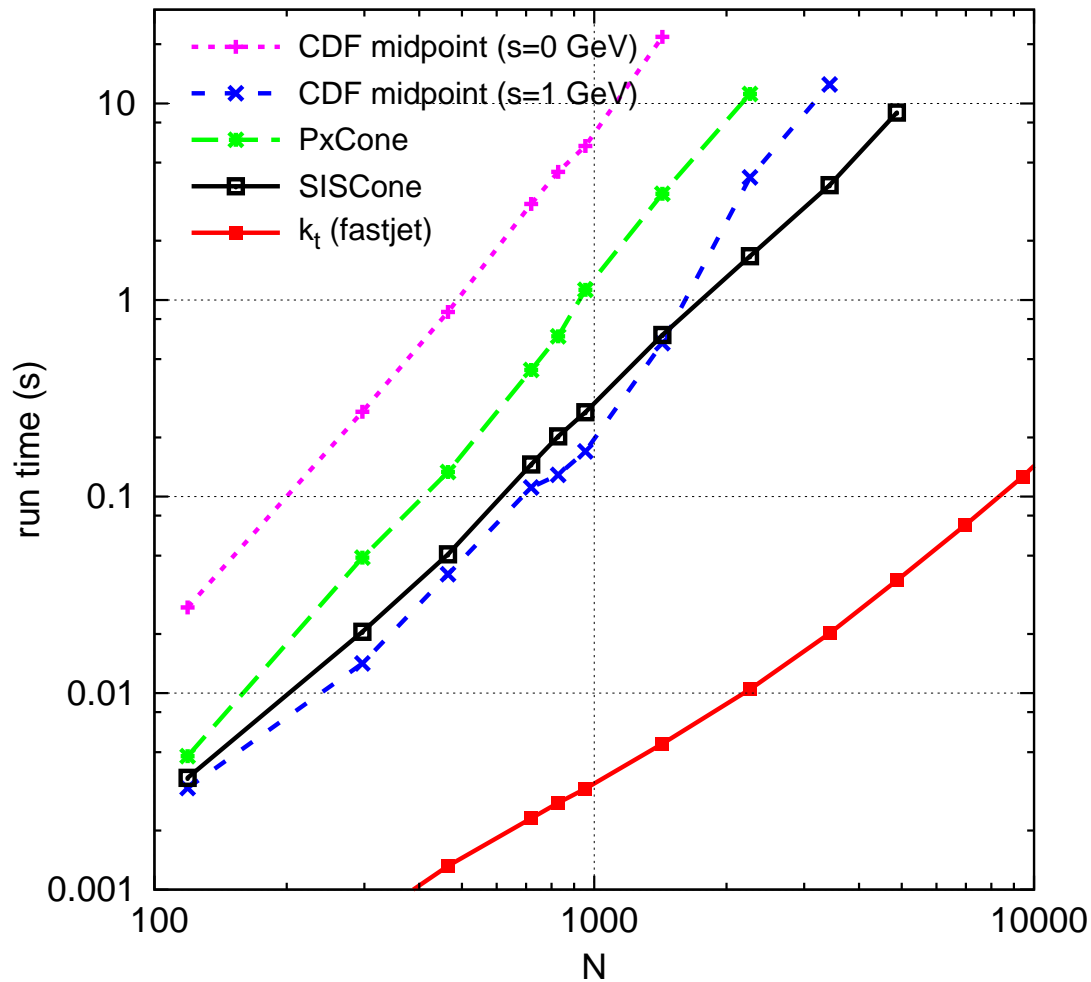




- faster than midpoint with no seed threshold and IR safe



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- same as midpoint with 1 GeV seed and collinear safe



- faster than midpoint with no seed threshold and IR safe
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- slower than  $k_t$ /FastJet affordable for practical usage e.g. at the LHC

# IR Unsafety failure rates

- Hard event: 2-10 particles
- Soft add-on: 1-5 particles
- Run:
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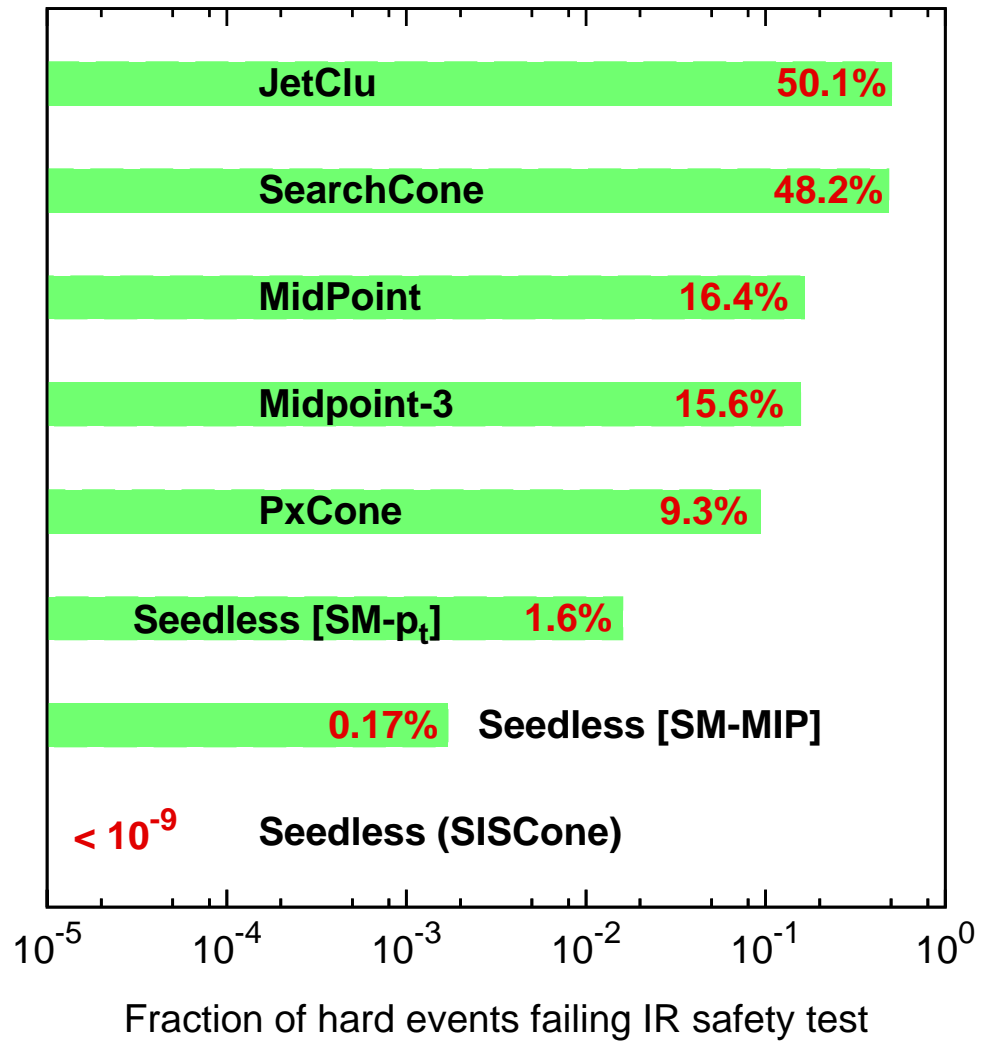
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<b>SISCone</b>	<b>IR safe !</b>



NB: small issues in the split-merge

# Consequences on observables

Physical impact: SISCone vs. midpoint( $s$ ) ?

IR unsafety of midpoint: 3 particles in the same vicinity + 1 to balance  $p_t$

⇒ starts at the  $2 \rightarrow 4$  level ( $\mathcal{O}(\alpha_s^4)$ )

Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
$W/Z/H + 1$ jet cross section	NNLO	NLO
3 jet cross section	NLO	LO
$W/Z/H + 2$ jet cross section	NLO	LO
jet masses in 3 jets	LO	none
masses in $W/Z/H + 2$ jets	LO	none

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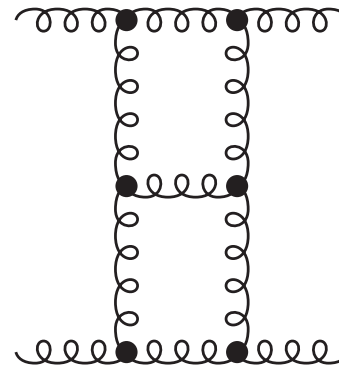
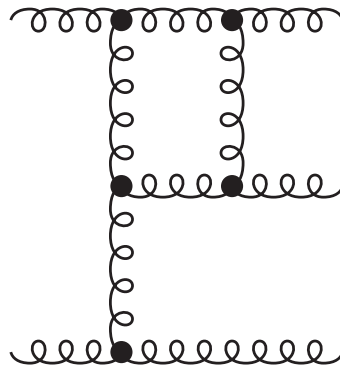
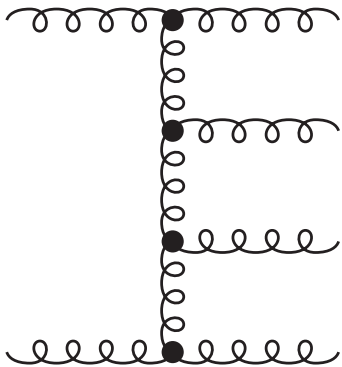
Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
$W/Z/H + 1$ jet cross section	NNLO	NLO
3 jet cross section	NLO	LO (NLO in NLOJet)
$W/Z/H + 2$ jet cross section	NLO	LO (NLO in MCFM)
jet masses in 3 jets	LO	none
masses in $W/Z/H + 2$ jets	LO	none

The IR-unsafety issue will matter at LHC

# Inclusive jet spectrum: perturbative exp.

SISCone vs. midpoint( $s$ ) in inclusive jet spectrum?

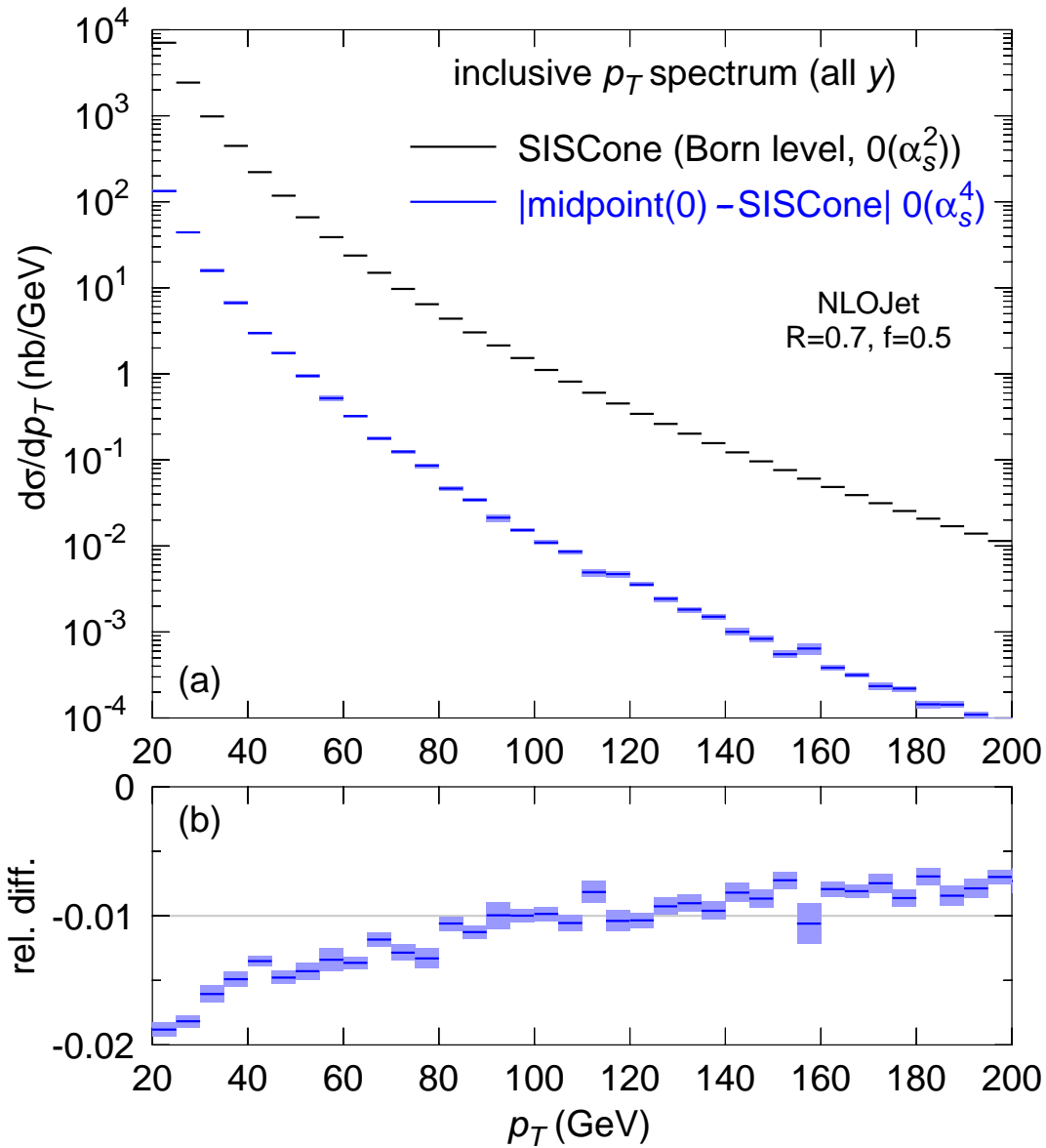
- IR unsafety of midpoint: 3 particles in the same vicinity + 1 to balance  $p_t$   
 $\Rightarrow$  starts at the  $2 \rightarrow 4$  level ( $\mathcal{O}(\alpha_s^4)$ )
- 3 contributions at this order:  
 $2 \rightarrow 4$  at LO (tree),  $2 \rightarrow 3$  at NLO (1 loop) and  $2 \rightarrow 2$  at NNLO (2 loops)



## SISCone vs. midpoint( $s$ ) in inclusive jet spectrum?

- IR unsafety of midpoint: 3 particles in the same vicinity + 1 to balance  $p_t$   
⇒ starts at the  $2 \rightarrow 4$  level ( $\mathcal{O}(\alpha_s^4)$ )
  - 3 contributions at this order:  
 $2 \rightarrow 4$  at LO (tree),  $2 \rightarrow 3$  at NLO (1 loop) and  $2 \rightarrow 2$  at NNLO (2 loops)
  - $2 \rightarrow 4$  at LO is IR divergent  
BUT the difference between SISCone and midpoint( $s$ ) is finite since it is 0 at the  $2 \rightarrow 2$  and  $2 \rightarrow 3$  levels
- ⇒ compute |SISCone-midpoint( $s$ )| for  $2 \rightarrow 4$  diagrams
- Compare with the  $2 \rightarrow 2$  (LO) spectrum to estimate effect

# Inclusive jet spectrum: perturbative exp.



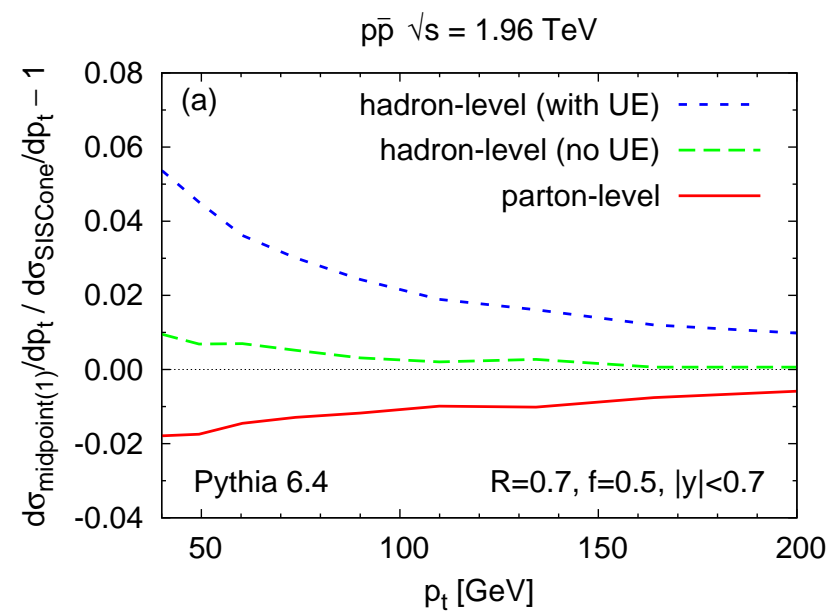
Differences of order 1-2 %



# Inclusive jet spectrum: hadron level

Including parton shower, hadronic corrections and/or underlying event:

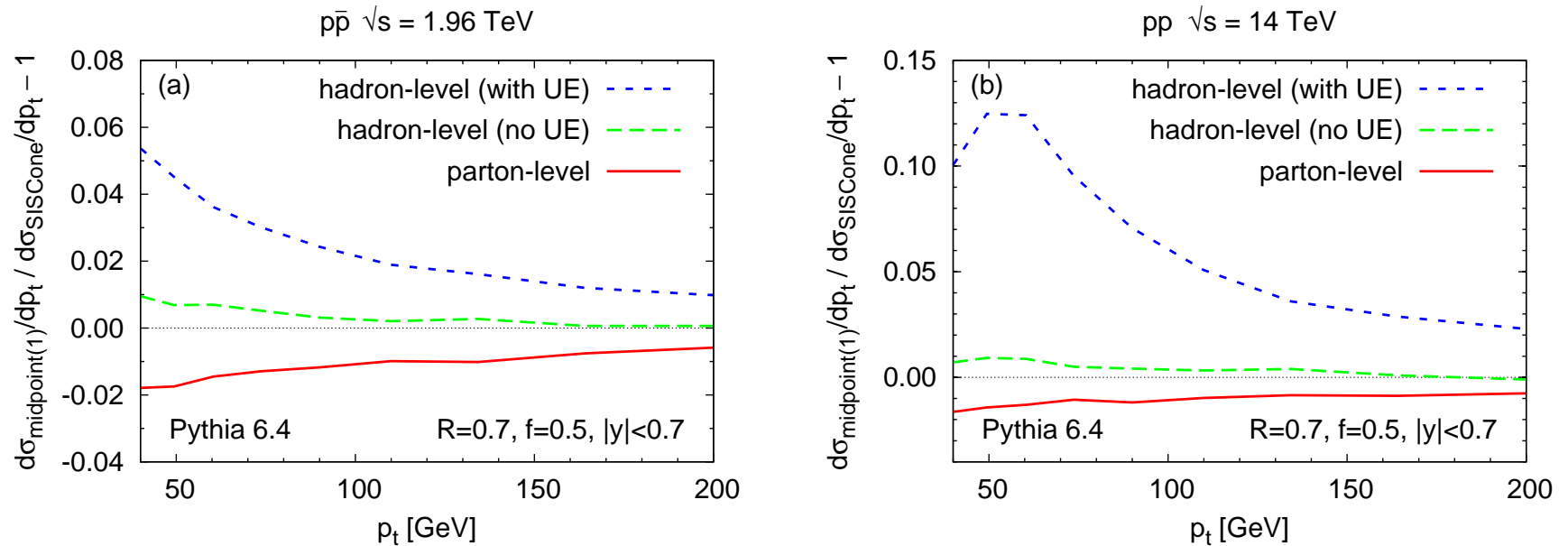
## Ratio midpoint/SISCone-1:



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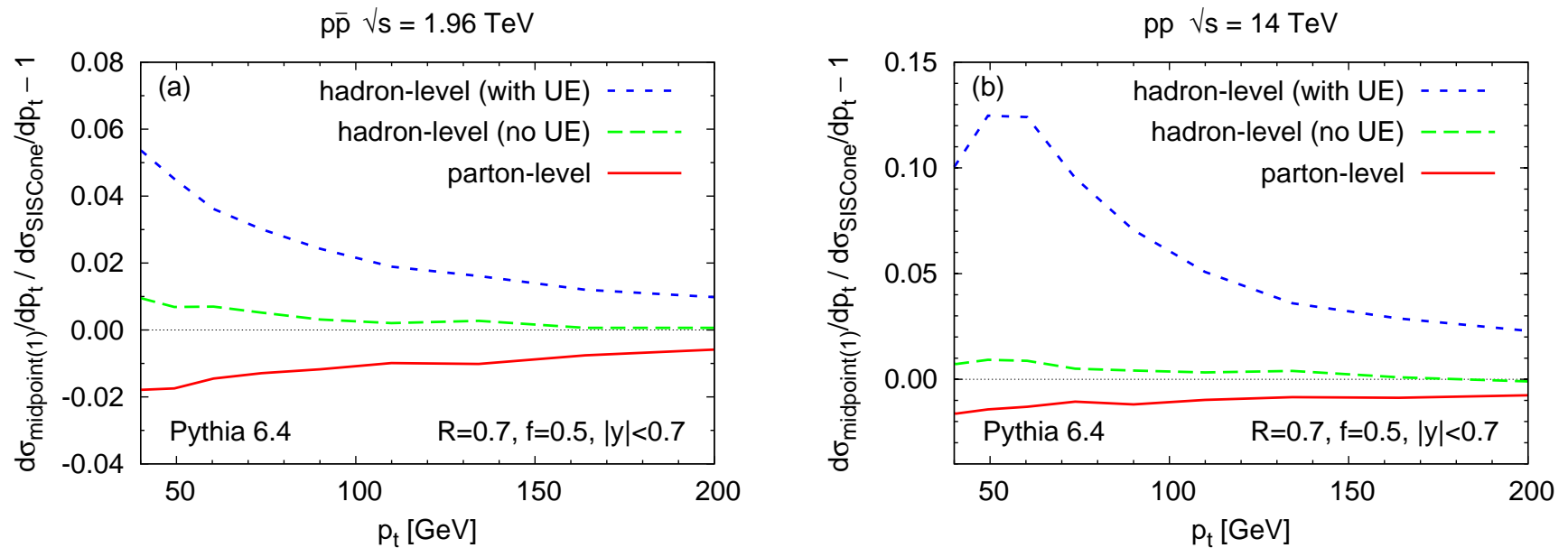


- Differences up to 5% (with a change of sign)
- Raise up to 10% at LHC energy!

# Inclusive jet spectrum: hadron level

Including parton shower, hadronic corrections and/or underlying event:

## Ratio midpoint/SISCone-1:



- Differences up to 5% (with a change of sign)
- Raise up to 10% at LHC energy!
- Less effect from underlying event in SISCone (i.e. better agreement with parton level)

## Inclusive jet spectrum

→ effect at NNLO i.e.  $\mathcal{O}(\alpha_s^2)$  w.r.t. LO

⇒ want to look at more exclusive processes

## Example: mass spectrum in 3-jet events (or W/Z/H+2j)

$$\left. \begin{array}{l} 2 \rightarrow 2 \text{ has only 2 jets} \\ 2 \rightarrow 3 \text{ has zero masses} \end{array} \right\} \Rightarrow \text{first contribution from } 2 \rightarrow 4$$

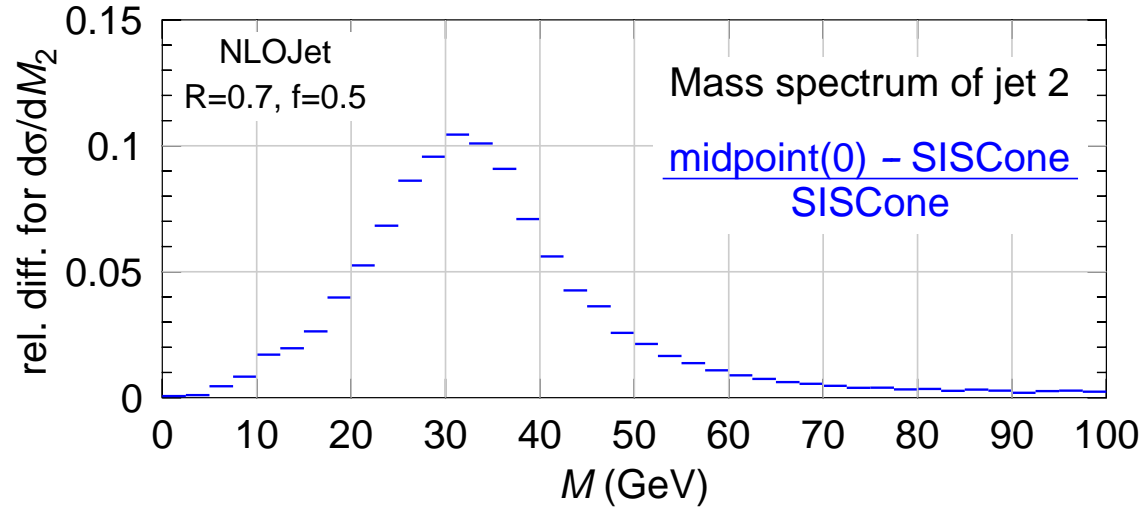
⇒ Expect modifications at LO!

Ratio  $\frac{\text{midpoint-SISCone}}{\text{SISCone}}$  for masses spectra in 3-jet events

cuts:  $p_{t,1} \geq 120 \text{ GeV}$ ,  $p_{t,2} \geq 80 \text{ GeV}$ ,  $p_{t,3} \geq 40 \text{ GeV}$

# Jet mass spectrum: perturbative level

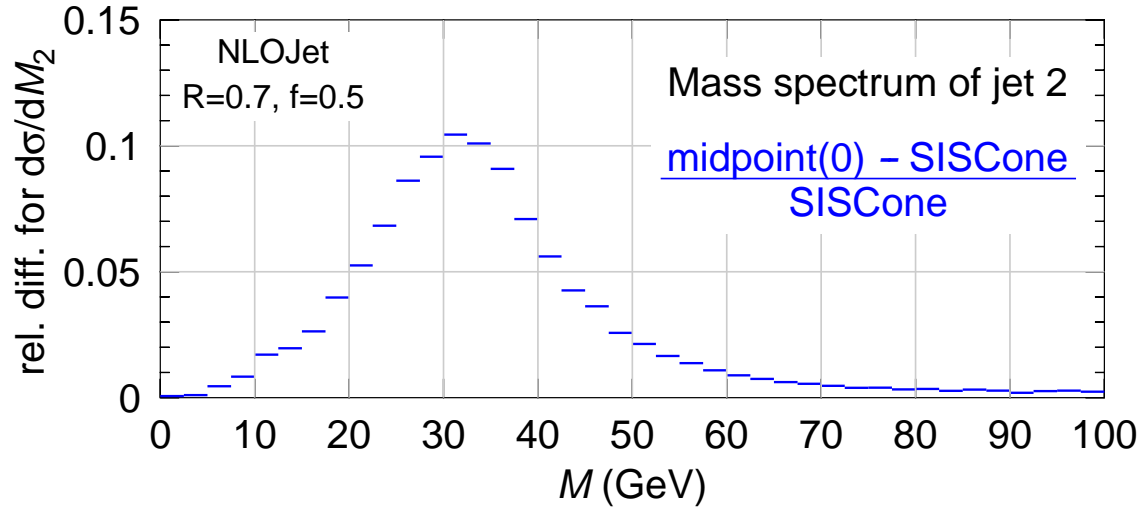
## 1. Fixed order computation (NLOJet, LO, $2 \rightarrow 4$ )



Differences up to 10 %

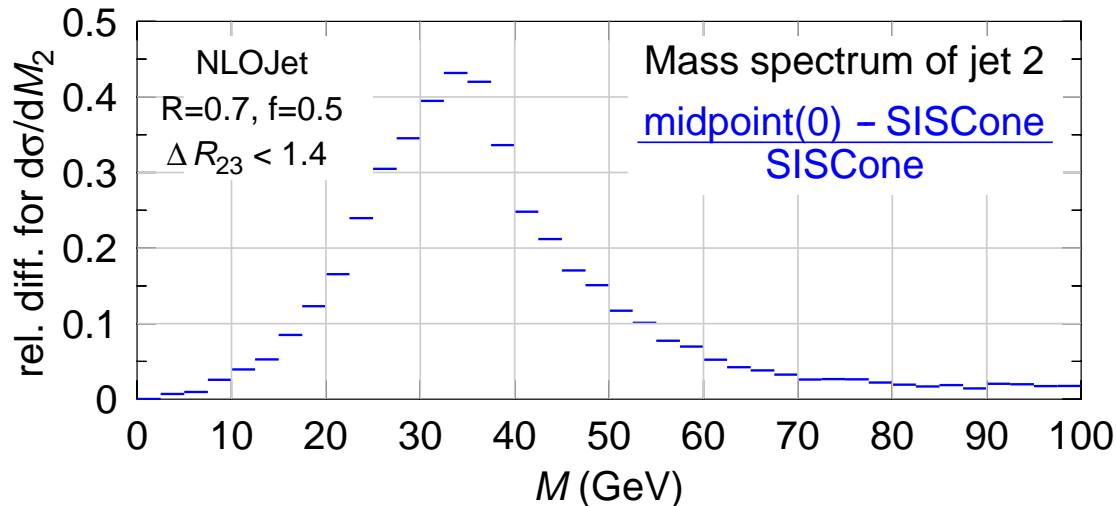
# Jet mass spectrum: perturbative level

## 1. Fixed order computation (NLOJet, LO, $2 \rightarrow 4$ )



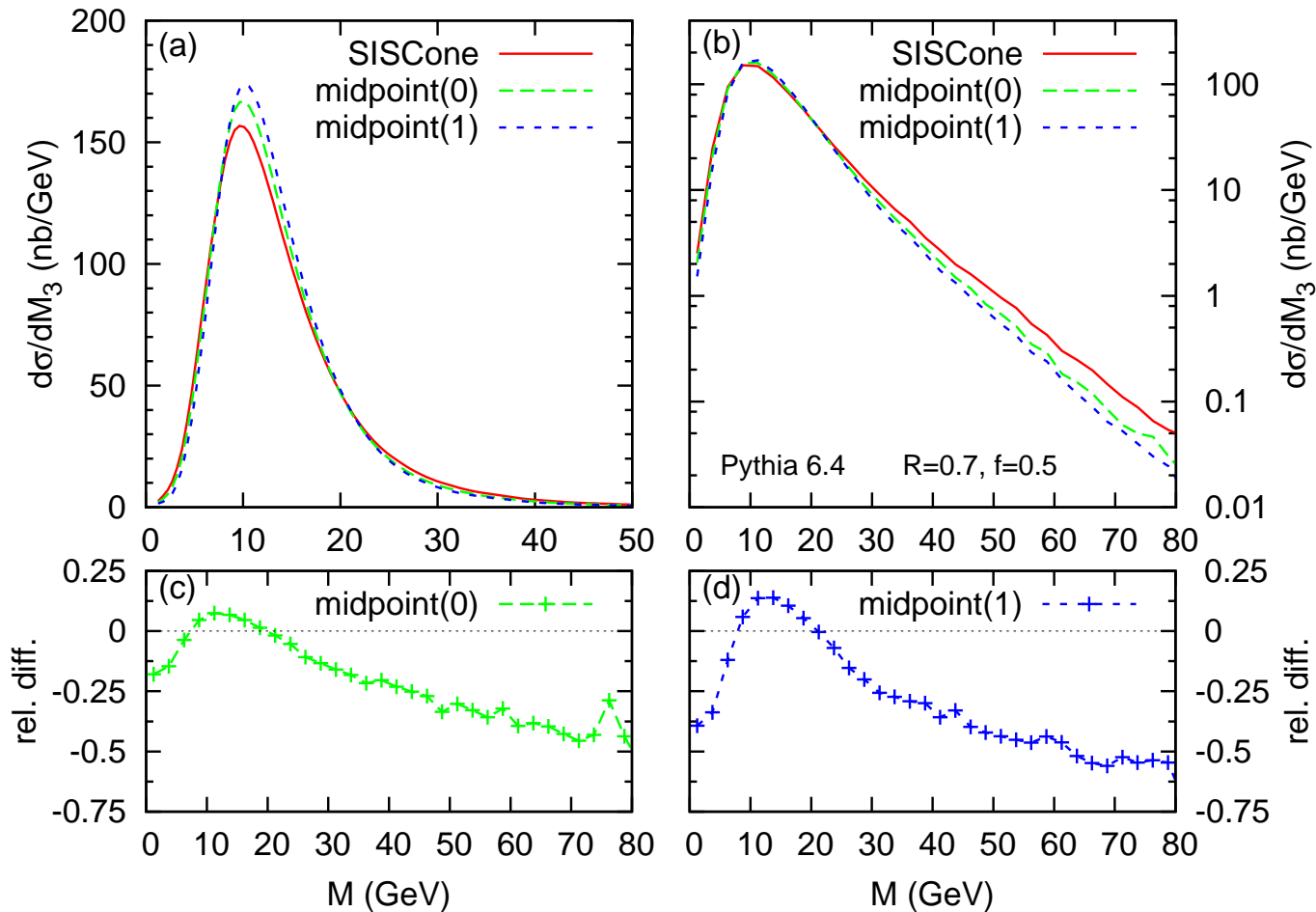
Differences up to 10 %

## 2. Also require jets 2 and 3 within distance $\leq 2R$



Differences up to 40 %

## 3. At hadron level (PYTHIA)



- ▷ Differences of order 10 %
- ▷ Larger effects in the tail
- ▷ seed threshold even worse

# SISCone conclusions

- Jets are present everywhere:  $k_t$  and cone are widely used
- seeded implementations are **IR unsafe** (sometimes **collinear unsafe**)  
IR safety is a prerequisite for perturbative QCD to make sense

We propose a [new cone algorithm](#) (SISCone):

- **IR safe** (and **collinear safe**)
- as **fast** as available cone implementations
- has **10% impact on jet mass spectra** (can be up to 40%)
- is **less affected by underlying events**



# *Jet area*

*Everyone has an idea of what a jet area is  
but can we define that properly?*

[M. Cacciari, G. Salam, G.S., in preparation]

[M. Cacciari, G. Salam, in preparation]

- Idea: add soft particle (**ghosts**)
  - with IR-safe algorithms such as  $k_t$ , **Aachen/Cambridge** and **SISCone**, clustering is unchanged
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add one ghost and look where it ends. repeat to cover the  $(y, \phi)$  plane

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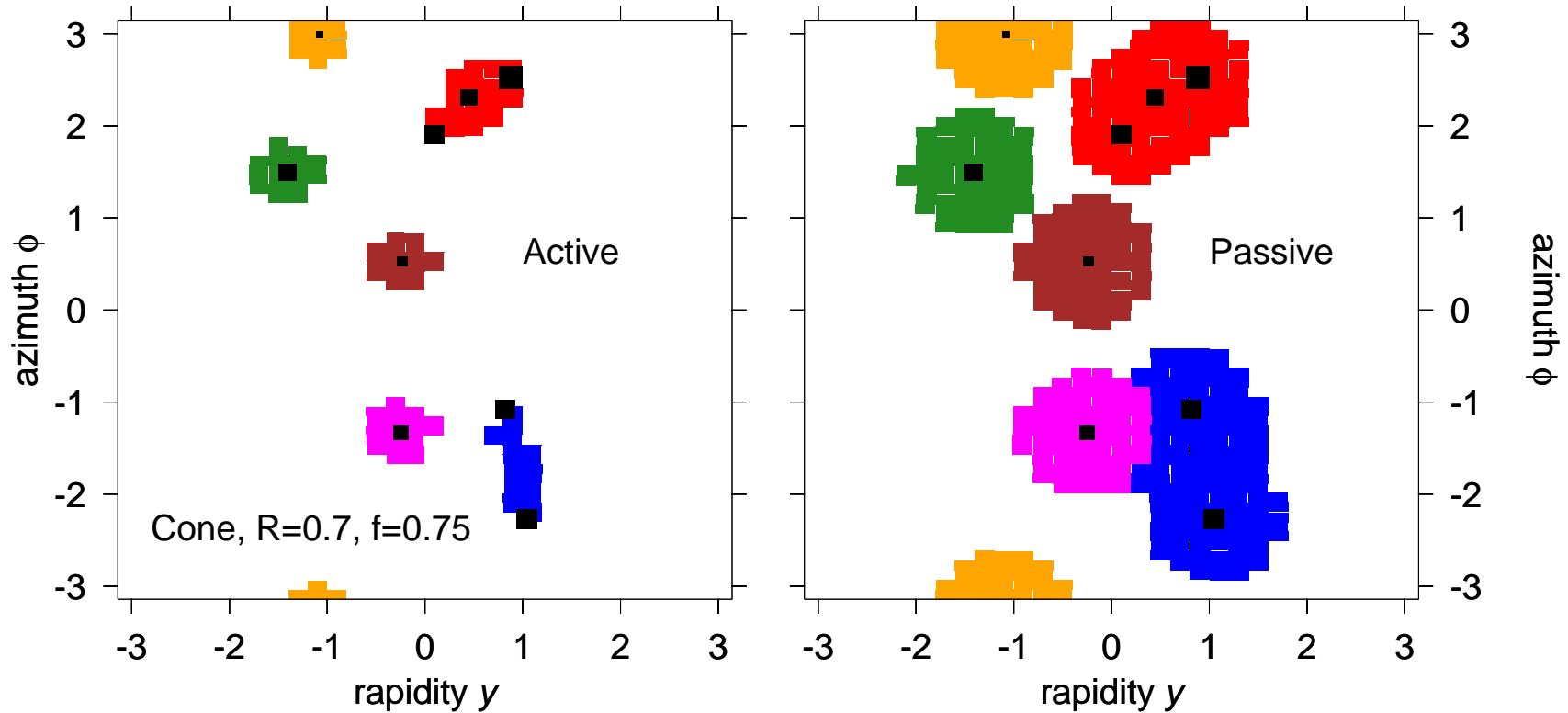
add one ghost and look where it ends. repeat to cover the  $(y, \phi)$  plane
- Active area

add a large amount of ghosts and cluster everything  
also gives purely ghosted jets
- Voronoi area

~ Area of the Voronoi cells

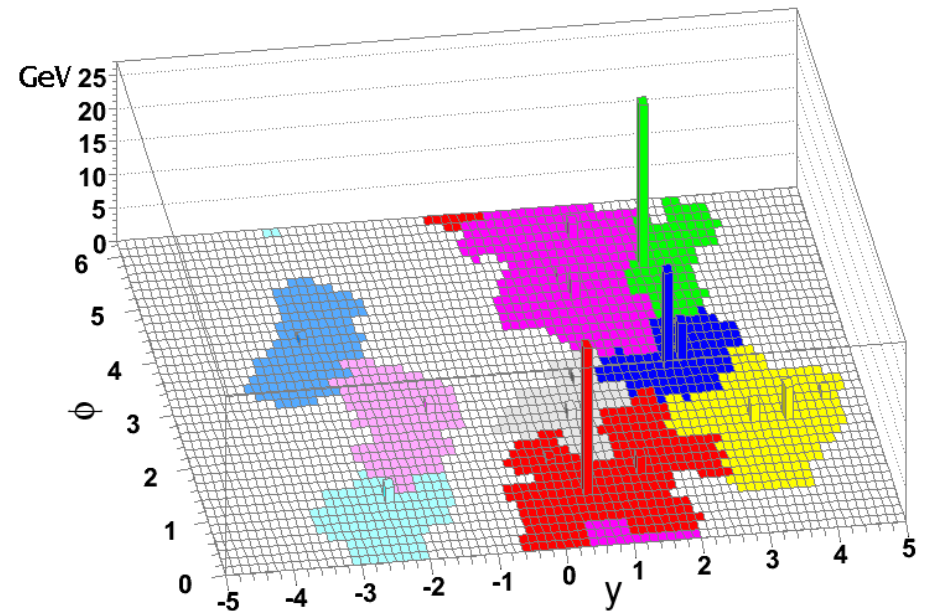
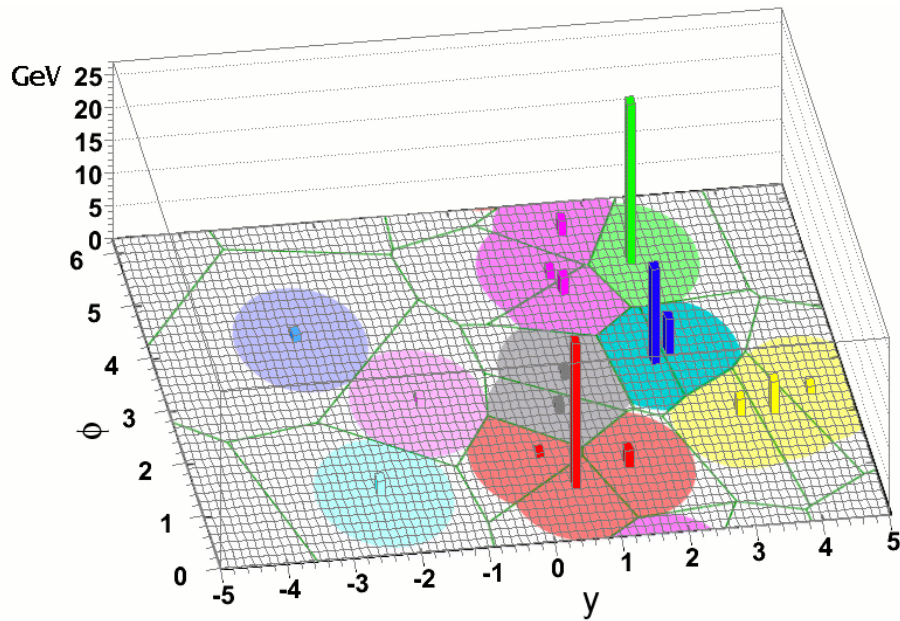
# Area definition

- Small  $N$ : active area is usually smaller than passive area (especially for the cone)



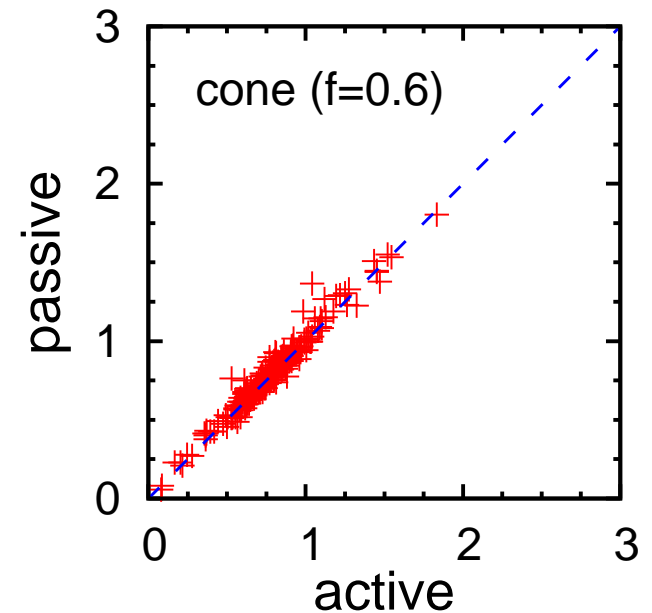
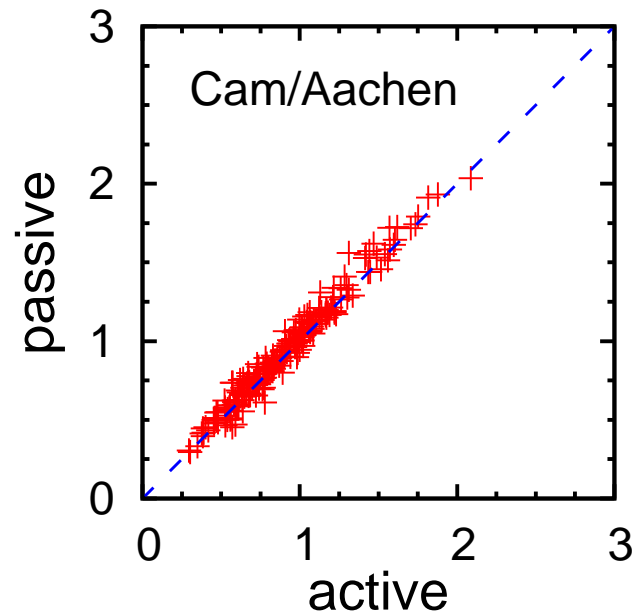
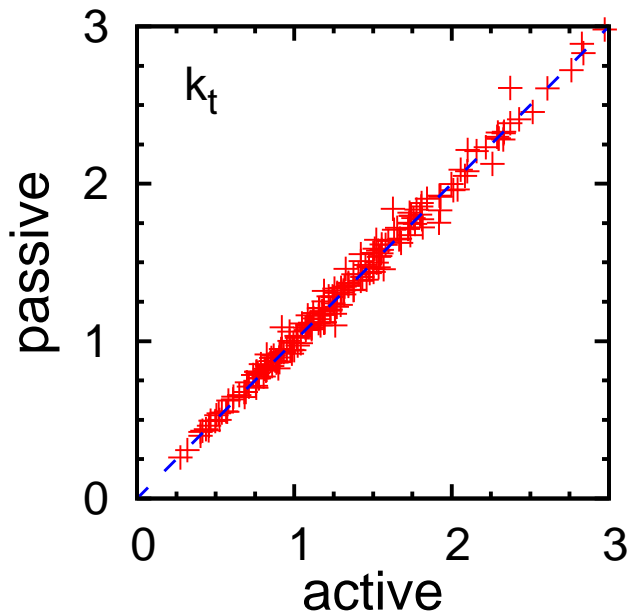
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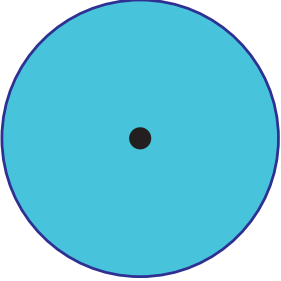
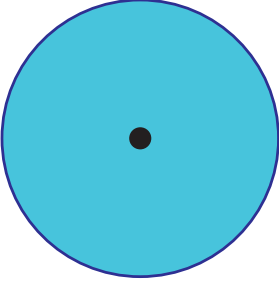




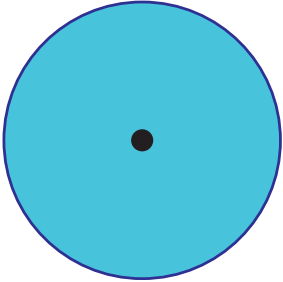
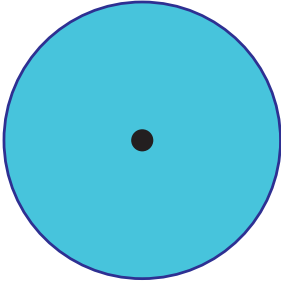
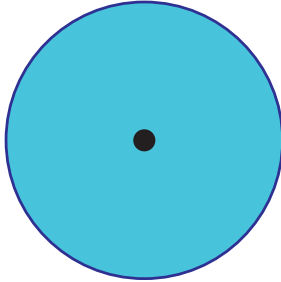
# Examples: 1-particle cases

	$k_t$	Aac/Cam	cone
Passive			
Active			

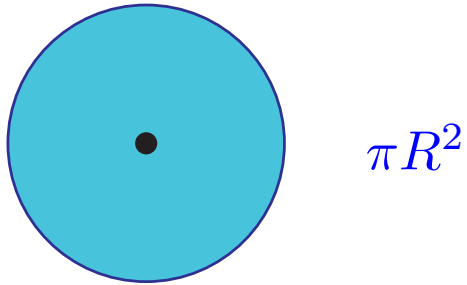
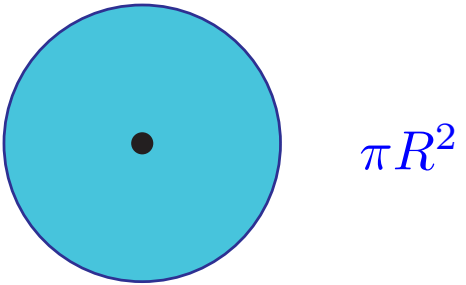
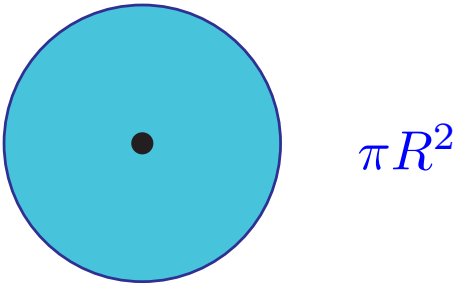
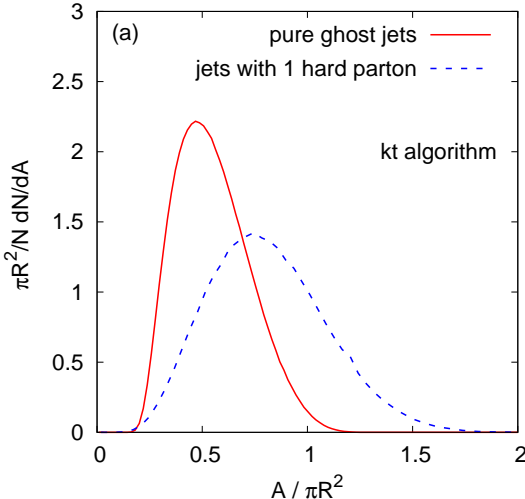
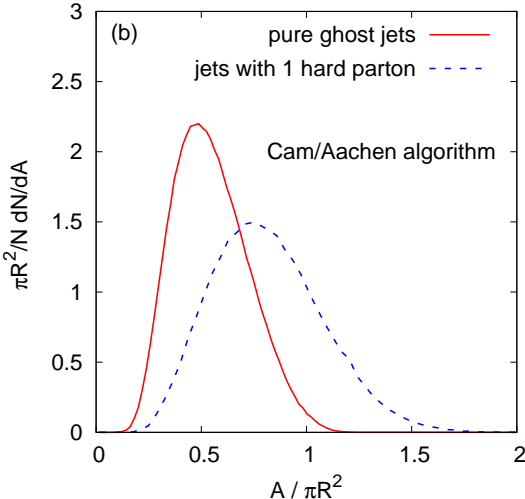
# Examples: 1-particle cases

	$k_t$	Aac/Cam	cone
Passive	 $\pi R^2$	 $\pi R^2$	
Active			

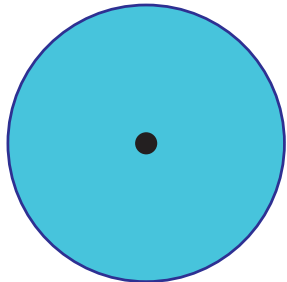
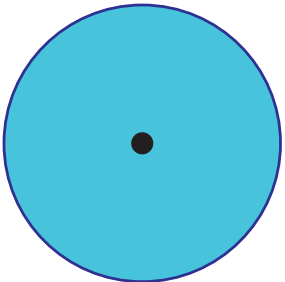
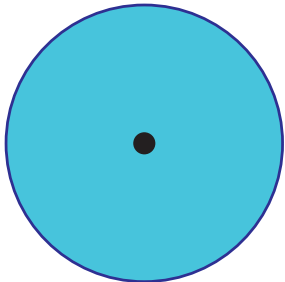
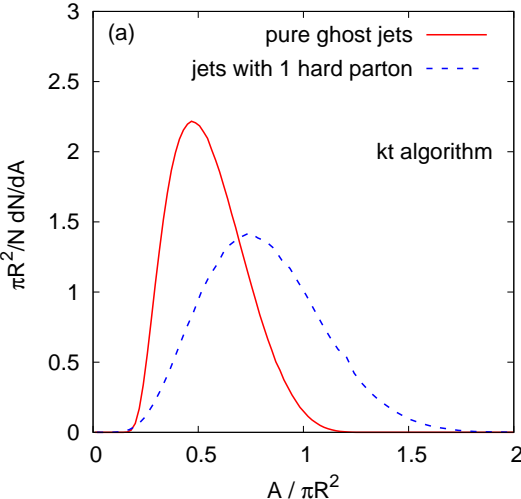
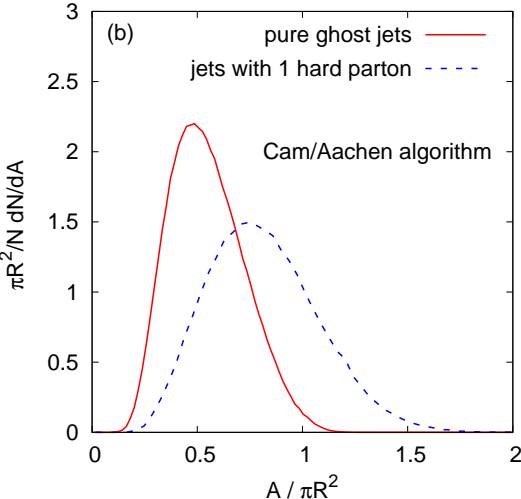
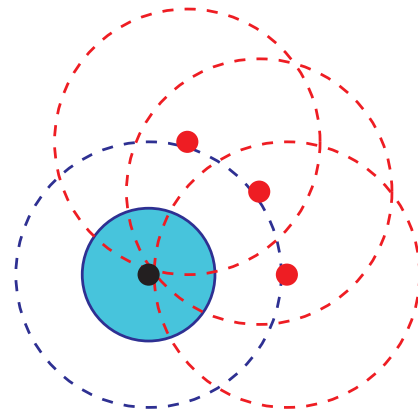
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Active			

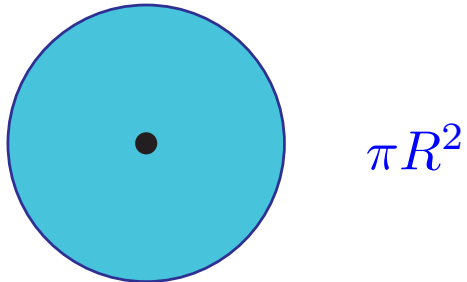
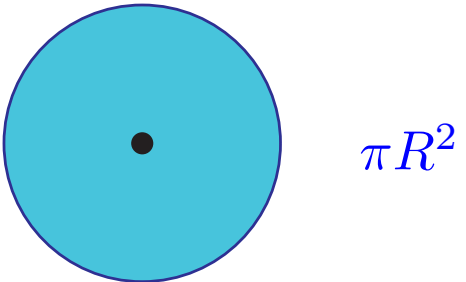
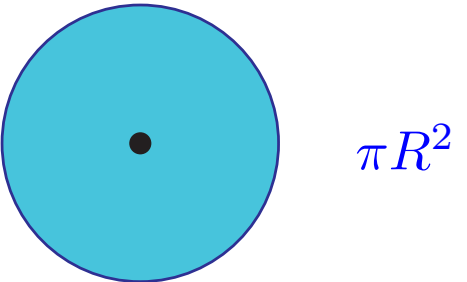
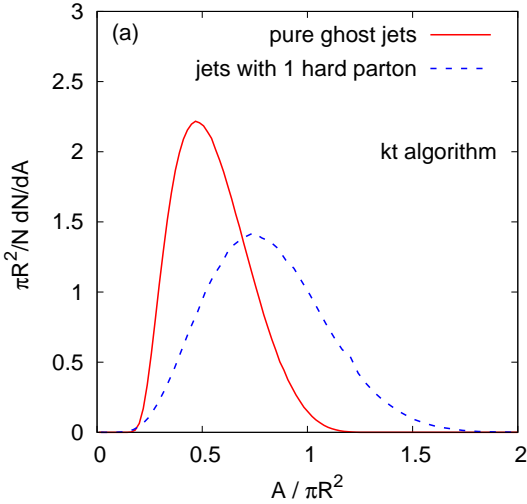
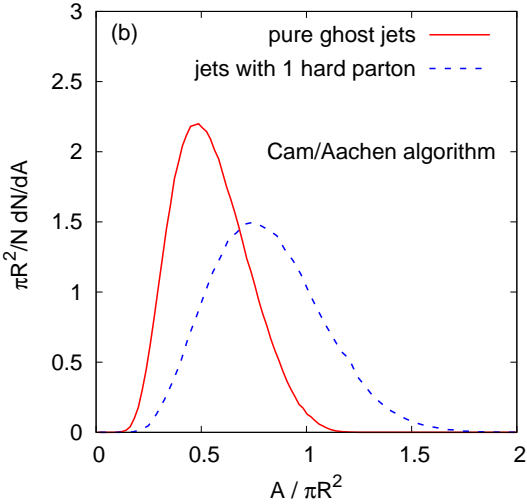
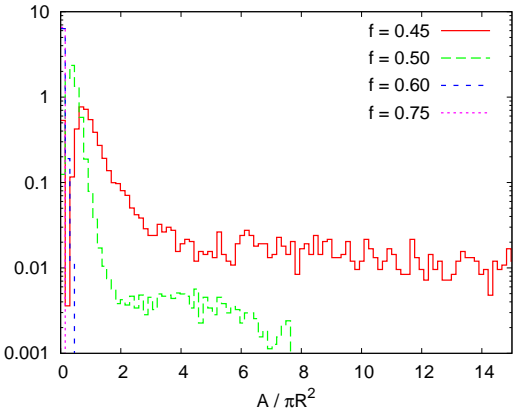
# Examples: 1-particle cases

	$k_t$	Aac/Cam	cone
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Active	<p>(a) <math>k_t</math> algorithm</p>  <p> <math>\frac{A_{\text{hard}}}{\pi R^2} \approx 0.812 \pm 0.277</math>  <math>\frac{A_{\text{ghost}}}{\pi R^2} \approx 0.554 \pm 0.174</math> </p>	<p>(b) Cam/Aachen algorithm</p>  <p> <math>\frac{A_{\text{hard}}}{\pi R^2} \approx 0.814 \pm 0.261</math>  <math>\frac{A_{\text{ghost}}}{\pi R^2} \approx 0.551 \pm 0.176</math> </p>	

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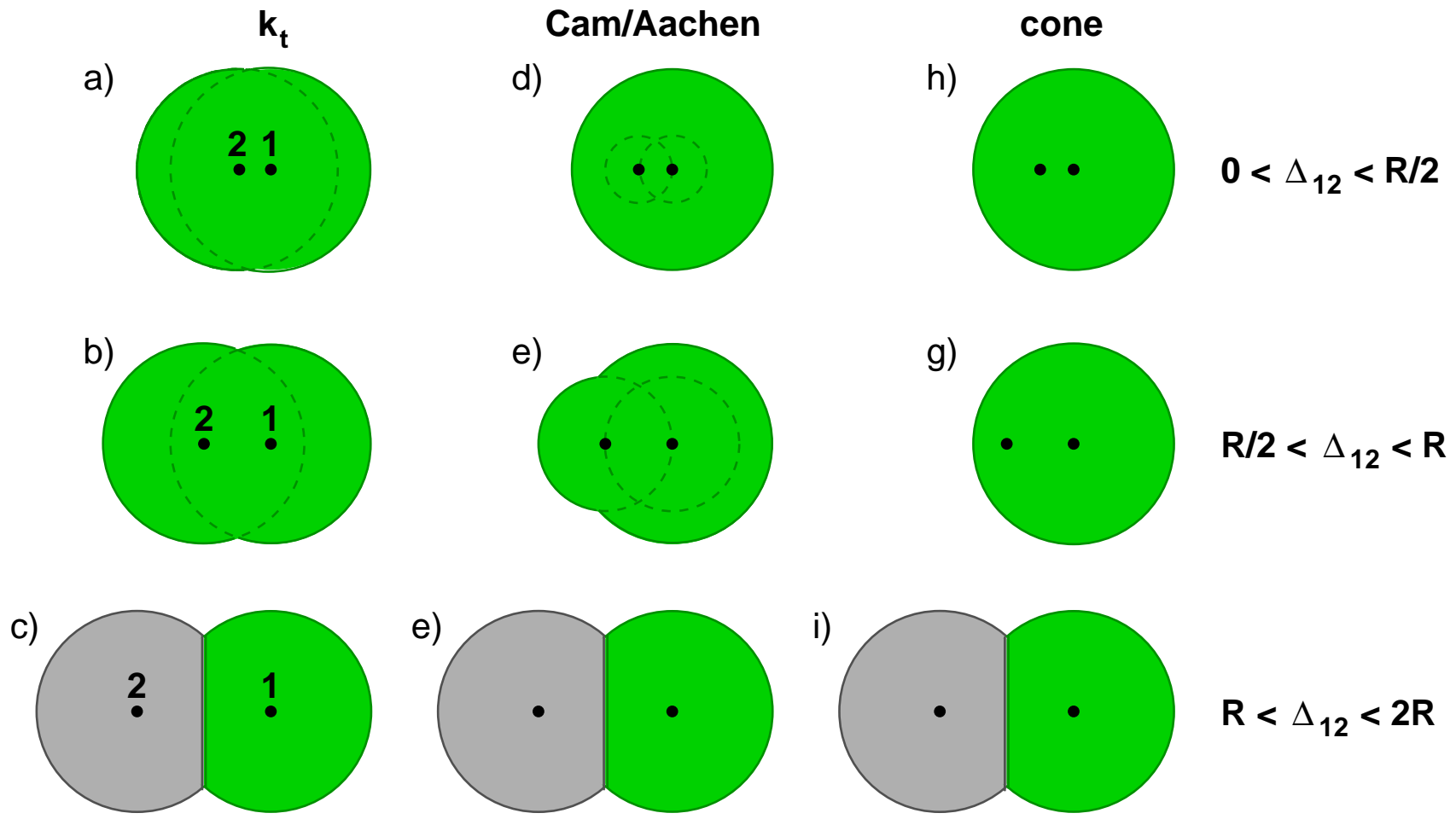
	$k_t$	Aac/Cam	cone
Passive	 $\pi R^2$	 $\pi R^2$	 $\pi R^2$
Active	<div style="display: flex; flex-direction: column; align-items: center;">  <div style="display: flex; justify-content: space-around; width: 100%;"> <div style="text-align: center;"> <math>\frac{A_{\text{hard}}}{\pi R^2} \approx 0.812 \pm 0.277</math>  <math>\frac{A_{\text{ghost}}}{\pi R^2} \approx 0.554 \pm 0.174</math> </div> <div style="text-align: center;"> <math>\frac{A_{\text{hard}}}{\pi R^2} \approx 0.814 \pm 0.261</math>  <math>\frac{A_{\text{ghost}}}{\pi R^2} \approx 0.551 \pm 0.176</math> </div> </div> </div>	<div style="display: flex; flex-direction: column; align-items: center;">  <div style="display: flex; justify-content: space-around; width: 100%;"> <div style="text-align: center;"> <math>\frac{A_{\text{hard}}}{\pi R^2} \approx 0.812 \pm 0.277</math>  <math>\frac{A_{\text{ghost}}}{\pi R^2} \approx 0.554 \pm 0.174</math> </div> <div style="text-align: center;"> <math>\frac{A_{\text{hard}}}{\pi R^2} \approx 0.814 \pm 0.261</math>  <math>\frac{A_{\text{ghost}}}{\pi R^2} \approx 0.551 \pm 0.176</math> </div> </div> </div>	<div style="display: flex; flex-direction: column; align-items: center;">  <math display="block">\frac{A_{\text{hard}}}{\pi R^2} = 0.25</math> </div>

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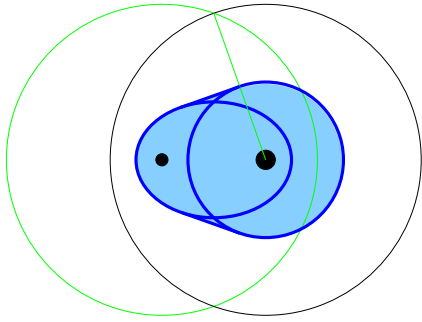
# 2-particle cases

Passive area: 1 hard particle + 1 soft

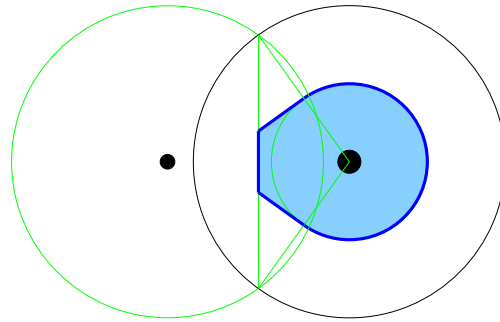


# 2-particle cases

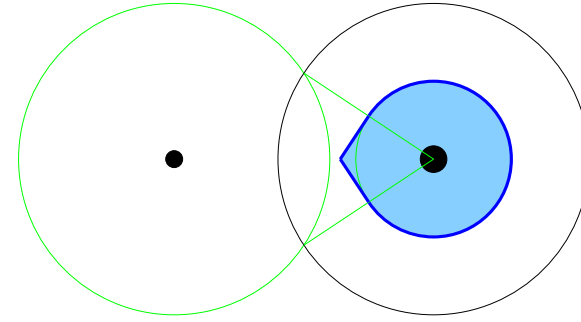
Active area: 1 hard particle + 1 soft: **analytic result for cone only**



$$d < R$$



$$R < d < \sqrt{2}R$$

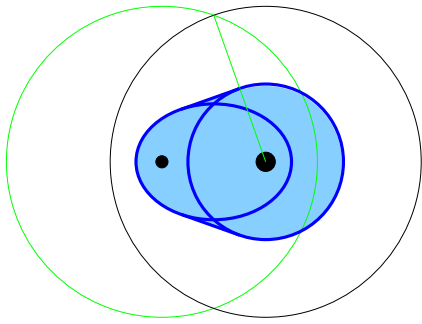


$$\sqrt{2}R < d < 2R$$

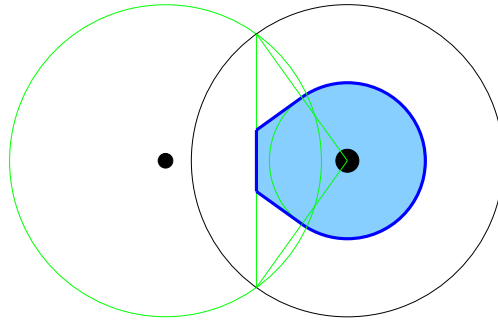


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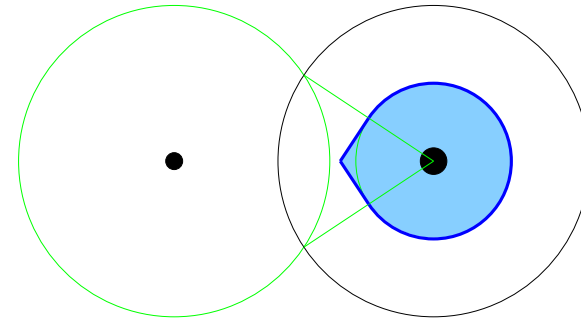
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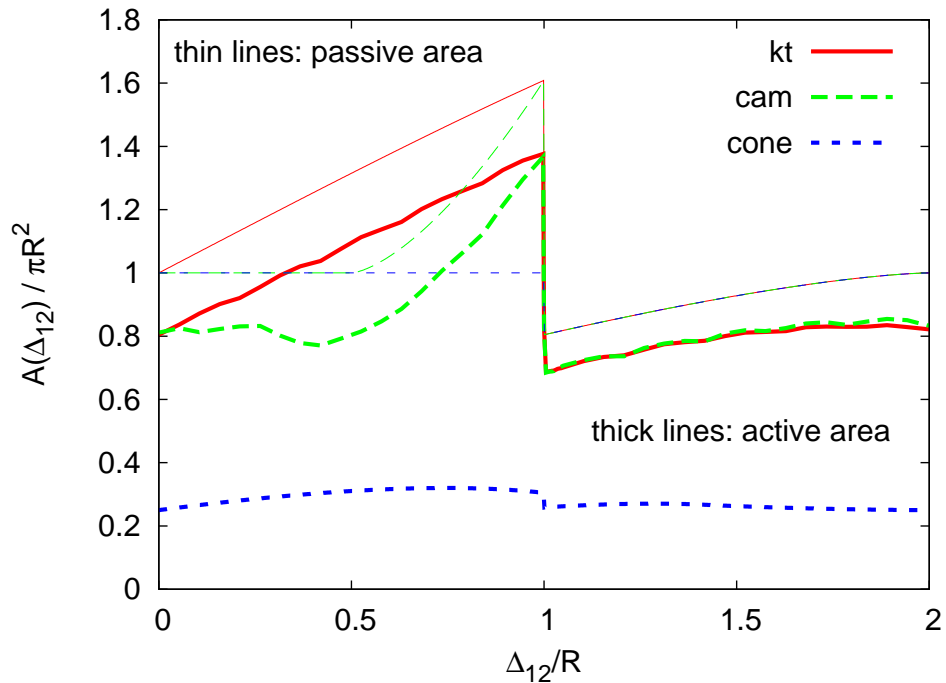
$$R < d < \sqrt{2} R$$



$$\sqrt{2} R < d < 2R$$

Alltogether, we have:

- Area  $\neq$  cst.  $\pi R^2$
- $\Delta_{12}$  dependence under control



# Area scaling violations

QCD probability of emitting a small-angle soft gluon:

$$\frac{dP}{d\Delta_{12} dp_{t,2}} = C_{F,A} \frac{2\alpha_s}{\pi} \frac{1}{\Delta_{12}} \frac{1}{p_{t,2}}$$

Hence the average area is

$$\langle \mathcal{A}(p_{t,1}, R) \rangle = \mathcal{A}_{\text{hard}}(R) + \int d\Delta dp_{t,2} \frac{dP}{d\Delta_{12} dp_{t,2}} [\mathcal{A}_{\text{hard}+1 \text{ soft}}(\Delta, R) - \pi R^2]$$

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- Scaling violation

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- Scaling violation
- gluon > quark
- with known LO anomalous dimension

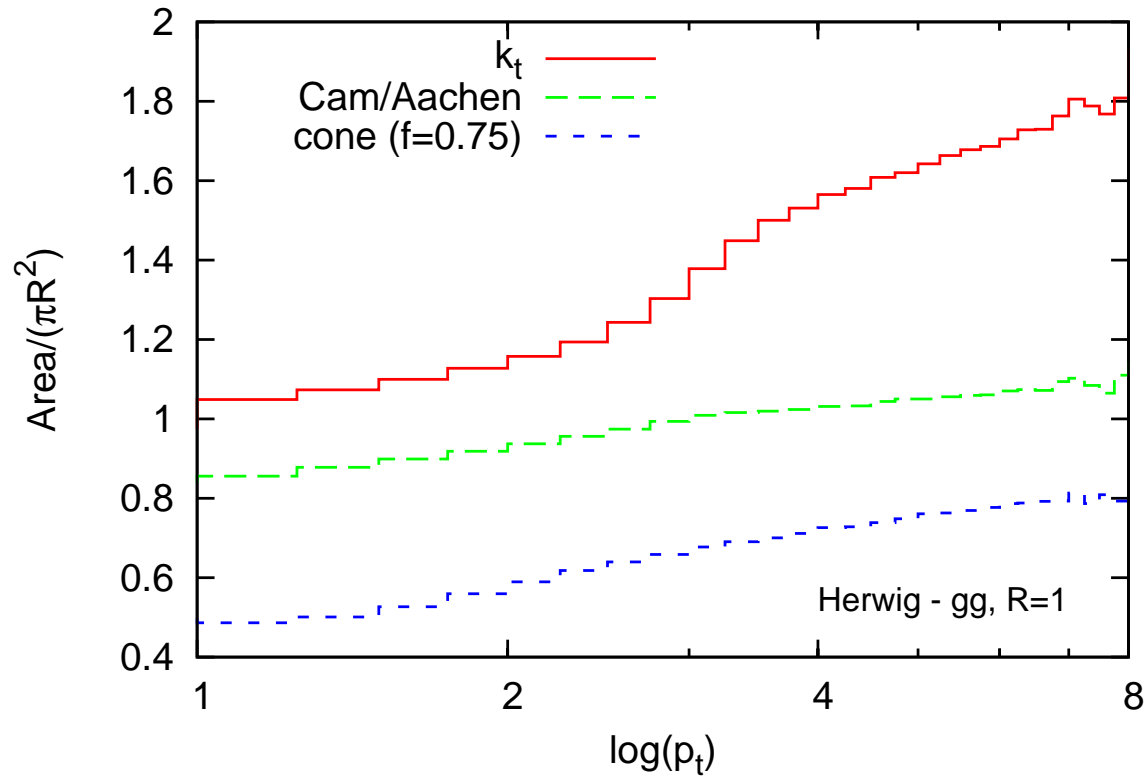
$d$	passive	active
$k_t$	0.5638	0.519
Cam	0.07918	0.0865
Cone	-0.06378	0.1246

# “Real-life” anomalous dimension

Herwig simulations of  $qq$  or  $gg$  processes at hadron level with underlying event:  
area vs.  $p_t$  of the jet

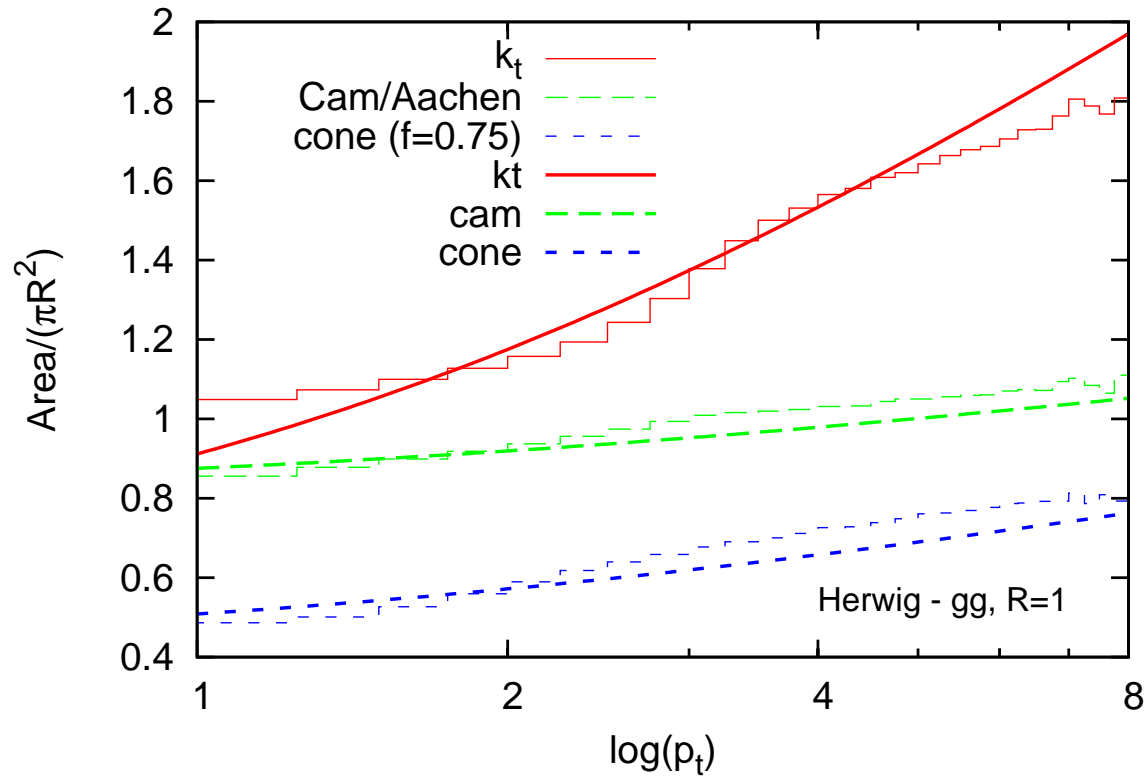
# “Real-life” anomalous dimension

Herwig simulations of  $qq$  or  $gg$  processes at hadron level with underlying event:  
area vs.  $p_t$  of the jet



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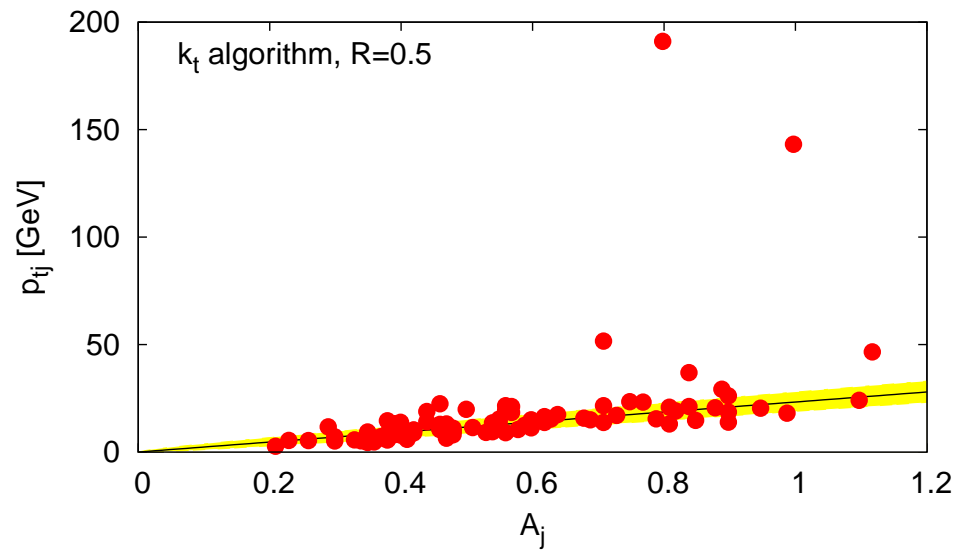


- good agreement with LO predictions
- $k_t$  bigger  $\Rightarrow$  NLO?



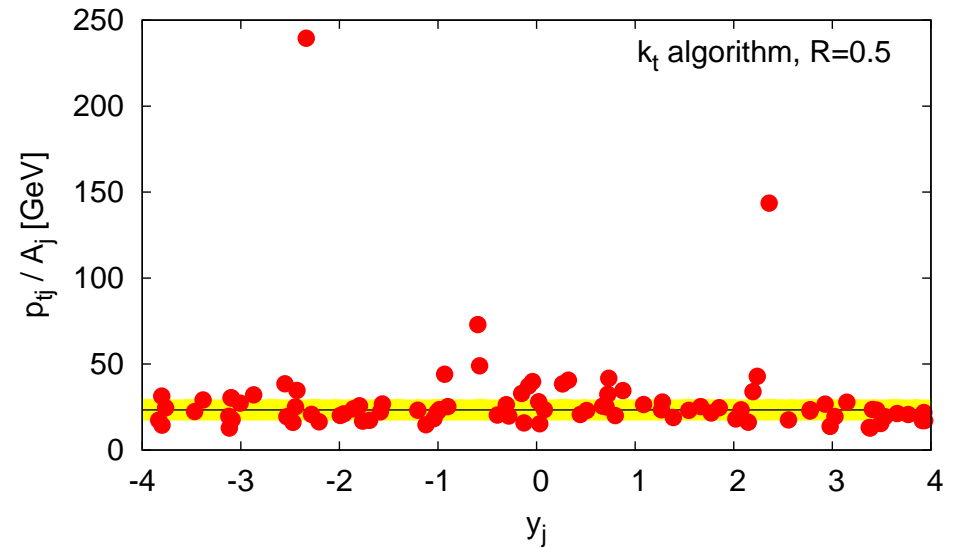
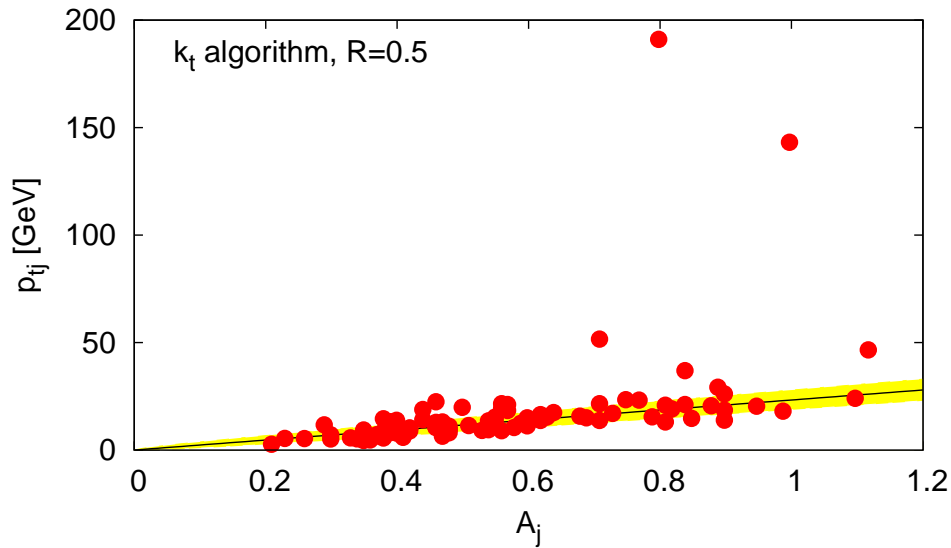
# What can area be used for?

Dense event with pile-up:



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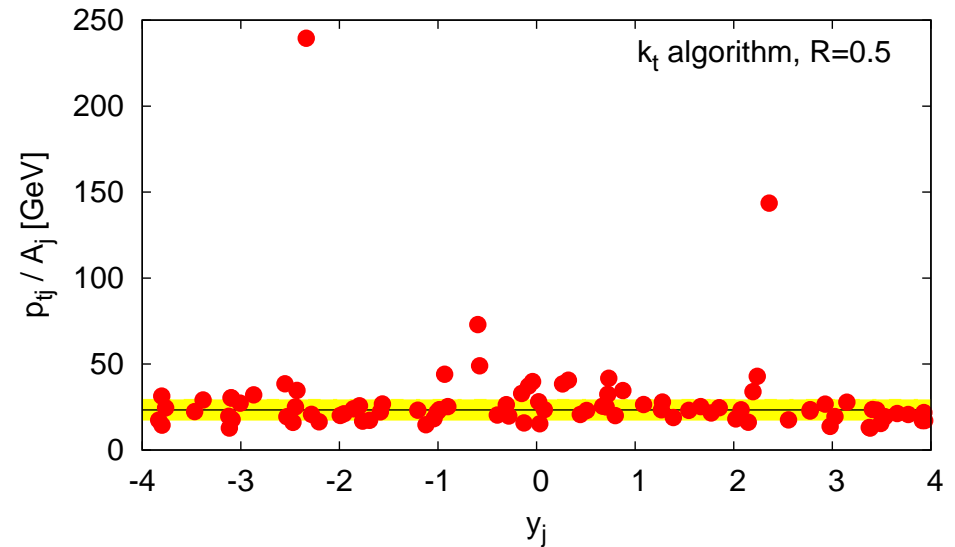
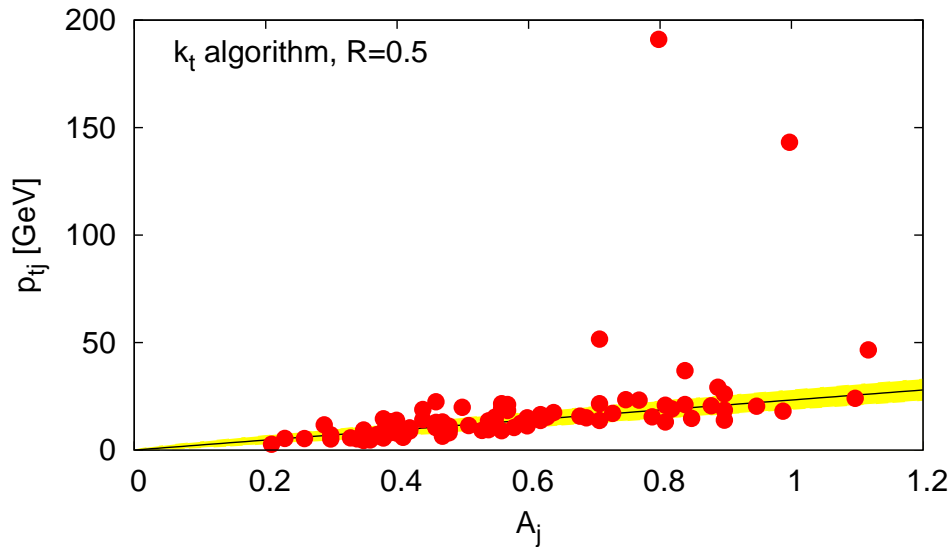
Dense event with pile-up:



- Area  $\propto p_t$  of the jet
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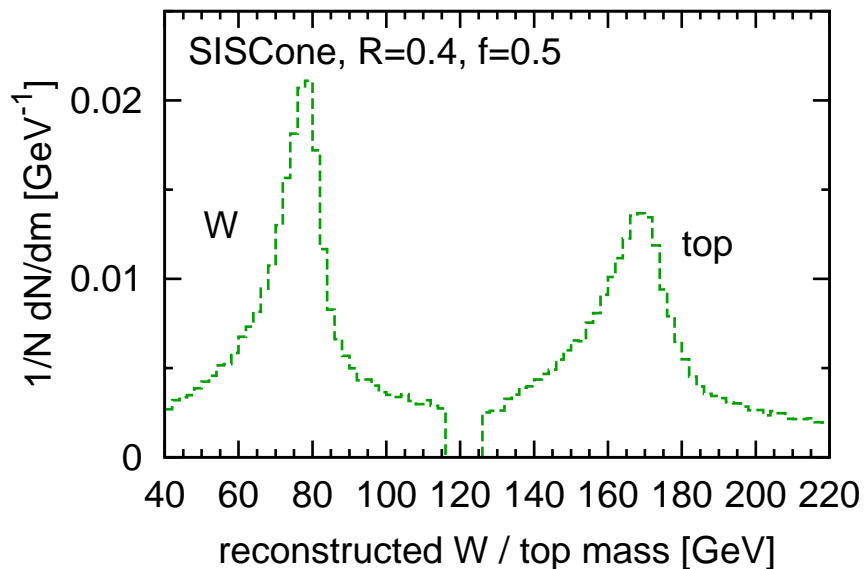
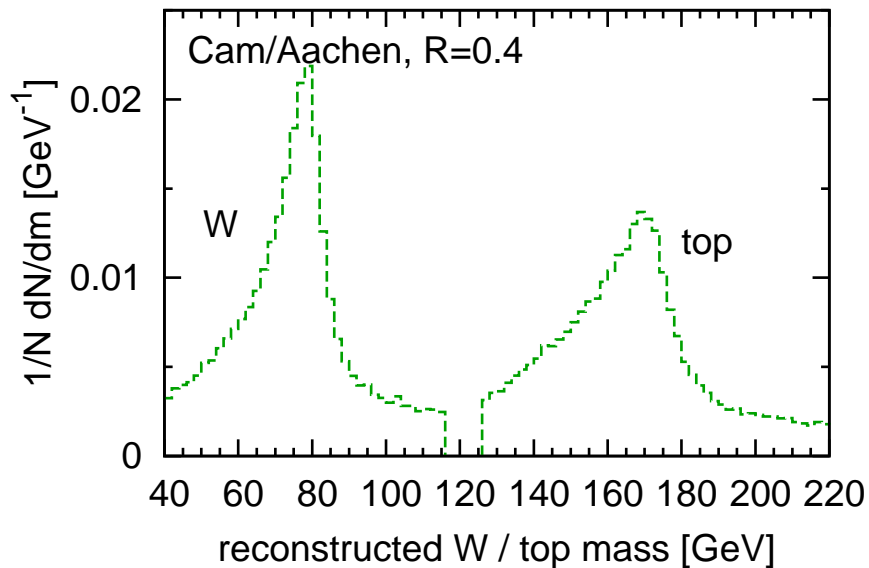
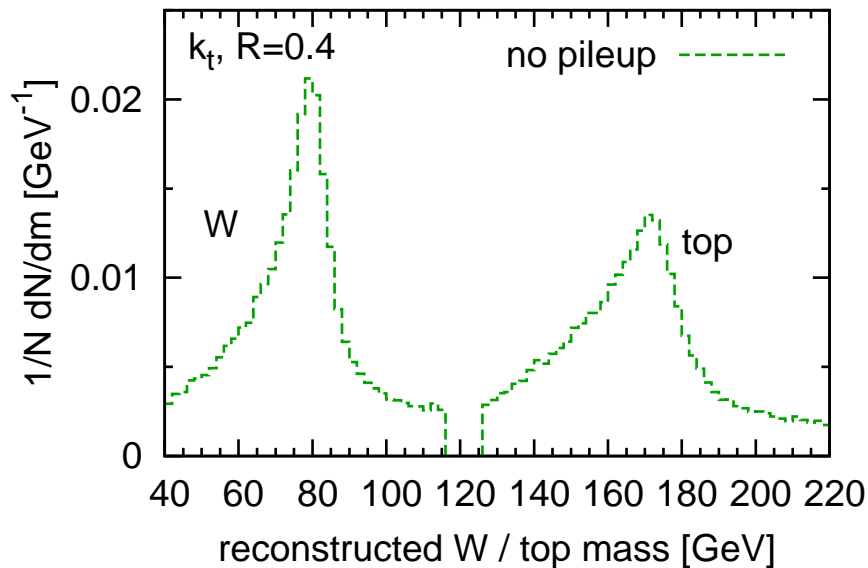
Area can be used to remove pileup pollution  
e.g. by removing  $\rho \cdot \text{area}$

# Subtraction in action

$t\bar{t} + W$

$(t\bar{t} \rightarrow \ell^+ \nu_{\ell} b + q\bar{q}b)$

$(W \rightarrow q\bar{q})$



LHC at high lumi

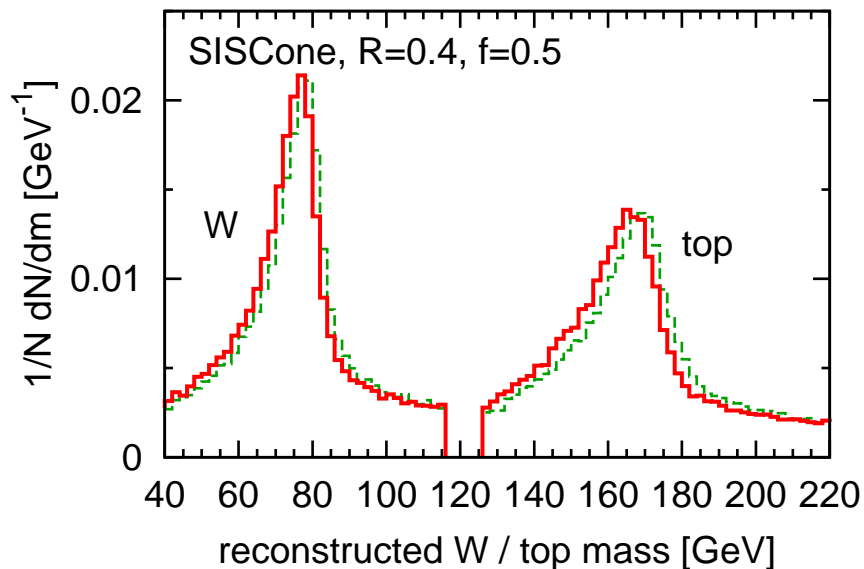
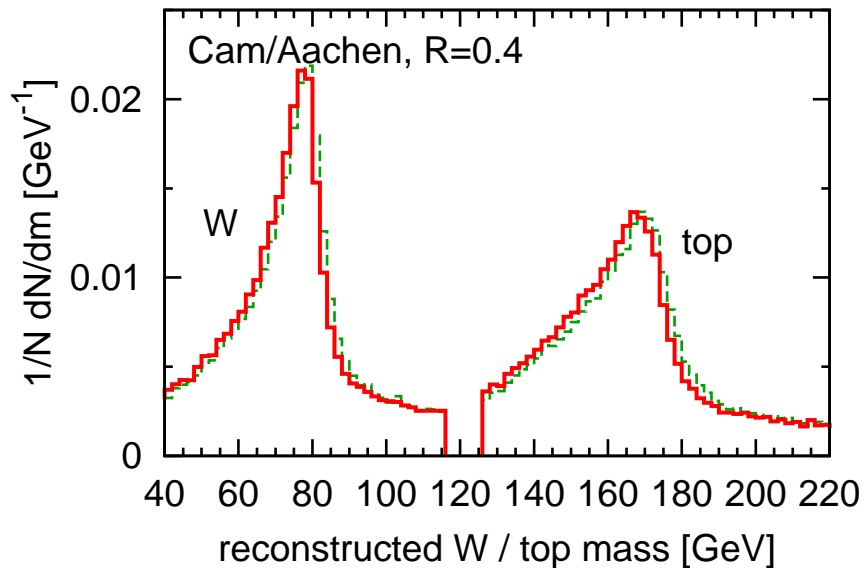
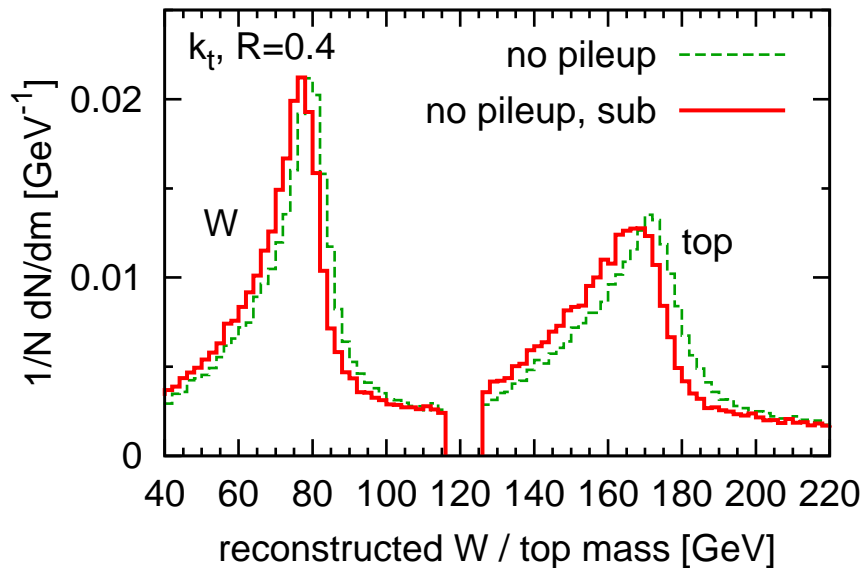
no pileup  $\Rightarrow$  good result

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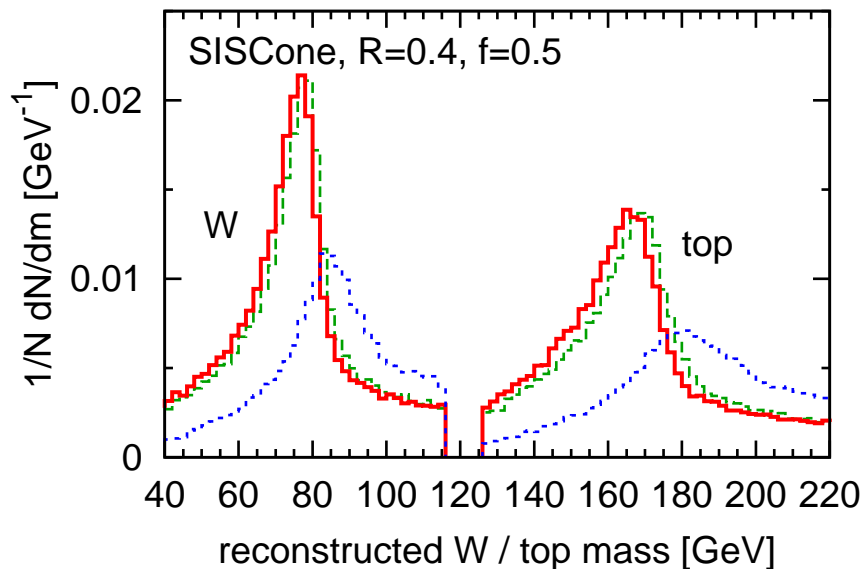
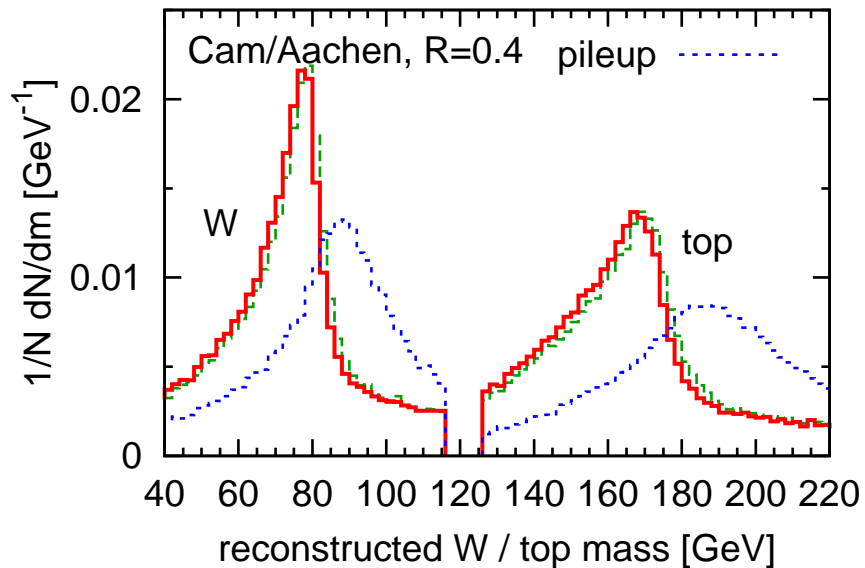
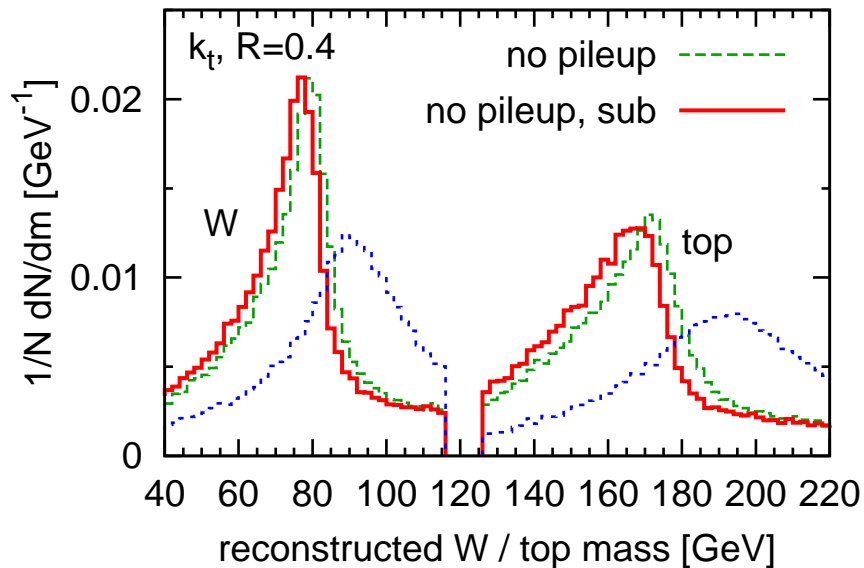
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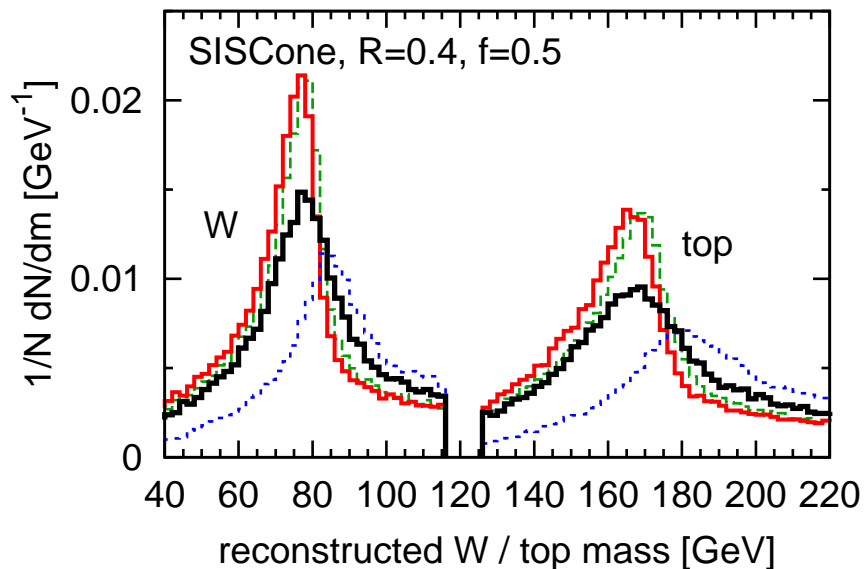
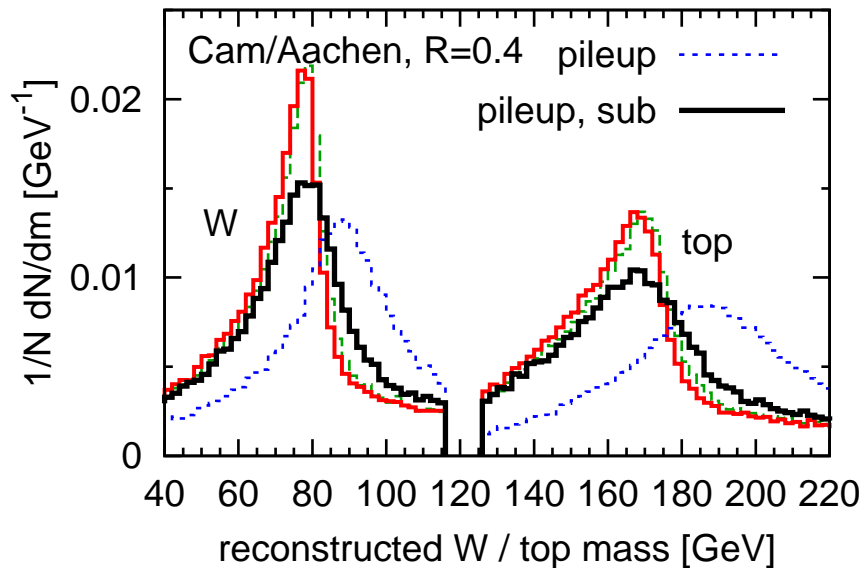
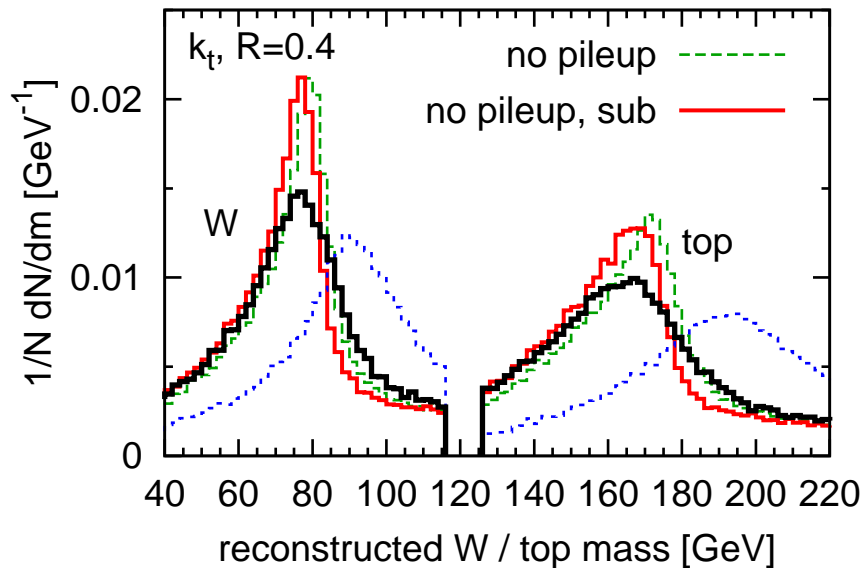
pileup  $\Rightarrow$  poor result

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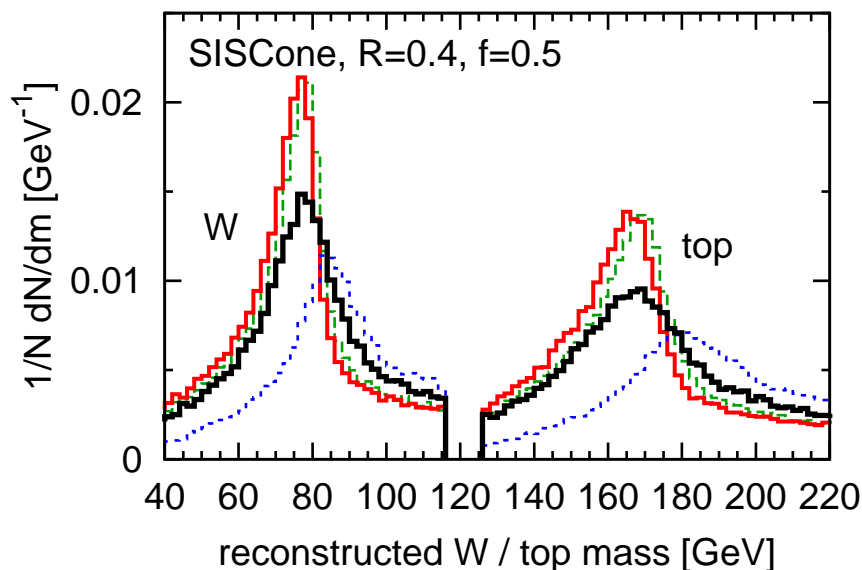
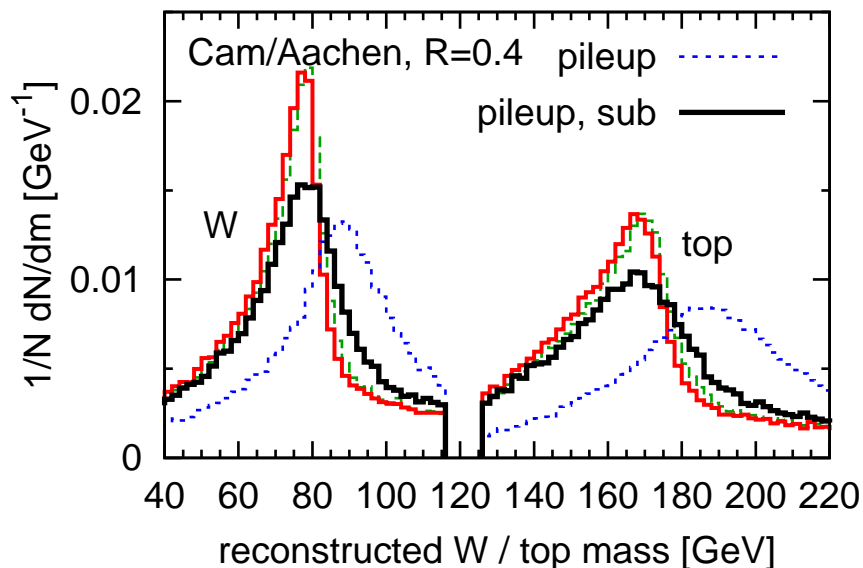
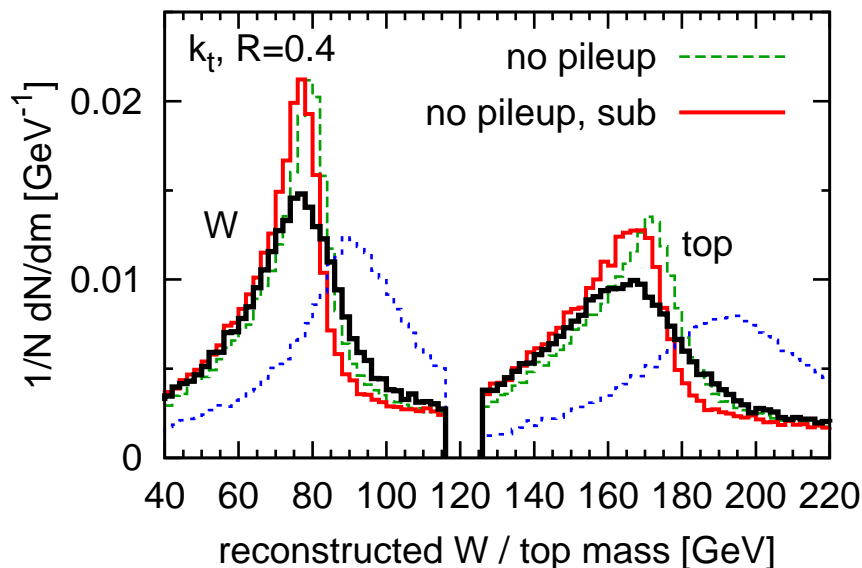
- no pileup  $\Rightarrow$  good result
- $\Rightarrow$  no subtraction effect
- pileup  $\Rightarrow$  poor result
- $\Rightarrow$  subtraction works

# Subtraction in action

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LHC at high lumi

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Background suppression in heavy ions!



# Conclusions

- SISCone: a new cone jet algorithm
  - first to satisfy requirements of the 90's!
  - mandatory for LHC
  - Get it at <http://projects.hepforge.org/siscone>  
or <http://www.lpthe.jussieu.fr/~salam/fastjet>

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# Conclusions and perspectives

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- **TODO**: in-depth study of  $k_t$ /Cam vs. cone.
- **New concept: the area of a jet**
  - active, passive and Voronoi
  - scaling violations & anomalous dimension
  - pileup effects subtraction, background subtraction in heavy ions
- **TODO**:
  - anomalous dimension resummation
  - only the beginning...