Fluctuation effects in high-energy QCD evolution

Gregory Soyez

Based on : G.S., hep-ph/0504129

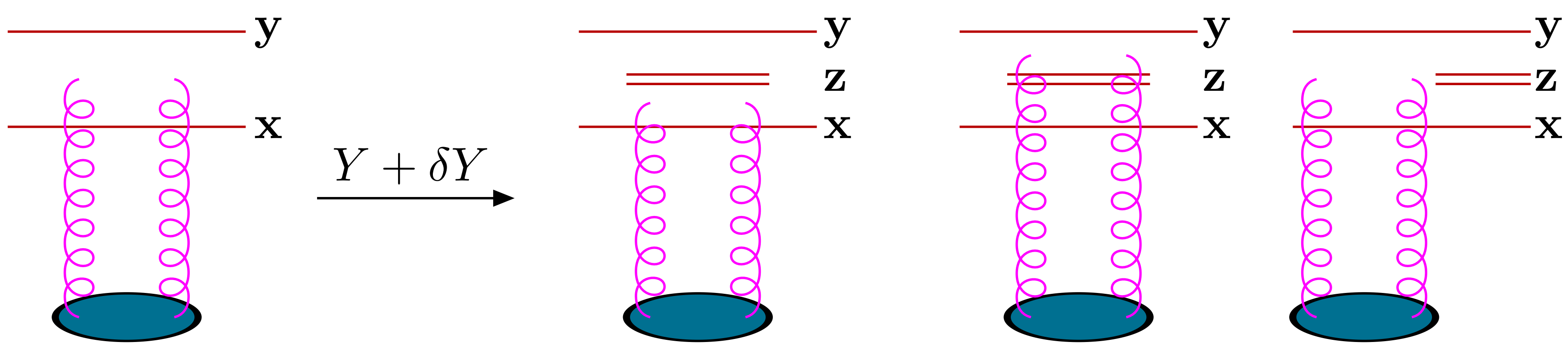
Outline

- Introduction:
  - BFKL and BK equations
  - Asymptotic solutions and geometric scaling
- Effects of fluctuations
  - Evolution equation: JIMWLK & fluctuations
  - Hierarchy (master eq.) vs. Langevin equation
  - Noise term: probability and front compacity
  - Saturation scale and geometric scaling violations
- Conclusions and perspectives

BFKL and BK evolution

Consider a $q\bar{q}$ dipole at large rapidity $Y = \log(s)$

BFKL: Rapidity increase $\Rightarrow$ Splitting into 2 dipoles

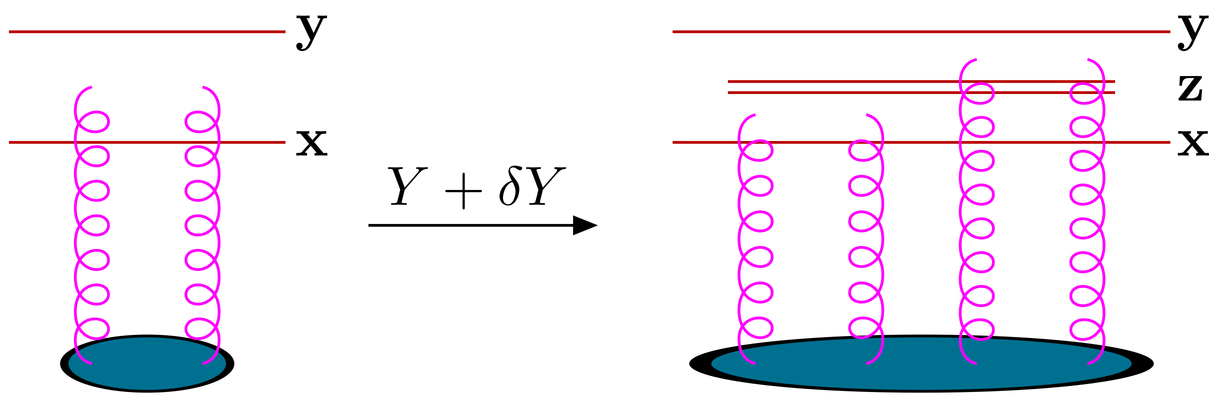


$$\partial_Y \langle T(\mathbf{x},\mathbf{y})\rangle = \bar{\alpha}\int_z \mathcal{M}_{\mathbf{xyz}}\left[\langle T(\mathbf{x},\mathbf{z})\rangle + \langle T(\mathbf{z},\mathbf{y})\rangle - \langle T(\mathbf{x},\mathbf{y})\rangle\right]$$

Solution: $T \propto e^{\omega Y}$ Violates unitarity $T(x,y) \leq 1$

Consider a $q\bar{q}$ dipole at large rapidity $Y = \log(s)$

BK: $T^2 \approx T \approx 1 \;\Rightarrow$ multiple scattering



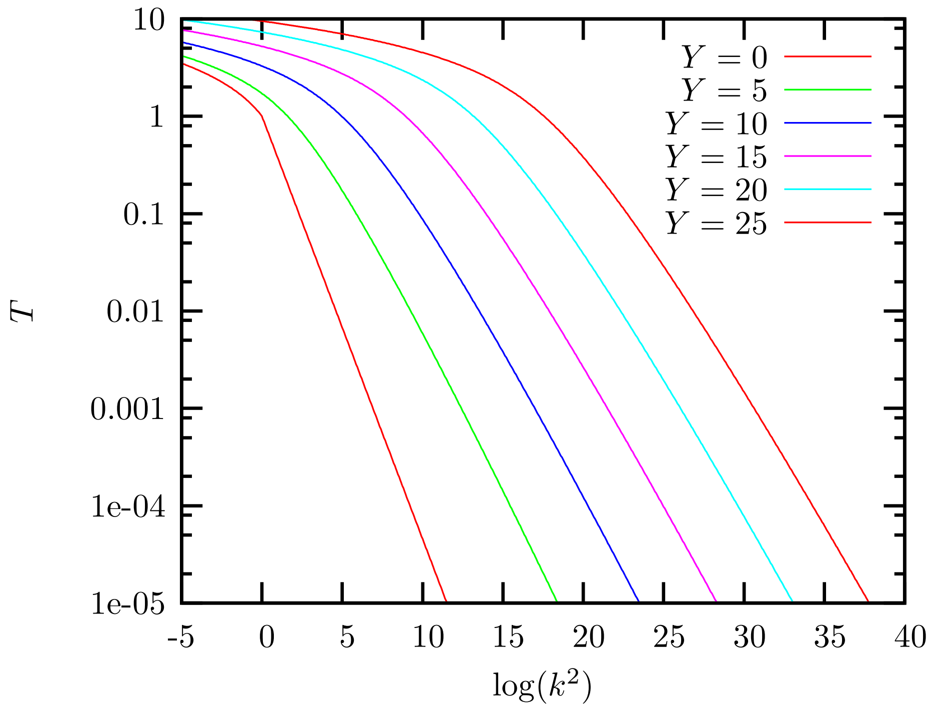
$$\partial_Y \langle T(\mathbf{x},\mathbf{y})\rangle = \bar{\alpha}\int_z \mathcal{M}_{\mathbf{xyz}}\left[\langle T(\mathbf{x},\mathbf{z})\rangle + \langle T(\mathbf{z},\mathbf{y})\rangle - \langle T(\mathbf{x},\mathbf{y})\rangle - \langle T(\mathbf{x},\mathbf{z})T(\mathbf{z},\mathbf{y})\rangle\right]$$

- $\langle T\rangle$, $\langle T^2\rangle$, ...: JIMWLK/Balitsky equations
- Mean-field approximation: $\langle T^2\rangle$, $\langle T\rangle^2$ (BK equation)

Asymptotic solutions

b-independent situation: momentum space

[S. Munier, R. Peschanski]



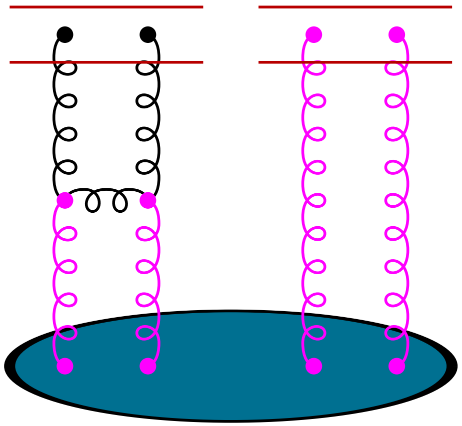
$$T(k) = \int \frac{d^2r}{r^2}\, \mathrm{e}^{i\mathbf{k}.\mathbf{r}}\, T(r)$$

$$T(k,Y) = T(\log(k^2) - v_c Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \approx \left|\frac{k^2}{Q_s^2(Y)}\right|^{-\gamma_c}$$ with $Q_s^2 \sim \exp(v_c Y)$

Geometric scaling (speed of the wave $\rightarrow$ energy dependence of Q_s^2)

[E. Iancu, D. Triantafyllopoulos]
Also A. Mueller, S. Munier, A. Shoshi,
W. van Saarloos, S. Wong

Consider correlations $\langle T^{(k)} \rangle$



- Usual BFKL ladder

$T^{(k)} \to T^{(k)}$

Fluctuations

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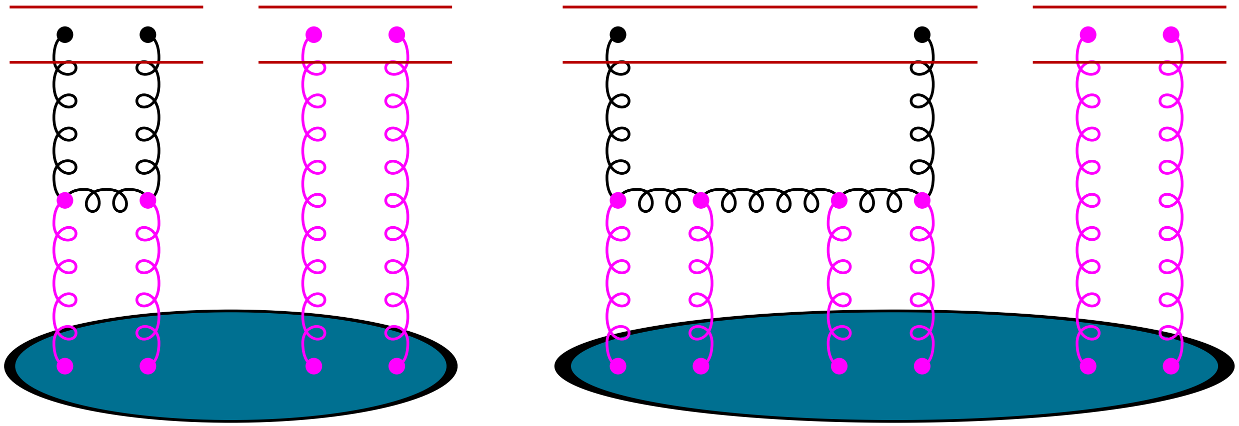

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- fan diagram $\longrightarrow$ saturation effects $T^{(k+1)} \to T^{(k)}$

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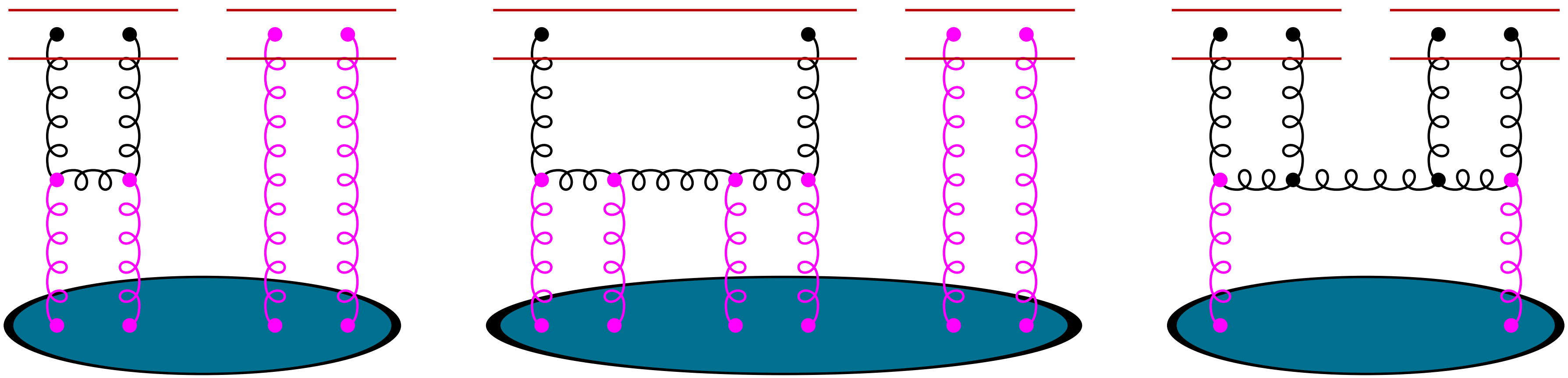


- Usual BFKL ladder $T^{(k)} \to T^{(k)}$
- fan diagram $\longrightarrow$ saturation effects $T^{(k+1)} \to T^{(k)}$
- splitting $\longrightarrow$ fluctuations, pomeron loops $T^{(k-1)} \to T^{(k)}$

$\Rightarrow$ complicated hierarchy

$$
\begin{aligned}
&\partial_Y T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2;Y)\\
&= \bar{\alpha}\int d^2z\,\frac{(\mathbf{x}_1-\mathbf{y}_1)^2}{(\mathbf{x}_1-\mathbf{z})^2(\mathbf{z}-\mathbf{y}_1)^2}\Big[T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{z};Y)+T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{z},\mathbf{y}_2;Y)\\
&\qquad -T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2;Y)-T^{(3)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{z};\mathbf{z},\mathbf{y}_2;Y)+(1\leftrightarrow 2)\Big]\\
&+ \bar{\alpha}\alpha_s^2\kappa\,\frac{(\mathbf{x}_1-\mathbf{y}_1)^2(\mathbf{x}_2-\mathbf{y}_2)^2}{(\mathbf{x}_1-\mathbf{y}_2)^2}\,T^{(1)}(\mathbf{x}_1,\mathbf{y}_2;Y)\,\delta^{(2)}(\mathbf{y}_1-\mathbf{x}_2).
\end{aligned}
$$

- Merging term: important when $T^{(2)}\sim T^{(1)}\sim 1$ i.e. at saturation
- Splitting term: important when $T^{(2)}\sim\bar{\alpha}^2T^{(1)}$ or $T\sim\bar{\alpha}^2$ i.e. in the dilute regime

Event evolution

Hierarchy $\equiv$ master equation

$\Rightarrow$ without b-dependence, equivalent to a Langevin equation

$$\partial_Y T(k,Y) = \bar{\alpha} K_{\text{BFKL}} \otimes T(k,Y) - \bar{\alpha} T^2(k,Y) + \bar{\alpha}\sqrt{\kappa \alpha_s^2 T(k,Y)}\,\nu(k,Y)$$

with

$$\langle \nu(k,Y) \rangle = 0 \qquad \langle \nu(k,Y)\nu(k',Y') \rangle = \frac{1}{\bar{\alpha}} \delta(Y-Y')\, k\delta(k-k')$$

Note: diffusive approximation $\rightarrow$ stochastic F-KPP equation

$$\partial_t u(x,t) = \partial_x^2 u + u - u^2 + \sqrt{2\kappa u}\,\nu(x,t)$$

- From noise term to fluctuations (local noise $\leftrightarrow$ no space dependence)

$$du = \sqrt{2\kappa u}\nu(t) \quad \overset{\text{Ito}}{\Rightarrow} \quad u_{j+1} = u_j + \delta t\sqrt{2\kappa u_j}\nu_j \qquad \text{with } \langle\nu_i\nu_j\rangle = \frac{1}{\delta t}\delta_{ij}$$

$$\Rightarrow \quad F(u_{j+1}) = F(u_j) + \delta t\sqrt{2\kappa u_j}\nu_j F'(u_j) + \delta t^2\kappa u_j \nu_j^2 F''(u_j)$$

$$\Rightarrow \quad \partial_t \langle F(u)\rangle = \kappa \langle uF''(u)\rangle$$

Note: $F(u) = u^n$ gives the hierarchy

- Associated probability

$$\langle F(u)\rangle = \int du\, F(u)\, P(u,t) \qquad \overset{\partial_t}{\Rightarrow} \qquad \partial_t P(u,t) = \kappa\partial_u^2 \left[uP(u,t)\right]$$

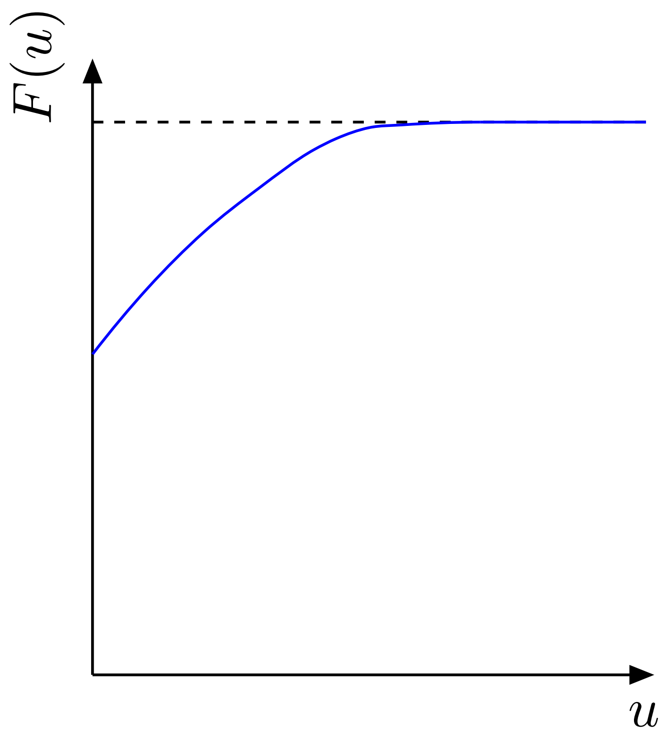
Including the initial contition $u(t=0) = u_0$, we get

$$P_t(u_0 \to u) \equiv \text{ probability to go from } u_0 \text{ to } u \text{ in a time } t.$$

Probability (2/2)

Define the cumulative probability

$$F_{u_0,t}(u) = \int_{0^-}^{u} dv P_t(u_0 \to v).$$



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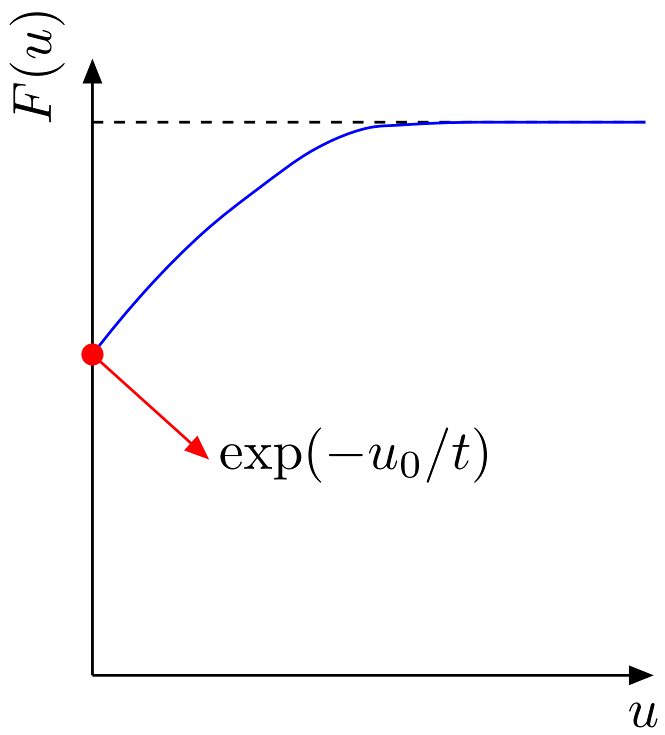

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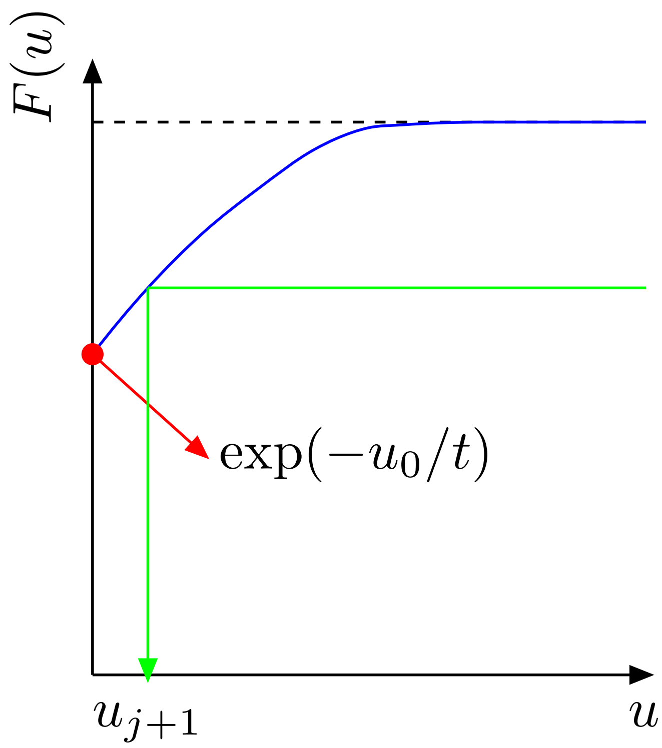

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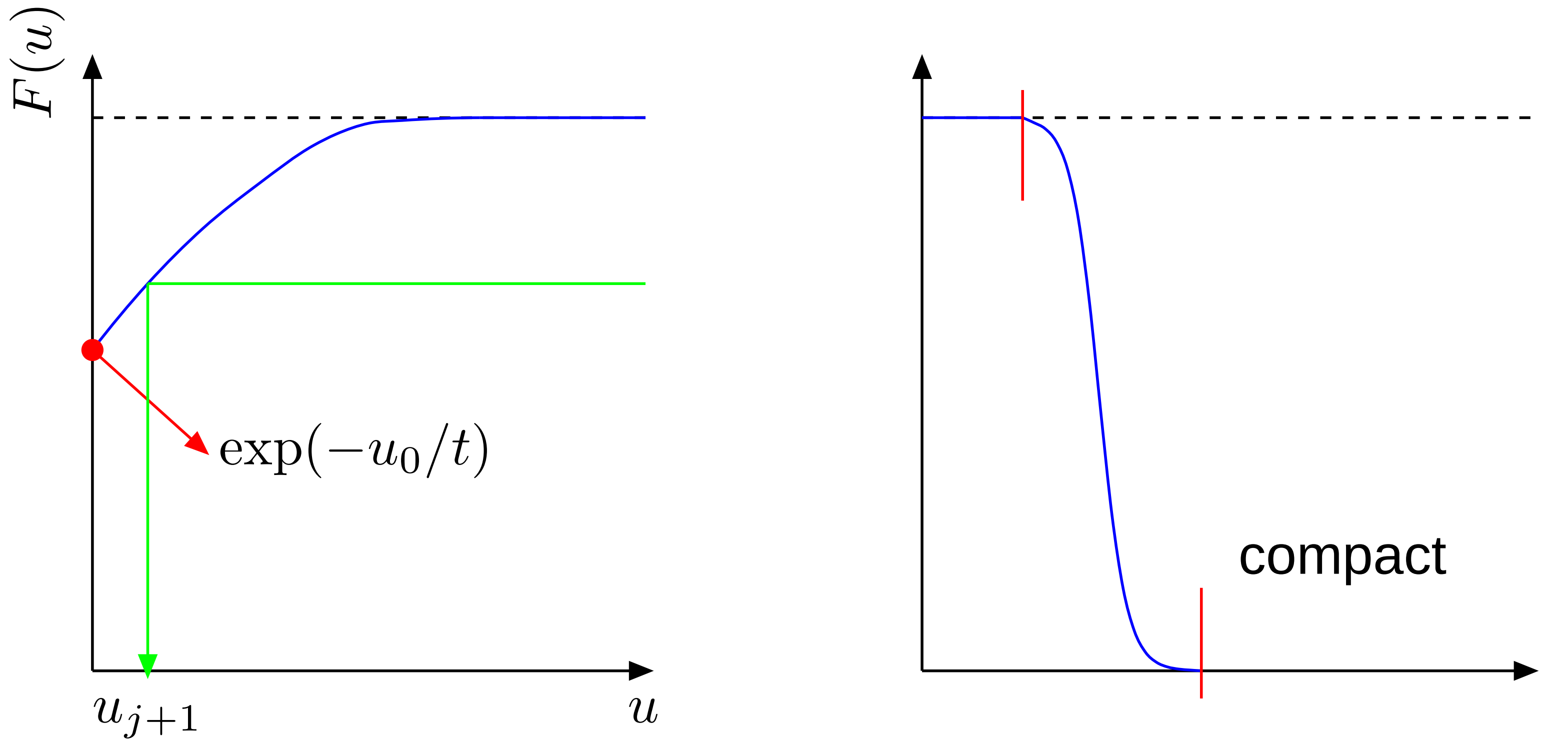


Numerical analysis

$$\partial_Y T(k,Y) = \bar\alpha K_{\text{BFKL}} \otimes T(k,Y) - \bar\alpha T^2(k,Y) + \bar\alpha\sqrt{\kappa\alpha_s^2 T(k,Y)}\,\nu(k,Y)$$

Rapidity step δY:

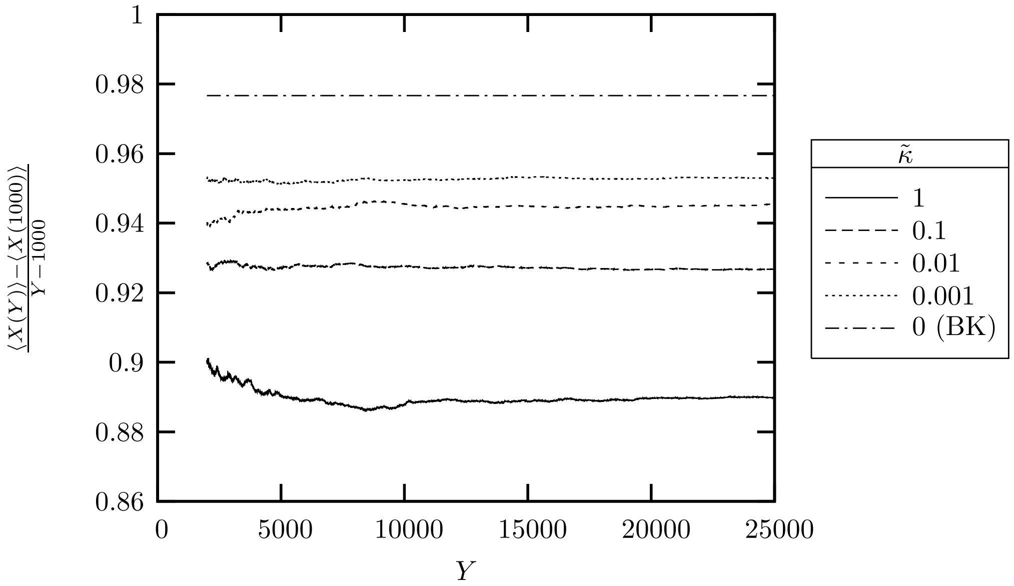
- Step 1: Use probability: $0 < y < 1$ uniform random variable

$$T_{\text{noise}}(k,Y) = F^{-1}_{T(k,Y),\delta Y}(y)$$

- Step 2: Apply the remaining equation

$$T(k,Y+\delta Y) = T_{\text{noise}}(k,Y) + \delta Y\left[\bar\alpha K_{\text{BFKL}} \otimes T_{\text{noise}}(k,Y) - \bar\alpha T^2_{\text{noise}}(k,Y)\right]$$

[G.S.]

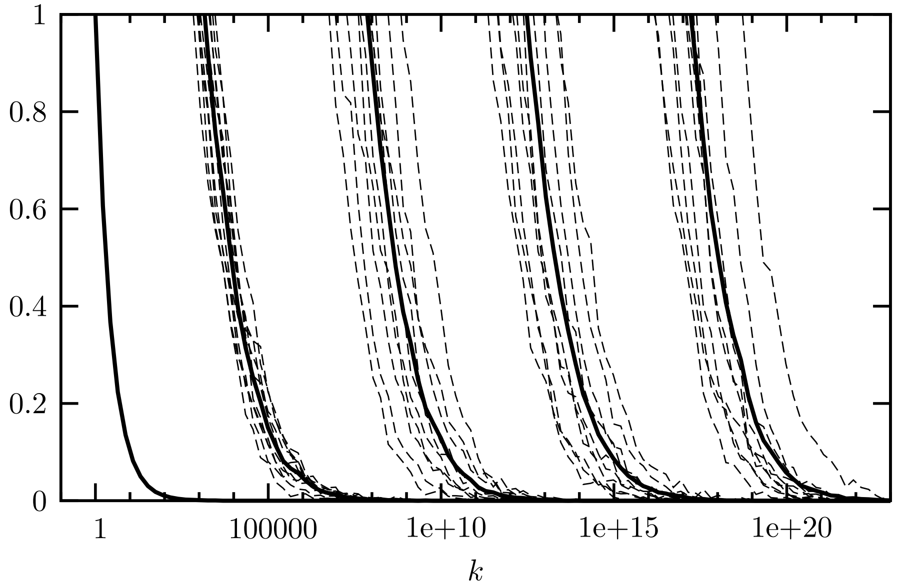


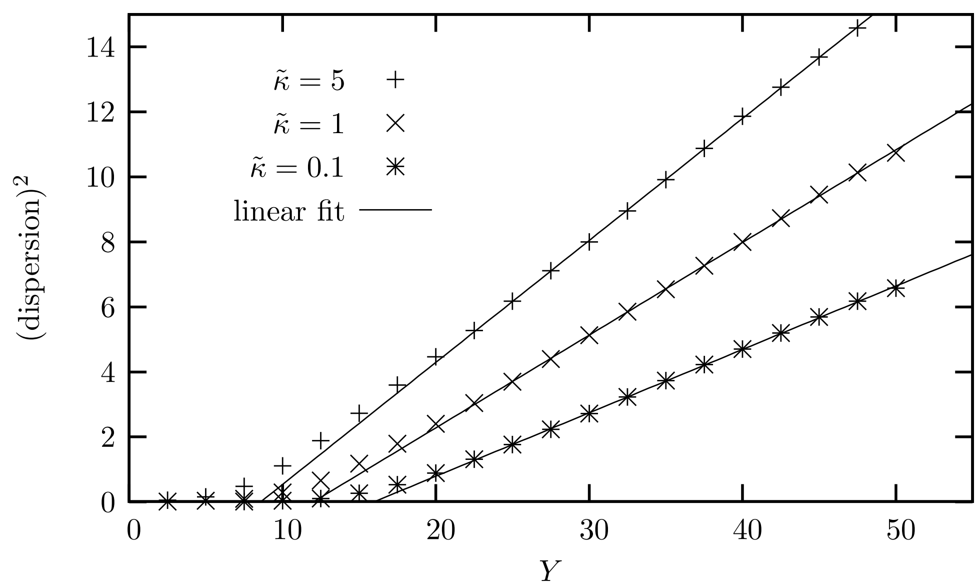
Decrease of the asymptotic velocity

For asymptotically small α_s (not true here)

$$v^* \underset{\alpha_s^2\kappa\to 0}{\longrightarrow} v_c - \frac{\bar{\alpha}\pi^2\gamma_c\chi''(\gamma_c)}{2\log^2(\alpha_s^2\kappa)}$$

[G.S.]





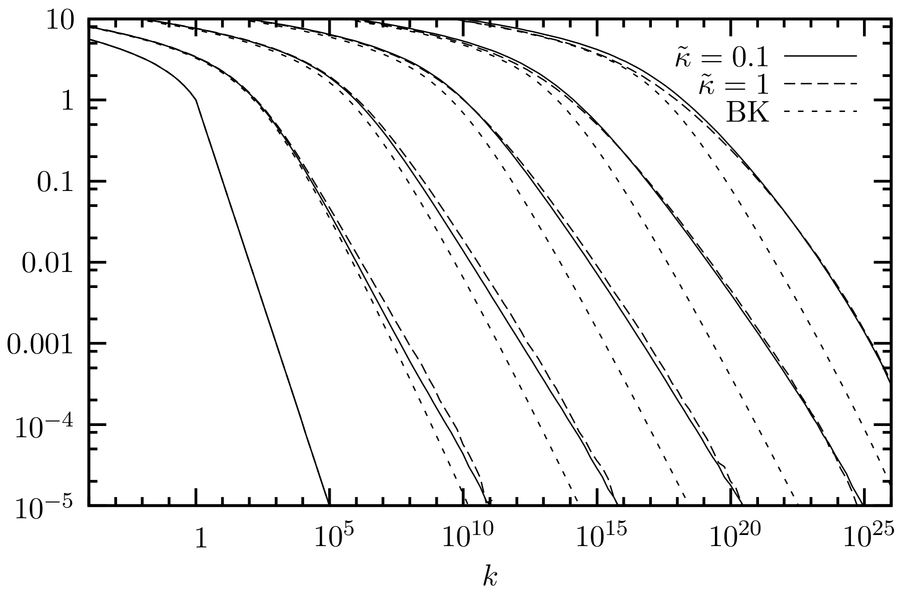
- Dispersion of the events

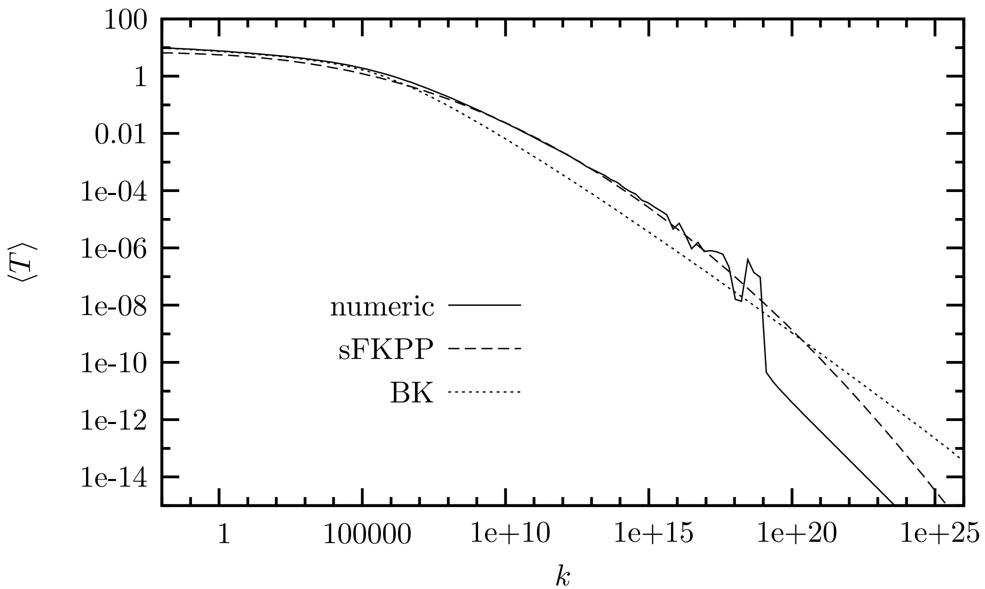
$$\Delta \log[Q_s^2(Y)] \approx \sqrt{D_{\text{diff}} \bar{\alpha} Y} \quad \text{with} \quad D_{\text{diff}} \underset{\alpha_s^2\kappa \to 0}{\sim} \frac{1}{|\log^3(\alpha_s^2 \kappa)|}.$$

- No important dispersion in early stages of the evolution !

Averaged amplitude

[G.S.]

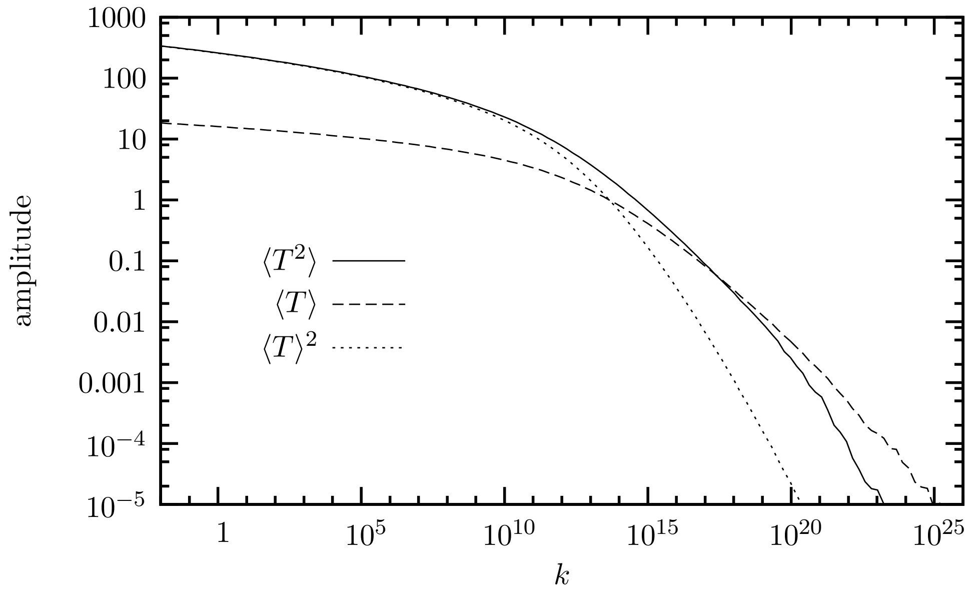




- Clear effect of fluctuations
- Violations of geometric scaling (not in early stages)
- Agrees with predictions

Correlators

[G.S.]



- Dense regime: $\langle T^2 \rangle \approx \langle T \rangle^2$
- Dilute regime: $\langle T^2 \rangle \approx \langle T \rangle$

Conclusion & perspectives

- Fluctuation effects: first numerical studies
  - slower speed, dispersion
  - violations of geometric scaling (maybe not so important!)

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Conclusion & perspectives

- Fluctuation effects: first numerical studies
  - slower speed, dispersion
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- phenomenological tests:
  - do we observe geometric scaling violations
- theoretical extensions:
  - include running coupling effects
  - include b-dependent fluctuations (under study)